Ensemble-Average Spectrum of Aharonov-Bohm Conductance Oscillations: Evidence for Spin-Orbit-Induced Berry's Phase

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We have experimentally investigated Aharonov-Bohm conductance oscillations in two-dimensional rings in the presence of a strong Rashba spin-orbit interaction. The peak of the ensemble average spectrum corresponding to the h/e oscillations is split in several samples. We argue that the splitting originates from the geometric phase acquired by an electron spin under the influence of the spin-orbit interaction. Our results demonstrate how the ensemble average spectrum provides information otherwise not easily accessible. [S0031-9007(97)05168-5]

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The study of the periodicity of the conductance oscillations observed in mesoscopic Aharonov-Bohm (AB) ring-shaped conductors [1] has played a key role in clarifying the distinction between sample specific and ensemble-average behavior of physical quantities. It is by now well established [1] that the Fourier spectrum of sample specific oscillations measured in a single AB ring exhibits a main peak corresponding to h/e fundamental periodicity (in magnetic flux through the ring area), whereas, upon ensemble average, this peak disappears (because of the random phase of the h/e oscillations) so that the fundamental peak in the Fourier spectrum of ensemble averaged oscillations corresponds to $h/2e$.

The suppression of the h/e peak upon ensemble averaging is a consequence of the way in which the average is performed. Specifically, the h/e peak disappears when one averages the conductance and then calculates the Fourier spectrum. However, one can also take an ensemble of rings, determine the amplitude of the Fourier spectrum [2] of each ring separately and sum all the amplitudes. The quantity thus obtained may be named "ensemble average Fourier spectrum." It is obvious that the h/e peak will be present in this quantity.

Investigations of the h/e versus $h/2e$ periodicity issue have resulted in a thorough analysis of the properties of the Fourier spectrum of the average conductance [1]. On the contrary, the average Fourier spectrum has never been studied extensively and the possibility that this quantity can contain new useful information has never been considered, neither theoretically nor experimentally.

In this paper we demonstrate that the analysis of the ensemble average Fourier spectrum can indeed provide new physical information, not easily accessible otherwise. Specifically, we have studied the ensemble average spectrum of the Aharonov-Bohm conductance oscillations measured in two-dimensional rings in the presence of strong Rashba spin-orbit interaction [3]. We have found that a splitting in the peak corresponding to the h/e periodicity is present in several samples. The splitting may be a manifestation of the geometric phase induced on the spin of an electron traversing the ring (whose relevance was first recognized by Loss and co-workers [4]) by the presence of spin-orbit interaction [4–9].

The rings (Fig. 1; diameter in between 0.9 and 2.1 μm; arms width ranging from 130 to 170 nm) have been realized using the two-dimensional electron gas (2DEG) present in an AlSb/InAs/AlSb heterostructure, the 2DEG being hosted in the InAs layer. Technological details have been described in Ref. [10], and here we only recall that reflection from the edges of the rings is specular. The electron density and the elastic mean free path are $N = 1.0 \times 10^{16}$ m^{-2} and $l_e \simeq 1$ μm. (Transport in the arms of the rings is not fully ballistic.)

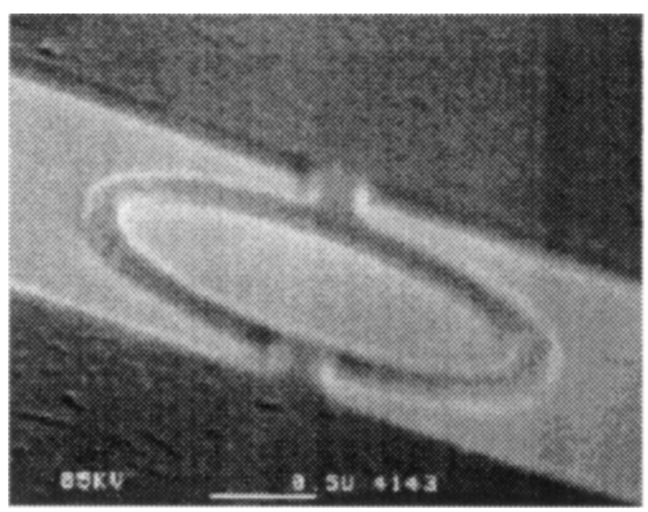

FIG. 1. One of the rings used in our investigations. The white bar shown is 0.5 μm long.

The presence of uniform spin-orbit Rashba interaction in this heterostructure produces a clear beating pattern in the Shubnikov–de Haas oscillations observed in the longitudinal magnetoresistance measurements performed in a standard Hall-bar geometry [11]. The analysis [12] of this beating pattern permits one to infer the strength of the interaction, which can be quantified by a parameter α (in the notation of Ref. [3]), approximately equal to 5.5×10^{-10} eV cm, with variations of $\approx$20% in different samples.

The AB effect in the rings has been investigated by measuring their resistance R as a function of a perpendicularly applied magnetic field B. The resistance is measured using a standard four probe lock-in technique in an essentially two terminal configuration (the rings are separated from the probes by a wide 2DEG area). The magnetic field, generated by a superconducting coil, is swept during a typical $R(B)$ measurement from $\simeq -0.35$ to $+0.35$ T. The data discussed here have been taken at 100 mK.

Figure 2 shows a $R(B)$ curve of one of the large ($\simeq$2.1 μm diameter, $\simeq$150 nm arms width) rings. Resistance oscillations are apparent, and their period in magnetic field ($\simeq$1.2 mT) corresponds to the one expected for h/e oscillations. The envelope of the curve shows sample specific conductance fluctuations (due to the random interference of the electronic wave) superimposed on a negative magnetoresistance due to the classical dynamics of electrons in a laterally confined geometry [13]. Around $B = 0$, a conductance dip is visible (observed in all the samples) due to weak antilocalization. A clear peak is present in the Fourier spectrum of $R(B)$ (inset of Fig. 2), whose width is in fair agreement with the value expected, because of the finite area enclosed by the arms of the ring. The rough shape of the peak is determined by the random sample specific nature of the h/e oscillations.

Our experiment aims at [9] eliminating the sample specific features superimposed on the peak by performing an ensemble average over measurements obtained from different microscopic realizations of the same ring. In order to explain how this has been accomplished in practice we start with pointing out that, even though we observe clear AB oscillations, the samples' electrical stability is not perfect, owing to switching events that slightly change their resistance. This effect is already apparent in the data of Fig. 2, which do not satisfy the reciprocity relation $R(B) = R(-B)$, mandatory for a two terminal measurement of a stable sample. The switching events are due to defects present in the heterostructure, but the specific mechanism at their origin is unclear.

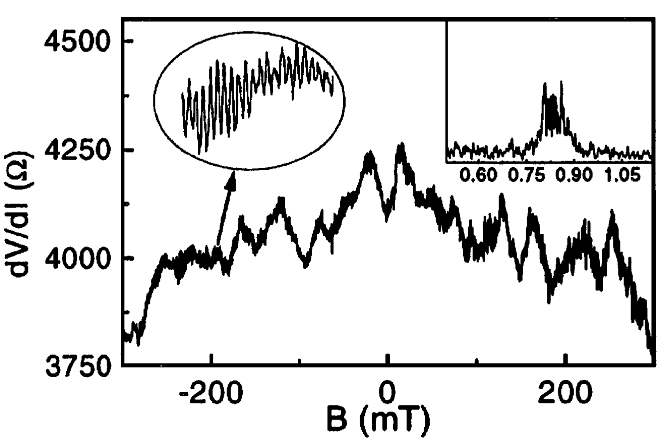

FIG. 2. Single magnetoresistance trace measured in the AB ring discussed in the text. The insets show an enlargement of the conductance oscillations and the peak in their Fourier spectrum (x axis units are in mT^{-1}).

We have monitored the samples' behavior by continuously measuring magnetoresistance traces (like the one of Fig. 2) for several days, and we have characterized how the switching affects electronic transport by analyzing the statistical properties of the set of measured $R(B)$ curves. It results that two $R(B)$ curves measured at a distance of days, with the sample constantly kept at 100 mK, differ by less than $\approx 0.2e^2/h$ *for any values of* B*;* the average (with respect to B, in the range -0.35 to $+0.35$ T) difference between the two curves is typically $\approx 0.05e^2/h$; the AB oscillations root mean square amplitude is essentially the same for all the $R(B)$ curves, $\simeq 0.02e^2/h$ [14].

It follows from these data that the magnitude of the difference between any two $R(B)$ curves in the set (measured at a distance of days) is essentially identical to the magnitude of the aperiodic conductance fluctuations observed as a function of B in a single magnetoresistance trace (Fig. 2). Since the aperiodic fluctuations are due to the random variations in the sample specific behavior of the rings (induced by changes in magnetic flux of the order of h/e through the arms of a ring, which affects the electronic interference), the above equality strongly suggests that the switching events are also modifying the ring sample specific properties only, and not its average behavior. We therefore assume that *the statistical properties of the set of* $R(B)$ *curves generated by the uncontrollable switching events are equivalent to those of curves that one would obtain by measuring different microscopic realization of the same sample* [15].

This assumption is supported by additional experimental evidence. For instance, the average $R(B)$ curve of Fig. 3, obtained by summing about 30 $R(B)$ curves like the one of Fig. 2, satisfies quite accurately the equality $R(B) = R(-B)$. This remarkable fact can be easily understood in terms of our hypothesis. In fact, switching events occurring in the course of a measurement induce "transitions" among different sample realizations and break the symmetry constraint $R(B) = R(-B)$ on a single magnetoresistance trace. The constraint is restored upon averaging, because switching events occur with equal probability at any value of B and they do not modify the average sample properties.

Further support is found by analyzing how the magnitude of the conductance oscillations observed in an

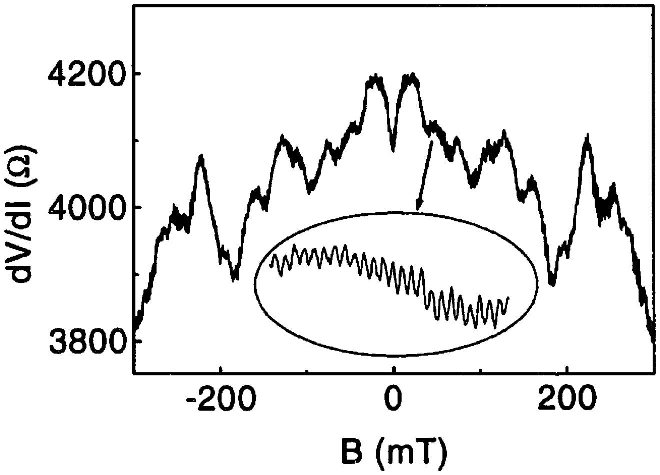

FIG. 3. Average of $\simeq 30 R(B)$ curves (the inset is an enlargement of the small part of the curve). Note how the reciprocity relation is rather accurately satisfied, far better than in the data of Fig. 2.

average $R(B)$ curve depends on the number N of traces over which the average is done. The magnitude of the oscillations is quantified by the height of the h/e peak in the amplitude of the Fourier spectrum. Figure 4 clearly shows that, for sufficiently large [16] N, this amplitude decays like $1/\sqrt{N}$, and that it extrapolates to 0 in the limit $N \rightarrow \infty$. This corresponds exactly to expected behavior of a true ensemble average [17].

On the basis of the above considerations we can now use the set of measured $R(B)$ curves to perform an average of the Fourier spectrum. Figure 5 shows the average Fourier spectrum of the h/e oscillations, obtained by summing the spectra of $\simeq 70$ different $R(B)$ curves like the one shown in the inset of Fig. 2. It is apparent how this procedure smooths the shape of the peak. The *averaged Fourier spectrum reveals that the h/e oscillations do not produce a single peak and that a clear splitting and structure on the sides is visible.*

A splitting emerging (the splitting is already visible after summing approximately 20 curves) from the averaging procedure has been observed in rings of different diameter; it has also been reobserved in the same ring, albeit with a different (but comparable) value, after thermal cycling the sample to room temperature. The values of the observed splittings are within a factor of $\simeq 2$ from each other.

We want to explicitly point out that we have carefully checked the procedure used to calculate the Fourier spectrum of our data and that we can rule out the possibility that the splitting is a product of such a procedure (e.g., due to aliasing or other effects). We have also considered the possibility for the splitting to be an artifact of experimental nature, but we could not find any specific mechanism capable of accounting for its presence.

The only theoretically proposed mechanism that we are aware of, capable of producing a splitting [18] in the frequency of the AB conductance oscillations, is related to the presence of spin-orbit interaction, which induces a geometric (Berry's or Aharonov-Anandan) phase on the spin of electrons traversing the AB ring [4–8]. In its essence the effect is the following. The spin of an electron in the 2DEG "sees" Rashba spin-orbit interaction as if an effective magnetic field was present, whose direction lies in the plane of the 2DEG, perpendicular to the electron momentum. For an electron propagating in a 1D ballistic ring, the effective magnetic field felt by its spin follows

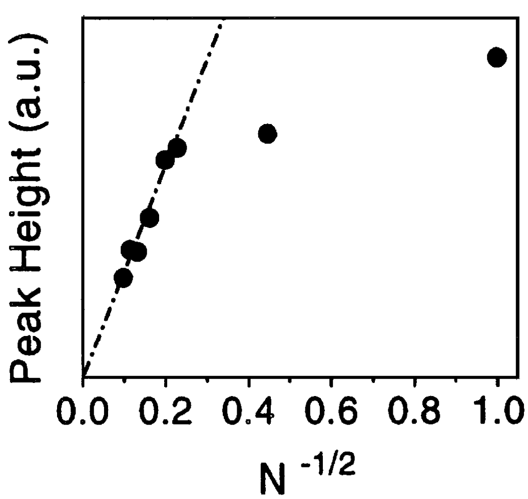

FIG. 4. Height of the peak in the Fourier spectrum of the average $R(B)$ curve as a function of the inverse square root of the number of traces involved in the average. The line shows that for sufficiently large N the height decays linearly with $N^{-1/2}$ and extrapolates to 0, as expected for an ensemble average. For small N this is not true because consecutively measured $R(B)$ traces are not completely uncorrelated.

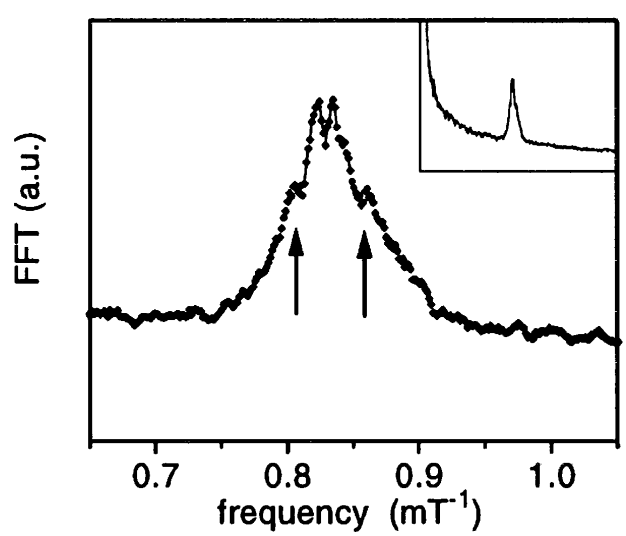

FIG. 5. The peak of the average Fourier spectrum: the splitting is evident, as well as some structure on the sides (pointed by the arrows). The inset shows the same curve on a larger frequency range: note the presence of $1/f$ noise resulting from the switching events.

the change of direction of the electron momentum. When the electron has completed a revolution inside the ring, the effective field has done the same. If the effective field is sufficiently strong [5], the electron spin follows its motion and undergoes a 2π rotation itself, thus acquiring a Berry's phase ($\pm\pi$, depending on the rotation direction).

The Berry's phase manifests itself when one measures the conductance of the AB ring as a function of an applied perpendicular magnetic field. The perpendicular field generates a magnetic flux piercing the ring and causing the AB conductance oscillations [1]. Besides, because of Zeeman interaction, the perpendicular field competes with the spin-orbit effective field and tends to align parallel to itself the electron spin, thus monotonously decreasing the (absolute value of the) geometric phase. Since the Berry's phase acquired by the spin couples to the orbital electronic motion in a similar way as an AB gauge flux does (see, however, Ref. [6] for relevant differences), the decrease of Berry's phase (from $\pm\pi \rightarrow 0$) with increasing perpendicular field results in a *splitting of the frequency of the AB oscillations* [18].

A word of caution should, however, be spent here, since the existing theories consider only highly idealized 1D ballistic rings [19], whereas our samples are multichannels (i.e., not 1D) and not fully ballistic, a fact that, in the presence of spin-orbit interaction, has important implications for the spin dynamics. For this reason it is also not possible to make a conclusive comparison between the value of the observed splitting and theory. Nevertheless, heuristic arguments based on the work of Refs. [5] and [7] indicate that in our samples an effect can be expected and give a theoretical estimate for the splitting value somewhat smaller than, but not incompatible with, the observed one.

In conclusion, our work shows the relevance of studying the ensemble average Fourier spectrum of the Aharonov-Bohm conductance oscillations. The study of this quantity permits us to discriminate small effects from features due to the random sample specific behavior of the rings. This fact is demonstrated by the observation of a splitting in the frequency of the AB oscillations, possibly due to a (Berry's or Aharonov-Anandan) geometric phase acquired by the electron spin.

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[1] See R. Washburn and R. Webb, Rep. Prog. Phys. **55**, 1311 (1992), and references therein.
[2] In order to avoid misunderstandings, we define explicitly the amplitude of the Fourier spectrum as the square root of the sum of the squares of the real and the imaginary parts of the Fourier transform.
[3] See, e.g., Yu. A. Bychkov, V. I. Mel'nikov, and E. I. Rashba, Sov. Phys. JETP **71**, 401 (1990) [Zh. Eksp. Teor. Fiz. **98**, 717 (1990)]. Rashba interaction is the spin-orbit interaction that electrons confined to move in a plane experience if a uniform electric field perpendicular to that plane is present.
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[14] The finite phase coherence length ($L_\phi \simeq 4$ to $5\ \mu$m, as inferred from weak antilocalization measurements performed on wires realized with the same material used in the present experiment) and the thermal length, $L_T = \sqrt{\hbar D/k_B T} \simeq 6\ \mu$m, are probably responsible for the small AB oscillations amplitude.
[15] Equivalent, apart from the fact that the reciprocity relation is *not* satisfied on single curves pertaining to the set, but only on average (see main text).
[16] For small N subsequent measurements are correlated. This means that one or few switching events are not sufficient to induce "transitions" between two completely independent microscopic sample realizations.
[17] A large N average of $R(B)$ also decreases the aperiodic conductance fluctuations.
[18] In real multimode rings this mechanism can affect the frequency of the AB oscillations in a more complex way (e.g., the frequency splitting can depend on the electronic mode). This point may be relevant to interpret the peak side structures shown in Fig. 5.
[19] The analysis of Ref. [8], valid in diffusive systems, does not consider the effect of Zeeman coupling of the spin to the external field, essential in the present context.