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Output of a Pulsed Atom Laser

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We study the output properties of a pulsed atom laser consisting of an interacting Bose-Einstein condensate in a magnetic trap and an additional rf field transferring atoms to an untrapped Zeeman sublevel. For weak output coupling we calculate the dynamics of the decaying condensate population, of its chemical potential, and the velocity of the output atoms analytically. [S0031-9007(97)04914-4]

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The experimental breakthrough to Bose-Einstein condensation with small numbers of atoms in magnetic traps [1] has raised much interest in the properties of mesoscopic quantum gases. Bose-Einstein condensates with atoms in a single magnetic sublevel have been studied experimentally and theoretically. Recently, interference between two independent Bose-Einstein condensates convincingly proved their macroscopic coherence [2,3]. Moreover, the laser-like coherence of the atoms is preserved in the presence of a matter-wave splitter based on rf transitions pumping the atoms into untrapped magnetic sublevels [4]. These states are either strong-field seeking or have no magnetic moment at all, and leave the trap. Alternatively, optical Raman transitions can be used for the transfer [5]. A scheme consisting of a coherent source of atoms and a controllable output coupler is suitably named "atom laser" [6–8] in an obvious analogy.

Continuing this analogy, one can distinguish between a cw laser, based on continuous refilling of the condensate, and a pulsed atom laser, where the condensate is periodically refilled and slowly released, similar to [4]. Whereas a continuous wave atom laser has been studied only theoretically [6–8], current Bose-Einstein condensation experiments are limited to the pulsed mode of operation. The closest approximation of a cw atom laser by a pulsed one can be reached in the limit of a weak coupling rf field. In this case we are able to describe the decay of the trapped condensate and its energy width analytically. Previous calculations addressed the opposite limit of strong coupling by numerical calculations [9] or neglected the crucial influence of atom-atom interactions [10].

The output coupler consists of a monochromatic resonant rf field of frequency ω_{rf} transferring ^{23}Na atoms into the $F = 1$ hyperfine state from the trapped $m = -1$ into the untrapped $m = 0$ and the repelled $m = 1$ magnetic sublevels. For simplicity an isotropic harmonic trap potential $V_{-1}(\mathbf{r}) = V_{\mathrm{off}} + M\omega_{\mathrm{T}}^2\mathbf{r}^2/2$, $V_{+1}(\mathbf{r}) = -V_{-1}(\mathbf{r})$ and $V_0(\mathbf{r}) \equiv 0$ are assumed while effects of gravity are neglected.

The three coupled coherent matter waves are described by a three-component Gross-Pitaevskii equation (GPE) with resonant excitation in rotating wave approximation first studied for a generic two-level system in Ref. [9].

In the following we adopt the point of view of *spontaneously* broken gauge symmetry for a Bose gas initially at zero temperature. The system of equations for the macroscopic wave function $\tilde{\psi}_m(t) = e^{-im\omega_{\mathrm{rf}}t}\langle\hat{\psi}_m(t)\rangle$ in rotating wave approximation for $m, m' \in \{-1, 0, +1\}$ now reads

$$i\hbar\,\frac{\partial}{\partial t}\,\tilde{\psi}_m(\mathbf{r},t) = \left(-\frac{\hbar^2\nabla^2}{2M} + V_m(\mathbf{r}) + \hbar m\omega_{\mathrm{rf}} + U\|\tilde{\psi}(\mathbf{r},t)\|^2\right)\tilde{\psi}_m(\mathbf{r},t) + \hbar\Omega\sum_{m'}(\delta_{m,m'+1} + \delta_{m,m'-1})\tilde{\psi}_{m'}(\mathbf{r},t)\,. \quad (1)$$

Here, we have replaced the atomic density by the modulus of the wave function

$$\langle \hat{n}(\mathbf{r},t)\rangle = \|\tilde{\psi}(\mathbf{r},t)\|^2 = \sum_m |\tilde{\psi}_m(\mathbf{r},t)|^2. \quad (2)$$

At zero magnetic field the symmetrized s-wave scattering matrix elements $U_{mm'} = 4\pi\hbar^2 a_{mm'}/M$ for an elastic collision of a pair of atoms in the sublevels $m, m' \in \{-1, 0, +1\}$ are all nearly equal to $\overline{a} = 53a_0$ (a_0 is the Bohr radius) according to preliminary calculations of Tiesinga and Julienne [11]. Since the steady state operation depends mainly on the initial condensate mean field, we assume in the following a diagonal scattering matrix $a_{mm'} = \delta_{mm'}\overline{a}$ for simplicity. Consequently, the Hartree mean-field potential for each spin component is equal to the total atom density $\langle \hat{n}(\mathbf{r},t)\rangle$ multiplied by $U = 4\pi\hbar^2\overline{a}/M$. The coupling constant $\hbar\Omega = g\mu_{\mathrm{Bohr}}|B|/\sqrt{2}$ denotes the Rabi frequency due to the rf field B for a Landé factor g_F.

The initial condition is chosen as the solution of the stationary GPE for the trapped ($m = -1$) condensate in the absence of the rf field, i.e., $\Omega = 0$ in Eq. (1). In the Thomas-Fermi approximation it reads

$$|\tilde{\psi}_{-1}(\mathbf{r},0)|^2 = \max\left[\frac{\mu_0 + V_{\mathrm{off}} - V_{-1}(\mathbf{r})}{U}, 0\right]. \quad (3)$$

Here, the initial chemical potential μ_0 follows from the normalization condition [see Eq. (8)]. For a small coupling strength ($\Omega \ll \omega_{\mathrm{T}}$) the process of atoms leaking out of the resonance points is faster than the Rabi oscillations. Therefore the coupling into state $m = +1$ can be neglected, since its population grows proportional to Ω^4. In the following only the states $m = -1$ and $m = 0$ are considered. After switching on the coupling due to the rf field, initial oscillations die out quickly, because the untrapped atoms leak out of the trap within less than one Rabi cycle. Other condensate atoms move into the resonance area replacing the leaving ones. Eventually a quasistationary state is reached, i.e., the $m = -1$ condensate wave function decays slowly without oscillations while the atoms coupled out of the condensate are expelled due to the mean-field potential and form a steady current [12]. A numerical solution of the two-component GPE shows the uniform decay of the trapped condensate. In Fig. 1 the sum of external and mean-field potentials $V_{\mathrm{eff}}(\mathbf{r},t) = V_{-1}(\mathbf{r}) - V_{\mathrm{off}} + U|\tilde{\psi}_{-1}(\mathbf{r},t)|^2$ is plotted. As this sum is nearly spatially independent inside the condensate, a description in terms of the Thomas-Fermi approximation with the time-dependent chemical potential $\mu(t)$ appears to be adequate. In the following we present an analytical calculation of the time-dependent output intensity in this quasistationary regime, assuming a three dimensional isotropic harmonic trap.

We first calculate the rate Γ of transitions from the condensate into the output. In the spirit of the Thomas-Fermi approximation we neglect the kinetic energy in a

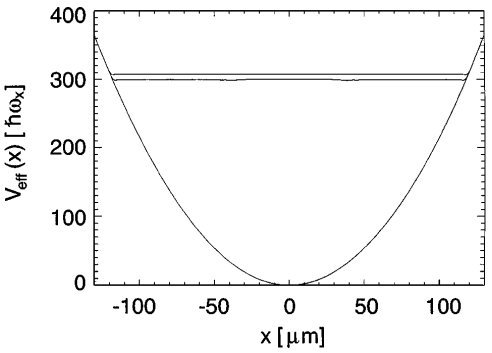

FIG. 1. The effective potential $V_{-1}(\mathbf{r}) - V_{\mathrm{off}} + U|\tilde{\psi}_{-1}(\mathbf{r},t)|^2$ stays spatially independent and equal to $\mu(t)$ inside the condensate for times $t = 0$ ms (upper line) and $t = 670$ ms (lower line); the parabola describes the external potential $V_{-1}(\mathbf{r}) - V_{\mathrm{off}}$. This shows the validity of the Thomas-Fermi approximation [Eq. (3)] also for later times with small coupling strength Ω. The numerical simulation was carried out for $\Omega = 12\ \mathrm{s}^{-1}$, $N_0 = 5 \times 10^6$, $\Delta(0) = 3100\ \mathrm{s}^{-1}$, $\omega_x = 2\pi \times 19$ Hz, and $\omega_{y,z} = 2\pi \times 250$ Hz in a quasi-one-dimensional setup.

two-component GPE corresponding to Eq. (1) and solve for the output density distribution

$$|\tilde{\psi}_0(r,t)|^2 = \frac{4\Omega^2 \sin^2[\frac{1}{2}\sqrt{\Delta^2(r) + 4\Omega^2}\,t]}{\Delta^2(r) + 4\Omega^2}\,|\tilde{\psi}_{-1}(r,0)|^2, \quad (4)$$

where $\hbar\Delta(r) = \hbar\omega_{\mathrm{rf}} - V_{-1}(r)$. The maximum amplitude of the Rabi oscillations is located at r_{res}, determined by the resonance condition

$$\hbar\Delta(r_{\mathrm{res}}) = 0\,. \quad (5)$$

Thus, the main contribution to the output coupling stems from a small shell around that resonance radius $r_{\mathrm{res}} = \sqrt{2\hbar\Delta(0)/(m\omega_T^2)}$.

The time derivative of the density gives the density transition rate. The total transition rate is obtained by integrating the position-dependent rate over the condensate volume. The transition rate is negligible outside a minute resonance shell and strongly peaked within that shell. Expanding the position-dependent detuning $\Delta(r)$ to first order, one obtains the time-dependent transition probability

$$\begin{aligned}\Gamma(t) &\equiv \frac{1}{|\tilde{\psi}_{-1}(r_{\mathrm{res}},0)|^2}\int_0^\infty 4\pi r^2\,\frac{\partial}{\partial t}\,|\tilde{\psi}_0(r,t)|^2\,dr \\ &\approx 4\pi r_{\mathrm{res}}^2\int_{-\infty}^{\infty} 2\Omega^2\,\frac{\sin[(\sqrt{\Delta'(r_{\mathrm{res}})^2 r^2 + 4\Omega^2}\,t)]}{\sqrt{\Delta'(r_{\mathrm{res}})^2 r^2 + 4\Omega^2}}\,dr \\ &= 8\pi^2 r_{\mathrm{res}}^2\,\frac{\Omega^2}{\Delta'(r_{\mathrm{res}})}\,J_0(2\Omega t)\,. \end{aligned} \quad (6)$$

The calculation of the above rate $\Gamma(t)$ does not account for the losses due to the leaving atoms. However, a rate equation allowing for these losses can be derived in the

limit of weak coupling by using the perturbational rate ($t \ll \pi/\Omega$)

$$\Gamma = 8\pi^2 \hbar \Omega^2 \frac{\sqrt{2\hbar\Delta(0)}}{(M\omega_{\mathrm{T}}^2)^{3/2}}, \tag{7}$$

with $\Delta'(r_{\mathrm{res}}) = 2\Delta(0)/r_{\mathrm{res}}$.

Because of the spatial localization of the output coupling, the decay of the condensate population,

$$N(t) \equiv \int d^3r |\tilde{\psi}_{-1}(r,t)|^2, \qquad N(0) = N_0, \tag{8}$$

depends solely on the density of the atoms around the resonance shell with radius r_{res}

$$\frac{dN(t)}{dt} = -\Gamma |\tilde{\psi}_{-1}(r_{\mathrm{res}},t)|^2. \tag{9}$$

In the quasistationary regime we assume the shape of the condensate density being equal to the Thomas-Fermi solution of the stationary GPE (cf. Fig. 1) with a slowly varying atom number $N(t)$,

$$N(t) = \frac{4\pi}{15U}[2\mu(t)]^{5/2}/(M\omega_{\mathrm{T}}^2)^{3/2}. \tag{10}$$

The condensate density at the resonance points is then given by

$$|\tilde{\psi}_{-1}(r_{\mathrm{res}},t)|^2 = [\mu(t) - \hbar\Delta(0)]/U. \tag{11}$$

Inserting this into the decay law (9) we obtain a nonlinear differential equation for the decay of the chemical potential,

$$\frac{d}{dt}\mu + \alpha \frac{\mu - \hbar\Delta(0)}{\mu^{3/2}} = 0, \tag{12}$$

where $\alpha = 3\Gamma(M\omega_{\mathrm{T}}^2)^{3/2}2^{-7/2}\pi^{-1}$. Integration yields

$$\left[2\hbar\Delta(0)\mu^{1/2} + \frac{2}{3}\mu^{3/2} - 2[\hbar\Delta(0)]^{3/2}\operatorname{arcoth}\sqrt{\mu/\hbar\Delta(0)}\right]_{\mu(t)}^{\mu(0)} = \alpha t. \tag{13}$$

With Eq. (10) this yields additionally the time evolution for the number of trapped atoms $N(t)$ as well as the flux and the velocity of the untrapped atoms.

During the depopulation of the condensate the chemical potential and the spatial extension of the condensate decrease until the resonance points lie on the surface of the shrinked condensate. At this point the flux out of the condensate vanishes, because $|\tilde{\psi}_{-1}(r_{\mathrm{res}},t)|^2 = 0$, and the chemical potential and the number of atoms remaining in the trap become constant in time,

$$\mu(\infty) = \hbar\Delta(0). \tag{14}$$

As an example, we chose a small value of $\Delta(0)$ and a trap frequency $\omega_{\mathrm{T}} = 2\pi \times 106$ Hz, the geometric mean of the values given in Ref. [4]. The resulting time evolution of the system variables according to the nonlinear differential equation is shown in Figs. 2(a) and 2(b). The chemical potential $\mu(t)$ and the number of trapped atoms reach their steady state after roughly 25 sec. Correspondingly the flux and the velocity of the untrapped atoms decrease to zero [cf. Figs. 2(c) and 2(d)]. The corresponding calculations were also carried out in one dimension in order to compare the analytical expressions with numerical simulations of the full coupled GPE's. The results showed excellent agreement (less than 5% deviation).

The finite duration of the atom pulse ejected from the trap leads to a finite energy width of both the condensate and the output beam that is called here the *natural energy width* δE of a pulsed atom laser, in analogy to the natural linewidth of spontaneously emitted photons. It can be calculated approximately by describing the initial stage of the output coupling process by an exponential decay of the condensate population,

$$\frac{dN}{dt}(t) = -\Gamma_{\mathrm{pop}}[N(t) - N(\infty)]. \tag{15}$$

The rate Γ_{pop} results in an energy uncertainty,

$$\delta E = \hbar\Gamma_{\mathrm{pop}} \approx \frac{15}{2}\pi\hbar^{5/2}\frac{\Omega^2\sqrt{\Delta(0)}}{\mu(0)^{3/2}}. \tag{16}$$

Whereas the transition rate Γ does not depend on μ and N_0, the population decay rate Γ_{pop} does so due to its dependence on $\tilde{\psi}_{-1}(r,0)$. We thus find a dependence of the energy width on the condensate number,

$$\delta E \propto \frac{\Omega^2\sqrt{\Delta(0)}}{N_0^{3/5}}. \tag{17}$$

The natural energy width becomes narrower for weaker coupling strength Ω, i.e., a slower output coupling process. The same effect can be achieved by choosing a smaller detuning $\Delta(0)$ which causes the sphere of the resonance points to shrink towards the center of the condensate. The energy width can be further reduced by starting out from a condensate with a large population N_0.

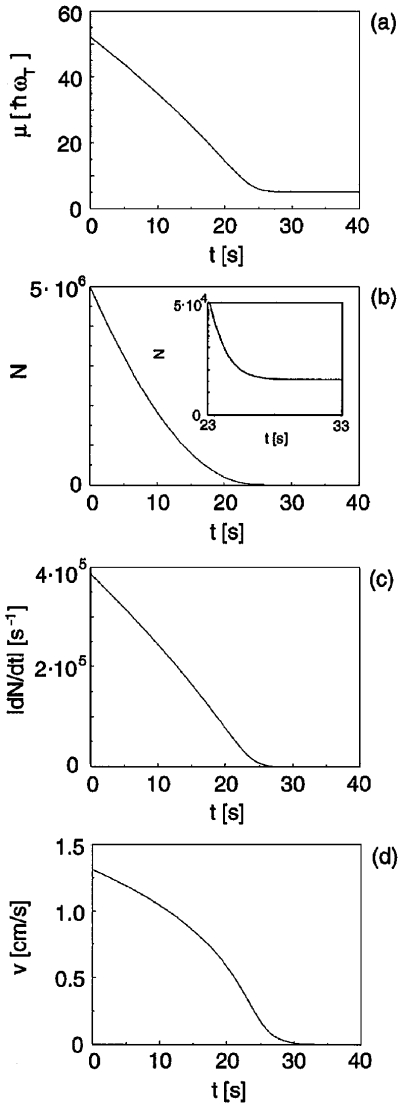

FIG. 2. Time evolution in the Thomas-Fermi approximation: (a) chemical potential, (b) number of particles in the trapped $m = -1$ state, (c) particle flux in the untrapped $m = 0$ state, and (d) velocity of atoms leaving the condensate. The parameters are $N_0 = 5 \times 10^6$, $\Delta(0) = 3500\ \mathrm{s}^{-1}$, and $\Omega = 20\ \mathrm{s}^{-1}$.

The energy width δE can also be understood as a velocity width $\delta v = \delta E/Mv$ of the untrapped atoms leaving the condensate. The parameters given in Fig. 2 result in $\langle v \rangle = 1.31$ cm/s for the velocity outside the condensate and $\delta v/\langle v \rangle \approx 10^{-6}$ for the relative velocity width.

An additional energy width is imposed by the temporal decay of the chemical potential $\mu(t)$. It leads to a decrease of the output velocity according to $Mv(t)^2/2 = \mu(t) - \hbar\Delta(0)$ thereby implying a frequency chirp of the output beam. This frequency chirp can be compensated, however, by imposing a chirp on the frequency ω_{rf} of the rf field such that the output velocity rather than the detuning $\Delta(0)$ becomes constant.

The output pulse can last for up to the order of 100 sec such that the causes of phase fluctuation which would also influence a cw-atom laser have to be considered as well. Additional broadening of the energy width of the output beam is caused by thermal excitations of the condensate wave function and technical noise of the output coupling mechanism such as fluctuations of the confining magnetic field. Thus, the *natural energy width* calculated above has to be understood as a lower limit. A comprehensive theoretical description of the fluctuations of the condensate wave function, particularly of its phase, is beyond the scope of this Letter.

It has been shown in [8] that the temporal decay of phase correlations transforms into a decay of the spatial coherence of the output beam along the mean classical trajectory that is being passed during the according correlation time of the condensate phase.

At present, random variations of the bias magnetic field at the mGauss level are among the most important experimental limitations to the coherence properties of the output, prior to fluctuations of the condensate wave function. They might contribute to a phase diffusion of the output beam in the 100 Hz range.

Pulsed atom lasers will play an important role in creating coherent matter waves as the size of the condensate may be largely increased in future experiments. The results of this Letter are based on the solutions of the coupled Gross-Pitaevskii equations and allow us to extract the relevant properties of the output analytically. The velocity of the untrapped atoms depends on the slowly decaying population of the trapped condensate fraction, leading to a slow chirp of the output frequency. While this frequency chirp might be compensated by a variation of the rf-field frequency, other sources of spectral width, such as fluctuations of the confining magnetic field, are surely more difficult to overcome.

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