REMARK ON ENERGY PEAKS IN MESON SYSTEMS

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Current high-energy experiments reveal the existence of a variety of pronounced peakings in the energy dependence of a number of strong particle reactions. In some instances it has been suggested that these peaks correspond to resonances, even where it has not as yet been ascertained that the appropriate phase shift goes through 90° In other cases, especially for 2π and 3π systems, such peaks have sometimes been connected with the possible existence of new particles. It is largely a matter of terminology whether one relates a peak to a resonance or to a particle. What is common to both views is the emphasis on the marked dominance in the peak region of a state with well-defined quantum numbers such as angular momentum, parity, isotopic spin, and hypercharge. As a function of energy the influence of such a state is to be centered on the peak with a characteristic half-width. In what follows we shall call such a state a particle by convention.

It is the purpose of this note to discuss the possible occurrence in meson systems of peaks which, even though quite sharp, cannot be called particles in the above sense for the following reasons. First, the effect does not occur in just one angular momentum state. Secondly, it may in general occur at the same energy in more than one isotopic spin state. Finally, the peak has a characteristic asymmetric distribution which cannot be described by a Breit-Wigner formula. Moreover, if the peak occurs in a three-particle state, there is a characteristic Dalitz-Fabri (DF) plot[1] associated with it.

Such peaks can be generated by a mechanism discussed by Peierls[2] in an attempt to relate the second to the first π-nucleon resonance. In general terms the idea is to consider the exchange scattering of two particles X and Y mediated by one particle Z, as drawn in Fig. 1. Let X also stand for the mass of particle X, etc. We are interested in the case that $X > Y + Z$. Then the two vertices of Fig. 1 are decay vertices and the scattering amplitude A has a pole at Z^2 in the appropriate invariant momentum transfer variable. Let $s = (E_X + E_Y)^2$ be the invariant energy square variable as expressed in the c.m. energies of X and Y and θ be the c.m. scattering angle. For fixed θ, A also has a pole in s, the real part of which varies from s_0 to s_1, where s_0, s_1 correspond to $\theta = 0$, π, respectively. If the width of particle X is not very large we will stay close to the physical region. This almost singular behavior of $A(s)$ for certain physical s causes the peaking effect to which we refer as an (X, Y, Z) peak.

Peierls[2] considers the second πN resonance N^{**} as possibly an (N^*, π, N) peak. He obtains an N^{**} value suggestively close to the observed one. He further shows that the (π, N^*) interaction takes place mainly in the S state which implies a correspondingly well-defined angular momentum for N^{**} It is readily seen that this is due to the fact that in this case s_1/s_0 is close to unity so that the (s_0, s_1) strip acts "almost" like a pole. We emphasize right away that for the 3π systems to be discussed below, s_1/s_0 will turn out to be $\gg 1$, which leads to a different angular momentum picture.

It is of course important in considerations of this kind that the partial amplitude due to a specific mechanism such as Fig. 1 is not completely masked by interference with a residual amplitude (supposed to be slowly varying in the peak region). We shall not consider such interference here and therefore do not claim to predict peakings that necessarily stand out clearly against background. Rather, we indicate where an effect could pos-

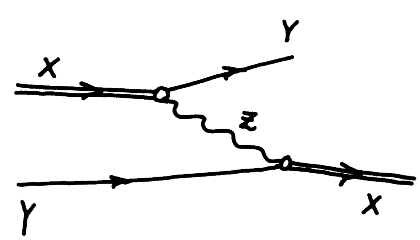

FIG. 1. The exchange scattering diagram.

sibly occur and what its properties would be.

As an example, consider now a fictitious meson X which can strongly decay into 2π. The (X, π) scattering amplitude is part of a $3\pi \to 3\pi$ scattering amplitude. Let us further suppose that X has $T = J = 1$. We now consider the exchange diagram Fig. 1 for (X, π) scattering.[3] In this approximation the probability ratios for the (X, π) contributions to 3π scattering in states with total isotopic spin T is

$$(T=0):(T=1):(T=2) = 4:1:1.$$

If we consider the scattering in a given T state we must perform the appropriate symmetry operations on the amplitude. We shall not do so now, but consider the symmetry effects later. Then the square of the amplitude, averaged over the "spin" directions of X, is given by[4]

$$f(s, \cos\theta) = \frac{[2s - 3X^2 - 6 - 4q^2(1-\cos\theta)]^2}{[s - 2X^2 - 1 - 2q^2(1-\cos\theta)]^2 + \Delta^2}, \tag{1}$$

and its integral over all scattering angles is

$$f(s) = \frac{(X^2-4)^2}{2q^2\Delta}\tan^{-1}\frac{4q^2\Delta}{\Delta^2 + s^{-1}(s-s_0)(s-s_1)} + \frac{X^2-4}{q^2}\ln\frac{(s-s_0)^2+\Delta^2}{s^{-2}(s-s_1)^2+\Delta^2} + 8, \tag{2}$$

where

$$s_0 = 2X^2 + 1, \tag{3}$$

and

$$s_1 = (X^2-1)^2,$$

$$4q^2 = s - s_0 - 1 + s_1/s,$$

$$\Delta = \frac{2X(X^2-1)}{s + X^2 - 1}\Gamma. \tag{4}$$

Γ is the full width[5] of particle X. We note the following qualitative properties.

(1) For small Γ and not too small $(X-2)$, $f(s)$ rises sharply just below $s = s_0$, reaches a maximum very near s_0, and then decreases somewhat less sharply. $f(s)$ does not vary appreciably near s_1. The resulting peak has a characteristic asymmetry of a different kind than the one of a Breit-Wigner distribution.

(2) Equation (1) shows that near s_0 the angular distribution contains many partial waves. This is due to the large recoil of the intermediate pion. Correspondingly s_1/s_0 is large.

(3) In a small neighborhood around s_0 the phase-space factor varies relatively little. In this region we may therefore directly relate $f(s)$ to the probability of 3π decay of a state with mass $s^{1/2}$. It follows from the foregoing that this state contains many angular momenta.

(4) One should therefore not analyze the corresponding DF plot for the "3π decay" of such a state in terms of a fixed total angular momentum. However, in our case this plot has other simple characteristics. It is inscribed in an equilateral triangle of height $s_0^{1/2} - 3$. Its structure is essentially three bands, each one parallel to one of the sides of the triangle at a distance $\simeq \frac{1}{2} s_0^{-1/2} \times \{[(s_0^{1/2}-1)^2 - X^2] \pm \Gamma X\}$. These bands have further to be weighted by the mass distribution around $s_0^{1/2}$ of the unstable 3π system itself.

As an illustration of these points we considered a recent result by a Saclay group.[6] This experiment is stated to give evidence for a $T=1$, $J=1$, 2π resonance (named ζ) at 575 ± 20 Mev with $\Gamma < 70$ Mev. We therefore applied our result to $X = \zeta = 4$ and took $\Gamma = \frac{1}{4}$ and $\frac{1}{8}$. The $f(s)$ distribution is given in Fig. 2 where the drawn curve refers to $\Gamma = \frac{1}{8}$.

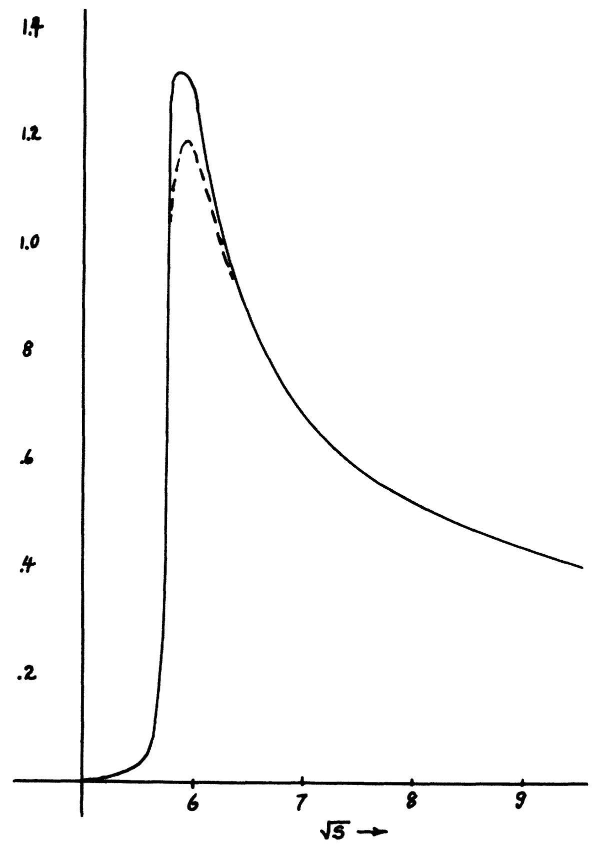

FIG. 2. $f(s)/2(X^2-4)^2$ versus $s^{1/2}$ for $X=4$, $\Gamma = \frac{1}{8}$ (drawn curve). The dashed line refers to $\Gamma = \frac{1}{4}$ (see text).

For $\Gamma=\frac{1}{4}$, $f(s)$, scaled down by a factor 2, gives practically the same distribution except just near the peak, as is indicated by the dashed line. The peak occurs at $s_0^{1/2}=5.75=805$ Mev. Thus our crude considerations lead us to contemplate the possible existence of a 3π peak most pronounced in the $T=0$ state and rather close in energy to the ω_0 which has[7] also $T=0$. One may ask if the (ζ,π,π) peak could possibly be the ω_0 itself. This is out of the question as can readily be seen by drawing the DF plot. This consists of three narrow bands (for $\frac{1}{8}\leqslant\Gamma\leqslant\frac{1}{4}$) in complete contrast[7] to the observed plot for ω_0. As these bands actually do not overlap it follows furthermore that we are entitled to use our formalism for the specific case $T=0$, because only the overlap regions are affected by the necessary total antisymmetrization. Regardless of any dynamical details, it will be interesting to see if some band effect due to ζ can be found in the ω_0 plot.

It will be an interesting test both for the role of ζ and for the usefulness of the Peierls mechanism to see if further experiments will show a peak near a mass $(2\zeta^2+1)^{1/2}$ which can be identified with the one analyzed here. As stated earlier we have nothing to say about the peak to background ratio.

The present remarks can of course be applied to other combinations and we quote some.[8] For states odd under G conjugation we have (ρ,π,η), (ρ,π,π), (ω,ζ,π) with $s_0^{1/2}=950$, 1090, 1370 Mev, respectively. For even G we have (ω,π,ζ) at 950 Mev. For strange particles we have (K^*,π,K) with $s_0^{1/2}=1170$ Mev which could be relevant in $S=\pm1$ channels, and (K^*,K,π) at 1410 Mev for $S=\pm2$. In addition there is the further interesting possibility that Z in Fig. 1 is replaced by a particle pair while the vertices retain their decay character. Finally we note that better approximations can perhaps be obtained by iterating[3] the diagram of Fig. 1.

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[1]R. Dalitz, Phil. Mag. 44, 1068 (1953); E. Fabri, Nuovo cimento 11, 479 (1954).

[2]R. F. Peierls, Phys. Rev. Letters 6, 641 (1961).

[3]The same graph is also mentioned by P. Carruthers, Cornell University (to be published).

[4]Apart from an over-all multiplicative factor which contains the fourth power of the effective strength for $X\rightarrow2\pi$ decay. The π mass has been taken as a unit.

[5]We have neglected some insensitive Γ-dependent terms.

[6]R. Barloutaud, J. Heughebaert, A. Leveque, J. Meyer, and R. Omnes, Phys. Rev. Letters 8, 32 (1962).

[7]B. Maglić, L. Alvarez, A. Rosenfeld, and L. Stevenson, Phys. Rev. Letters 7, 178 (1961).

[8]For ρ see A. Erwin, R. March, W. Walker, and E. West, Phys. Rev. Letters 6, 628 (1961). For η see A. Pevsner, R. Kraemer, M. Nussbaum, C. Richardson, P. Schlein, R. Strand, T. Toohig, M. Block, A. Engler, R. Gessaroli, and C. Meltzer, Phys. Rev. Letters 7, 421 (1961). For K^* see M. Alston, L. Alvarez, P. Eberhard, B. Cook, W. Graziano, H. Ticho, and S. Wojcicki, Phys. Rev. Letters 6, 300 (1961).