RARE DECAY MODES OF THE ω (η) MESON*

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We wish to consider the decay modes of the $\omega(\eta)$ meson into two charged particles: $e^+ + e^-$, $\mu^+ + \mu^-$, and $\pi^+ + \pi^-$ In particular we point out that the $\pi^+\pi^-$ mode of the ω meson, which involves $(\frac{1}{137})$ twice in the decay rate, may turn out to be relatively frequent; this rather unexpected behavior arises because the ω meson "rides on" the $T=1$, $J=1$ $\pi\pi$ resonance (ρ) peak so that the final-state interaction between the two charged pions in a p state strongly enhances the $\pi^+\pi^-$ mode. Quantitatively speaking, we can calculate the partial-decay rates for the e^+e^- and $\mu^+\mu^-$ modes in terms of (a) the coupling constant of the $\omega(\eta)$ meson to the nucleon and (b) the $\omega(\eta)$ meson contribution to the isoscalar charge form factor. For a quantitative estimate for the partial-decay rate for the $\pi^+\pi^-$ mode, in addition to (a) and (b), we must know rather precisely (c) the mass and the width of the ρ meson.

Recent experiments have established the existence of an unstable neutral meson called ω, which has a mass ~780 Mev, isospin 0, and decays into $\pi^+ + \pi^- + \pi^0$.[1,2] Most likely it is a vector particle. There also is evidence for another neutral meson (η) of similar nature with a lower mass, around 550 Mev, although its quantum numbers have not been firmly established.[2] A vector meson ρ of isospin one, decaying into 2π, had been found earlier at ~750 Mev.[3]

As is well known, the search for such particles has been largely stimulated by theoretical speculations, especially in connection with the electromagnetic form factors of the nucleon which seem to require vector mesons of isospin zero and one.[4,5] Now that these vector mesons have been found, we may turn the question around and ask: To what extent do they really contribute to the form factors? This question is particularly interesting for ω and η since it is likely that they both have isospin zero and are candidates for the isoscalar form factors. A very preliminary analysis indicates that their contributions to the charge form factor are more or less of the same order of magnitude.[6] (C, to be defined below, is -0.7 for ω and 1.2 for η.)

We would like to emphasize that if ω (or η) participates in the form factor experiments at all, it would automatically mean that ω (η) is coupled both to the nucleon and to the electron.[7] The coupling of ω (η) to the electron should be actually mediated by a virtual photon, so that it would be of the order of e^2. But then the same coupling mechanism should also exist between ω (η) and any charged particle. This means that ω (η) is capable of decaying weakly (compared to the 3π mode) as follows:

$$\omega\ (\eta) \to e^+ + e^- \qquad (1a)$$

$$\to \mu^+ + \mu^- \qquad (1b)$$

$$\to \pi^+ + \pi^-. \qquad (1c)$$

If the electron and the muon have no difference except for their masses, as seems to be the case, then the first two processes will have essentially the same coupling. The last one occurs via violation of isospin conservation. In view of the strong interaction of ω with baryon pairs, its intrinsic coupling strength can be different from the former, but at least it should exist and be comparable. What is more, the nearby ρ-meson level can lead to an enormous enhancement of the decay rate through the final-state interaction of the two pions in a p state.

Let us denote the vector (charge) couplings of ω to the nucleon and to the electron by G and g, respectively. Further let the contribution of ω in the dispersion theoretic expression for the isoscalar charge form factor $F_1(q^2)$ be of the form $Cm^2/(q^2+m^2)$ (in other words, the residue of F_1 at $q^2=-m^2$ is Cm^2 where m is the ω mass). Then it is easy to see that[8]

$$Gg = \tfrac{1}{2}e^2 \frac{q^2+m^2}{q^2} F_1(q^2)\bigg|_{q^2=-m^2} = -\tfrac{1}{2}e^2 C,$$

or

$$\frac{g^2}{4\pi} = (\tfrac{1}{4}\alpha C^2)\Big/\left(\frac{G^2}{4\pi}\right). \qquad (\alpha = \tfrac{1}{137}) \qquad (2)$$

Assuming the same g, the partial widths for the three processes in Eq. (1) are given by

$$\Gamma(\omega \to l^+ + l^-) = \frac{g^2}{4\pi}\frac{m}{3}\left(1+\frac{2m_l^2}{m^2}\right)\left(1-\frac{4m_l^2}{m^2}\right)^{1/2},$$

$$(m_l = m_e \text{ or } m_\mu) \qquad (3a, b)$$

$$\Gamma(\omega \to \pi^+ + \pi^-) = \frac{g^2}{4\pi}\frac{m}{12}\left(1 - \frac{4m_\pi^2}{m^2}\right)^{3/2} |F_\pi(-m^2)|^2, \quad (3c)$$

where $F_\pi(q^2)$ is the pion electromagnetic form factor expressed in terms of the ρ-meson mass m_ρ and the width Γ_ρ:

$$F_\pi(q^2) = \frac{m_\rho^2}{q^2 + m_\rho^2 - i m_\rho \Gamma_\rho}. \quad (4)$$

Combining Eqs. (2) and (3) we obtain

$$\Gamma(\omega \to e^+ + e^-) = \Gamma(\omega \to \mu^+ + \mu^-) = 3\times10^{-3} \times \frac{C^2}{G^2/4\pi}\ \text{Mev},$$

$$\Gamma(\omega \to \pi^+ + \pi^-) = 7\times10^{-4} \times \frac{C^2}{G^2/4\pi} \times |F_\pi(-m^2)|^2\ \text{Mev}. \quad (5)$$

It has been estimated[5,6] that $C \approx -0.7$ and $G^2/4\pi \approx 2$-5. The enhancement factor for the $\pi^+\pi^-$ mode is very sensitive to the mass difference $m - m_\rho$ and the width Γ_ρ. Using the calculations of Bernstein and Feinberg,[9] we obtain $|F_\pi(-m^2)|^2 \approx 40$ where m_ρ and Γ_ρ are taken to be 750 Mev and 100 Mev, respectively. Thus

$$\Gamma(\omega \to e^+ + e^-) = \Gamma(\omega \to \mu^+ + \mu^-) \approx 5\times10^{-4}\ \text{Mev},$$

$$\Gamma(\omega \to \pi^+ + \pi^-) \approx 5\times10^{-3}\ \text{Mev}. \quad (6)$$

In order to obtain the branching ratios for the rare modes, we must know the partial width for the "strong" decay mode $\omega \to \pi^+ + \pi^- + \pi^0$. Experimentally only its upper limit ($\Gamma < 20$ Mev) has been given.[1,2] One way to guess it is to compare the ω width to the η width. The η width can, in turn, be estimated if the $\eta \to \pi^0 + \gamma$ mode (whose partial width we can "calculate" from the π^0 lifetime using the method of Gell-Mann and Zachariasen[10]) is shown to compete favorably with the $\pi^+\pi^-\pi^0$ mode. Experiments to detect neutral modes of the η meson are currently in progress.[2,11] For the sake of argument we may set $(\pi^0\gamma)/(\pi^+\pi^-\pi^0) \approx 3$. Now the Gell-Mann–Zachariasen estimate for the π^0 partial width for η is about 0.03 Mev,[6] so that the $\pi^+\pi^-\pi^0$ width for η is expected to be in the neighborhood of 0.01 Mev. To compare the $\pi^+\pi^-\pi^0$ mode of the ω meson with the $\pi^+\pi^-\pi^0$ mode of the η meson we first recall that, if other conditions are equal, the $\pi^+\pi^-\pi^0$ decay rate of a neutral vector meson varies as Q^4 where Q is the Q value of the decay[12]; this leads to a factor of about 50 for ω. This may not be the whole story. If both ω and π are even under hypercharge reflection R of Gell-Mann,[13] as conjectured earlier,[14] the ω width may be reduced by as much as a factor of 10. Finally we obtain

$$\Gamma(\omega \to \pi^+ + \pi^- + \pi^0) \approx 0.05\ \text{Mev}. \quad (7)$$

Needless to say, the above estimate may well be wrong by an order of magnitude. In any case, (6) together with (7) implies that the leptonic modes of ω may take place about 1% of the time whereas the $\pi^+\pi^-$ mode may occur about 10% of the time.[15] We again wish to emphasize that these numbers are suggestive rather than quantitative.

As for the η meson, the same general formulas (2) and (3) can be applied, except that we do not have the benefit of the enhancement factor for the two-pion mode. The decay rates become

$$\Gamma(\eta \to e^+ + e^-) = \Gamma(\eta \to \mu^+ + \mu^-) = 2\times10^{-3}\frac{C^2}{G^2/4\pi}\ \text{Mev},$$

$$\Gamma(\eta \to \pi^+ + \pi^-) = 3\times10^{-4}\frac{C^2}{G^2/4\pi}\ \text{Mev}. \quad (5')$$

With $C \approx 1.2$, $G^2/4\pi \approx 2$, and $3\Gamma(\eta \to \pi^+ + \pi^- + \pi^0) \approx \Gamma(\pi^0 + \gamma) \approx 0.03$ Mev, we obtain $\approx 3\%$ for the e^+e^- and the $\mu^+\mu^-$ mode and $\approx 0.5\%$ for the $\pi^+\pi^-$ mode.

Summarizing, we conclude:

(1) There exist Goldberger-Treiman type relationships for the ω (η) meson. They predict the rare decay modes $\omega \to \pi^+ + \pi^-$ and $\to e^+ + e^-$ or $\mu^+ + \mu^-$ with branching ratios of $\approx 10\%$ for $\pi^+\pi^-$ and $\approx 1\%$ for the lepton pairs provided that the ω width is as narrow as 0.05 Mev. The $\pi^+\pi^-$ decay rate is very sensitive to the ω - ρ mass difference and the ρ width. For the η meson, the branching ratios would be of the order of a few % for lepton pairs and a fraction of a percent for $\pi^+\pi^-$ provided that the $\pi^+\pi^-\pi^0$ partial width is so small that the $\pi^0\gamma$ mode competes favorably with the $\pi^+\pi^-\pi^0$ mode.

(2) If the $\omega \to \pi^+ + \pi^-$ mode is unusually enhanced, and if the ω width turns out to be very narrow, we should expect a spurious ρ peak at the ω mass for the $\pi^+\pi^-$ Q-value distribution but not for the $\pi^\pm\pi^0$ Q-value distribution. The possible existence of a spurious ρ^0 peak in $p + \bar{p} \to 2\pi^+ + 2\pi^- + \pi^0$ is currently being investigated.[16] Similarly for $\pi^+\pi^-$ pairs in the reaction $\pi^- + p \to \pi^+ + \pi^- + p$, a spurious ρ^0 peak may be present; such a peak should become less pronounced when only "peripheral" events are selected since the production mechanism for the ω meson is not of the one-pion-exchange type.

(3) In spite of their small branching ratios, the leptonic decay modes might be observable, say, in a spark chamber. If ω and/or η are really re-

sponsible for the form factors, one should expect the corresponding peaks in the Q-value distribution of e^+e^- and $\mu^+\mu^-$ pairs.[17]

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[7]For earlier considerations along this line see R. W. Huff, Phys. Rev. 112, 1020 (1958); I. Iu. Kobzarev, L. B. Okun, and I. Ia. Pomeranchuk, J. Exptl. Theoret. Phys. (U.S.S.R.) 41, 495 (1961) [translation: Soviet Phys.–JETP (to be published)].

[8]It is amusing that Eq. (2) resembles the Goldberger-Treiman relation for pion decay to the extent that the coupling constant g that characterizes the decay of ω into charged particles is inversely proportional to the strong interaction constant G.

[9]J. Bernstein and G. Feinberg (to be published) discuss a situation very similar to ours in connection with the decay of the W particle (intermediate vector boson responsible for weak interactions) into two pions.

[10]M. Gell-Mann and F. Zachariasen, Phys. Rev. 124, 953 (1961).

[11]Private communications from D. Miller (K^-p experiments) and R. March (π^-p experiments).

[12]See, e.g., I. Iu. Kobzarev and L. B. Okun, J. Exptl. Theoret. Phys. (U.S.S.R.) 41, 499 (1961) [translation: Soviet Phys.–JETP (to be published)], and P. T. Matthews in the Proceedings of the Aix-en-Provence Conference on Elementary Particles, 1961 (to be published).

[13]M. Gell-Mann, California Institute of Technology Report CTSL-20, 1961 (unpublished).

[14]J. J. Sakurai, Phys. Rev. Letters 7, 426 (1961).

[15]In this connection recall that the $\pi^0+\gamma$ and the $2\pi^0+\gamma$ mode may be "forbidden" by R invariance whereas the $\pi^++\pi^-+\gamma$ is forbidden by R invariance for $\pi^+\pi^-$ in an even relative state and by charge conjugation invariance for $\pi^+\pi^-$ in an odd relative l state.[14] Under such circumstances the $\pi^+\pi^-$ mode may be the most favorable "electromagnetic" decay mode of ω.

[16]B. C. Maglić's report in the 1961 Thanksgiving Meeting of the American Physical Society [Bull. Am. Phys. Soc. 6, 435(T) (1961)], based on work of M. L. Stevenson, G. R. Kalbfleisch, B. C. Maglić, and A. H. Rosenfeld, Lawrence Radiation Laboratory Report UCRL-9814 (to be published).

[17]Of course the real ρ^0 meson will also have intrinsic decay rates for $\rho^0 \to e^+ + e^-$ and $\rho^0 \to \mu^+ + \mu^-$ which are comparable to the ω case if ρ^0 is responsible for the isovector form factors [Eqs. (2) and (3a,b) apply here equally]. But the branching ratio $\Gamma(\rho^0 \to l^+ + l^-)/\Gamma(\rho^0 \to \pi^+ + \pi^-)$ would be much less favorable than for ω because of the presumably large width $\Gamma(\rho^0 \to \pi^+ + \pi^-) \approx 100$ Mev (see the text).