N. Schmitz, in Proceedings of the 1960 Annual International Conference on High-Energy Physics at Rochester (Interscience Publishers, Inc., New York, 1960), p. 58; J. A. Anderson, V. X. Bang, P. G. Burke, D. D. Carmony, and N. Schmitz, Phys. Rev. Letters 6, 365 (1961); J. A. Anderson, V. X. Bang, P. G. Burke, D. D. Carmony, and N. Schmitz, Revs. Modern Phys. 33, 431 (1961).

[3]D. D. Carmony and R. T. Van de Walle, Lawrence Radiation Laboratory Report UCRL-9933, 1962 (unpublished).

[4]G. F. Chew and F. E. Low, Phys. Rev. 113, 1640 (1959).

[5]The formulas used in calculating $\cos\theta_{\pi\pi}$ from laboratory quantities can be found in reference 4 [formulas (1.13), (1.14), and (1.15)].

[6]G. F. Chew and S. Mandelstam, Phys. Rev. 119, 467 (1960).

HYPERON DECAY PARAMETERS AND THE $K\Lambda N$ PARITY*

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With the results of recent experiments on the production[1] and decay[2] of hyperfragments, Dalitz and Liu[3] have shown that a determination of the $K\Lambda N$ parity may under certain assumptions be obtained from a measurement of the ratio of s-wave to p-wave amplitudes in the decay $\Lambda \rightarrow p + \pi^-$ A measurement of this ratio is reported here along with further results on the parameters for the decays $\Sigma^+ \rightarrow p + \pi^0$ and $\Lambda \rightarrow p + \pi^-$. The parameters of interest are usually defined as

$$\alpha = \frac{2\,\mathrm{Re}(S^*P)}{|S|^2+|P|^2}, \quad \beta = \frac{2\,\mathrm{Im}(S^*P)}{|S|^2+|P|^2}, \quad \gamma = \frac{|S|^2-|P|^2}{|S|^2+|P|^2},$$

where S and P are the amplitudes for s- and p-wave decay, respectively.

Recently we have reported (in a paper hereinafter referred to as I) a measurement of $\alpha_0(\Sigma^+ \rightarrow \pi^0 + p)$,[4] together with a measurement of the sign of $\alpha_\Lambda(\Lambda \rightarrow \pi^- + p)$.[5] The experimental arrangement is described in I. Hyperons were produced by the reactions $\pi^+ + p \rightarrow \Sigma^+ + K^+$, $\pi^+ + n \rightarrow \Lambda + K^+$ in hydrogen and lithium deuteride, respectively. The reactions were identified by counter and spark chamber techniques, and the polarization of the decay protons was measured by means of p-C^{12} scattering in a carbon-plate spark chamber. A photograph of an event and the orientation of the equipment in space are shown in Fig. 1 (see also Fig. 1 of I).

The polarization of the proton from the decay of a spin-$\frac{1}{2}$ hyperon can be shown to be[6]

$$\vec{\mathrm{P}} = [1/(1-\alpha\bar{p}\cos\theta)] \times \{[-\alpha + \bar{p}(1-\gamma)\cos\theta]\hat{k}_p + \gamma\bar{\vec{\mathrm{p}}} + \beta\bar{\vec{\mathrm{p}}}\times\hat{k}_p\}, \quad (1)$$

in a nonrelativistic approximation (which is adequate for our purposes, since relativistic effects are presumably small), and the angular distribution is

$$D(\cos\theta) = \tfrac{1}{2}(1-\alpha\bar{p}\cos\theta); \quad (2)$$

$\hat{k}_p$ is a unit vector parallel to the momentum of the proton in the rest frame of the hyperon; $\cos\theta = \hat{k}_p\cdot\bar{\vec{\mathrm{p}}}/\bar{p}$; $\bar{\vec{\mathrm{p}}}$ is the average polarization of the hyperon, and is perpendicular to the K-hyperon production plane. The proton polarization perpendicular to the K-π^+ plane, averaged over an interval in $\cos\theta$, is, then,

$$\langle P\rangle_\pm = [1/(1\mp\alpha\bar{p}|\langle\cos\theta\rangle|] \times\{\mp\alpha|\langle\cos\theta\rangle| + \bar{p}(1-\gamma)\langle\cos^2\theta\rangle + \gamma\bar{p}\}, \quad (3)$$

the + signs corresponding to a proton being produced with momentum up with respect to the K-π^+ plane ($\cos\theta > 0$ is defined as "up"), and − signs corresponding to down.

The proton polarization data for Λ decay were divided into two groups. Group 1 included 121 events for which the decay pion was detected (i.e., counted in the U or D counter; see I). Group 2 included 340 events for which the decay pion was not detected. For each group, the factors $\langle\cos\theta\rangle$ and $\langle\cos^2\theta\rangle$ were calculated from the counter geometry. The calculation included the effects of the internal momentum distribution of the target neutron, but was insensitive to the details of the distribution. The factor $\alpha\bar{p}$ was computed from the measured up-down asymmetry of the decay pions to be $\alpha\bar{p} = 0.35 \pm 0.05$. The errors in the factors $\langle\cos\theta\rangle$, $\langle\cos^2\theta\rangle$, and $\alpha\bar{p}$ are small and have been neglected. From Eq. (3), the measured polarizations $P_\pm$ for each group are then functions of α and γ only, and (3) can be solved for α and γ

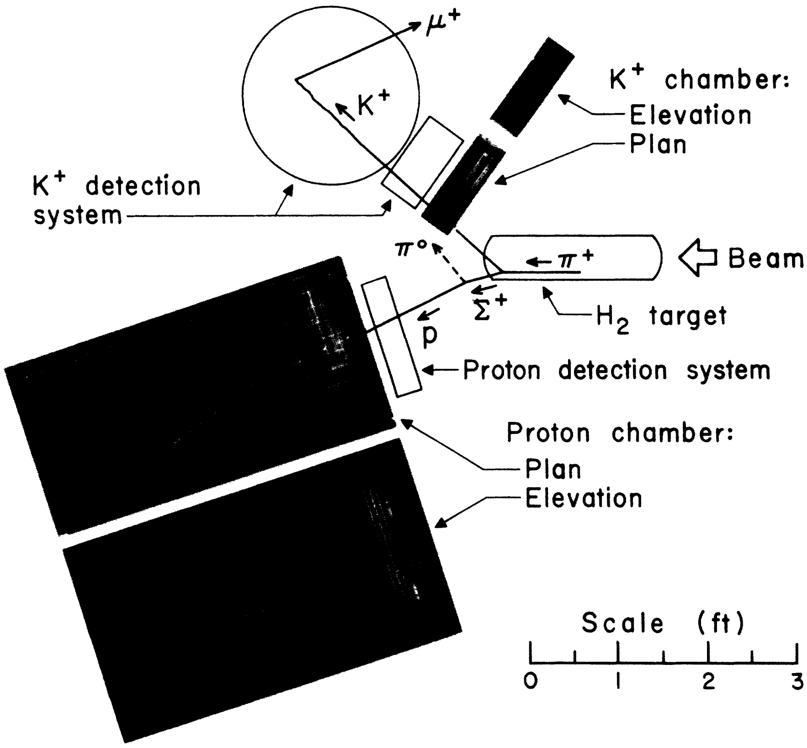

FIG. 1. Schematic drawing and photograph of a typical example of $\pi^+ + p \rightarrow \Sigma^+ + K^+$, $\Sigma^+ \rightarrow p + \pi^0$, $p + C^{12} \rightarrow p + C^{12}$, $K^+ \rightarrow \mu^+ + \nu$. The unphotographed tracks have been distorted for the sake of clarity. This particular proton scattering has $\cos\phi_S > 0$ (to the left) and $\sin\phi_S > 0$ (up).

as functions of P_+ and P_-. We neglect β^2 (if time-reversal invariance and the $|\Delta \vec{T}| = \frac{1}{2}$ rule holds, then the known π-N phase shifts give $\beta^2 = 0.02$); it is then easily seen that $\alpha^2 + \gamma^2 = 1$, and $P_\pm$ are functions only of γ and the algebraic sign of α.

The dependence of $P_\pm$ on γ and $\alpha/|\alpha|$ in Eq. (3) was corrected for a background of 12% for the sample comprising Group 1, and 20% for the sample comprising Group 2. A likelihood function of the form,

$$L = \left\{ \prod_i^{\text{all}+} [1 + P_+(\gamma, \alpha/|\alpha|)A_i \cos\phi_{Si}] \prod_j^{\text{all}-} [1 + P_-(\gamma, \alpha/|\alpha|)A_j \cos\phi_{Sj}] \right\},$$

was computed separately for Group 1 (L_1) and for Group 2 (L_2). Here, A_i is the analyzing power of carbon and ϕ_{Si} is the angle between the scattering plane and the K-π^+ plane for the ith event. The sign convention is such that the scattering shown in Fig. 1 has $\cos\phi_S > 0$. The maximum value of L as a function of γ and $\alpha/|\alpha|$ corresponds to the most likely configuration of γ and $\alpha/|\alpha|$. The likelihood function L_1 for Group 1 predicts that α_Λ is negative; on the other hand, L_1 is relatively insensitive to γ_Λ. The likelihood function L_2 for Group 2 also predicts that α_Λ is negative, but in addition predicts that γ_Λ is positive. Because L_1 and L_2 were found to be consistent, the product $\mathcal{L} = L_1 L_2$ is used to represent the results [Fig. 2(a)]. The likelihood that our observed sample arose from a negative α can be seen to be approximately 10^3 times as high as if α were positive. If the conclusion $\alpha < 0$ is accepted, then the most likely value of $\sigma_\Lambda = |P|^2/(|S|^2 + |P|^2) = [1 - \gamma_\Lambda]/2$ is 0.13, corresponding to $\alpha_\Lambda = -0.67$.

The results of the calculations by Dalitz and Liu[3] of the branching ratio,

$$R(\sigma) = \frac{{}_\Lambda H^4 \rightarrow \pi^- + He^4}{{}_\Lambda H^4 \rightarrow \pi^- \text{ (all modes)}},$$

for the two cases $J({}_\Lambda H^4) = 0$ and $= 1$ are shown in Fig. 2(b). The shaded strip shows the limits

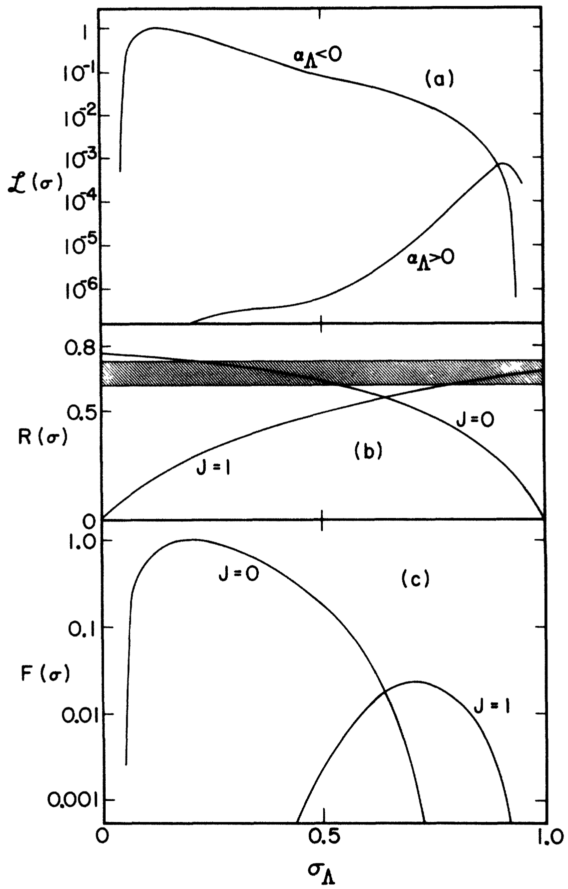

FIG. 2. (a) The likelihood function $\mathcal{L}$ of σ_Λ $\{=[1-\gamma_\Lambda]/2 \approx [1 \pm (1-\alpha_\Lambda{}^2)^{1/2}]/2\}$ and of the sign of α_Λ. (b) The ratio $R(\sigma) = [{}_\Lambda H^4 \to \pi^- + He^4]/[{}_\Lambda H^4 \to \pi^-$ (all modes)] as a function of σ_Λ. The two curves $J=0$ and $J=1$ are those calculated by Dalitz and Liu (reference 3). The horizontal strip shows the result by Ammar et al. (reference 2), $R=0.67^{+0.06}_{-0.05}$. (c) The function $F=L_A\mathcal{L}$ for $\alpha_\Lambda<0$. The two cases correspond to $J=0$ and $J=1$.

(± one standard deviation) on R determined in the emulsion experiment by Ammar et al.[2] (They found $R=\frac{46}{63}$.) A consistent way of combining our results with those of Ammar et al. is to construct the likelihood function $L_A(R[\sigma])$ for their results [using the curves $J=0$ or $=1$ in Fig. 2(b)], and then to use the product function $F(\sigma)=L_A\mathcal{L}$ (for $\alpha<0$). This is plotted in Fig. 2(c) for the two cases $J=0$ and $J=1$. The ratio of the peak values of F is 43/1, strongly favoring $J=0$.

We may summarize our conclusion on Λ-decay parameters as follows:

1. Our results combined with those of Ammar et al.[2] indicate, on the basis of the calculations by Dalitz and Liu,[3] that the spin of ${}_\Lambda H^4$ is 0. If this is so, then the occurrence of the reaction[1] $K^- + He^4 \to {}_\Lambda H^4 + \pi^0$ proves that the $K\Lambda N$ parity is odd, provided only that the ${}_\Lambda H^4$ is produced in its ground state. This argument is independent of the initial angular momentum state of the production reaction.

2. The negative sign of α_Λ, based on results from this experiment, from Birge and Fowler[7] and from Leitner et al.,[8] is now established. It follows[9] that $\alpha<0$ also for the decay mode $\Lambda \to n + \pi^0$. The magnitude of α_Λ, however, is still not well known. Our experiment gives $\alpha_\Lambda = -0.67^{+0.18}_{-0.24}$, and $\gamma_\Lambda = +(1-\alpha_\Lambda{}^2)^{1/2} = +0.74^{+0.13}_{-0.32}$. [The errors correspond to the $1/e$ points on the likelihood function $\mathcal{L}$ in Fig. 2(a).[10] Note that $\mathcal{L}$ falls off more slowly than a Gaussian distribution in the direction of small γ_Λ.] This is in agreement with the latest published result by Crawford et al.,[11] namely $|\alpha_\Lambda| > 0.66 \pm 0.13$.

Regarding the decay $\Sigma^+ \to p + \pi^0$, the result $\alpha_0 = 0.75 \pm 0.17$ was reported in I on the basis of an approximate (one-parameter) likelihood analysis. A more extensive and rigorous analysis of the polarization data using a three-parameter likelihood function, which takes into account the dependence of the polarization on β_0 and γ_0 [Eq. (1)], predicts $\alpha_0 = +0.73^{+0.16}_{-0.11}$, and $-0.3 < \alpha\bar{p} < 0.3$, where the limits on α_0 and on $\alpha\bar{p}$ correspond to the $1/e$ points on the likelihood surface. An uncertainty of 5% in the analyzing power A has been included in the errors; all other uncertainties are believed to be negligible. It should be noticed that the inclusion of the extra parameters in the maximum-likelihood estimation modifies slightly the value of α_0 given in I and, in particular, leads to somewhat smaller errors. Because $\bar{p}$ was small, in agreement with the measurements by the Yale group,[12] very little of significance can be inferred about β_0 and γ_0.

A more restricted result is obtained if time-reversal invariance and the $|\Delta\vec{T}| = \frac{1}{2}$ rule are assumed. Then, using the known π-N phase shifts, one can show that the quantity $\tan\psi = \beta_0/\alpha_0$ must have either of the values -0.22 or +0.04. Likelihood contours for the two cases are plotted against $\alpha\bar{p}$ and $|S_0/P_0|$ in Fig. 3 (for $\alpha_0 > 0$). The four maxima give the same value for α_0 within 1%, namely

$$\alpha_0 = +0.78^{+0.11}_{-0.13} \text{ (for } \tan\psi = -0.22, +0.04).$$

The limits given by the errors correspond to the extreme values on the $1/e$ curves.

An important consequence of the small value of the Σ-hyperon polarization found in this experi-

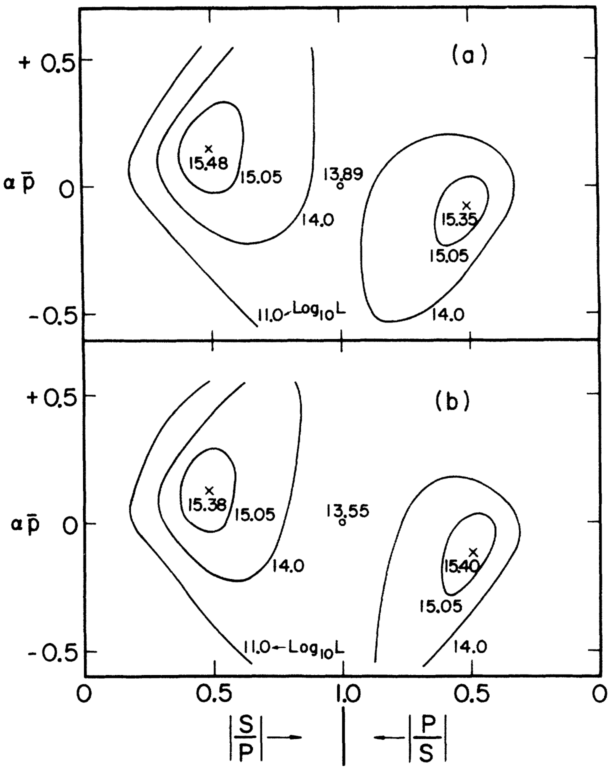

FIG. 3. Common logarithm of the likelihood as a function of $\alpha\bar{p}$ and $|S|/|P|$ for $\Sigma^+ \to p + \pi^0$ (for $\alpha_0 > 0$) for (a) $\tan\psi = -0.22$, (b) $\tan\psi = +0.04$.

ment and in the Yale experiment[12] should be noted. Both experiments involved the reaction $\pi^+ + p \to \Sigma^+ + K^+$ at a pion momentum $p_\pi = 1.2$ Bev/c. It has often been assumed that the small asymmetry ($\alpha\bar{p} = 0.01 \pm 0.17$) found in the decay of Σ^- hyperons produced in the charge-symmetric reaction[13] $\pi^- + n \to \Sigma^- + K^0$ at $p_\pi = 1.2$ Bev/c is evidence for $\alpha_- \approx 0$. This conclusion, which is valid only if $\bar{p}$ is large, tacitly assumes that the high polarization found[9] (for Σ^+) at $p_\pi = 1.13$ Bev/c persists at 1.2 Bev/c. The new polarization results therefore weaken the arguments for $\alpha_- \approx 0$ and make it highly desirable to measure $\alpha_-\bar{p}$ at $p_\pi = 1.13$ Bev/c. For the moment the principal evidence for $\alpha_- \approx 0$ rests on the assumption of the $|\Delta\vec{T}| = \frac{1}{2}$ rule and time-reversal invariance.[9]

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[1]M. M. Block, E. B. Brucker, C. C. Chang, R. Gessaroli, T. Kikuchi, A. Kovacs, C. M. Meltzer, A. Pevsner, P. Schlein, R. Strand, H. O. Cohn, E. M. Harth, J. Leitner, L. Monari, L. Lendinara, and G. Puppi, in Proceedings of the 1960 Annual International Conference on High-Energy Physics at Rochester (Interscience Publishers, Inc., New York, 1960), p. 419.

[2]R. G. Ammar, R. Levi Setti, W. E. Slater, S. Limentani, P. E. Schlein, and P. H. Steinberg, Nuovo cimento 19, 20 (1961).

[3]R. H. Dalitz and L. Liu, Phys. Rev. 116, 1312 (1950).

[4]E. F. Beall, Bruce Cork, D. Keefe, P. G. Murphy, and W. A. Wenzel, Phys. Rev. Letters 7, 285 (1961).

[5]Details of this experiment are described by E. F. Beall in his thesis, Lawrence Radiation Report UCRL-9919, 1961 (unpublished).

[6]See, for example, R. H. Dalitz, Societa Italiana di Fisica, Rendiconti della Scuola Internazionale di Fisica "Enrico Fermi," Corso XI, p. 299 (unpublished).

[7]R. W. Birge and W. B. Fowler, Phys. Rev. Letters 5, 254 (1960).

[8]J. Leitner, L. Gray, E. Harth, S. Lichtman, M. Block, B. Brucker, A. Engler, R. Gessaroli, A. Kovacs, T. Kikuchi, C. Meltzer, H. O. Cohn, W. Bugg, A. Pevsner, P. Schlein, M. Meer, N. T. Grinellini, L. Lendinara, L. Monari, and G. Puppi, Phys. Rev. Letters 7, 264 (1961).

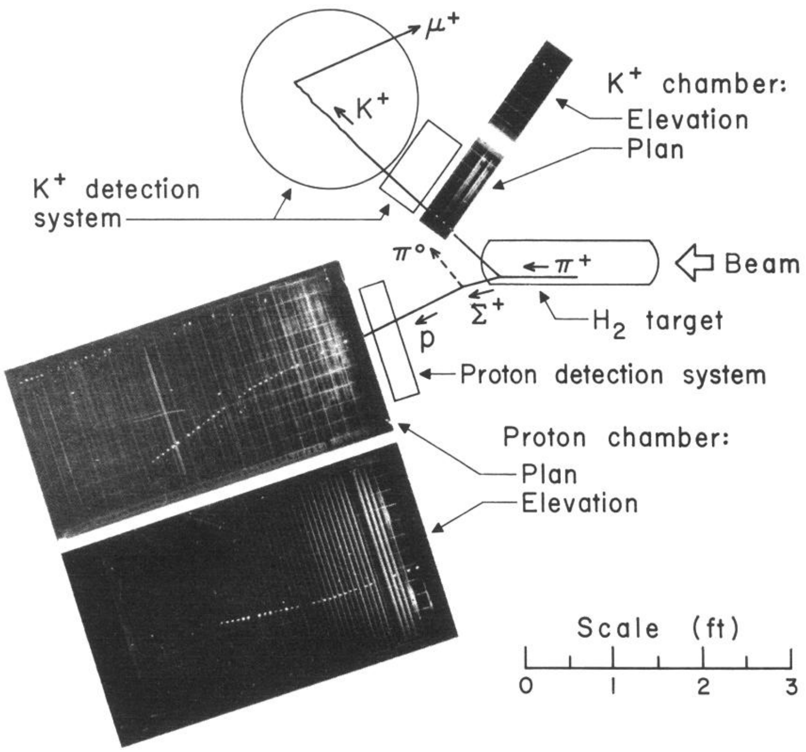
[9]B. Cork, L. T. Kerth, W. A. Wenzel, J. W. Cronin, and R. L. Cool, Phys. Rev. 120, 1000 (1960).

[10]The measure of uncertainty in a maximum-likelihood estimate is often defined differently by different authors. For a discussion of this point see, for example, M. Annis, W. Cheston, and H. Primakoff, Revs. Modern Phys. 25, 818 (1953). These authors suggest an alternative measure [see their Eq. (47)].

[11]F. S. Crawford, Jr., M. Cresti, M. L. Good, F. T. Solmitz, and M. L. Stevenson, Phys. Rev. Letters 2, 11 (1959).

[12]C. Baltay, H. Courant, W. J. Fickinger, E. C. Fowler, H. L. Kraybill, J. Sandweiss, J. R. Sanford, D. L. Stonehill, and H. D. Taft, Revs. Modern Phys. 33, 374 (1961).

[13]P. Franzini, A. Garfinkel, J. Keren, A. Michelini, R. Plano, A. Prodell, M. Schwartz, J. Steinberger, and S. E. Wolf, Bull. Am. Phys. Soc. 5, 224 (1960); also M. Schwartz, Columbia University (private communication).



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