ward the bisectrix from the binary axis, respectively. If we take into account that our experimental values may always correspond to the lower one of two shear modes indicated with a solid curve in the figure, the absolute values of the velocity and the general shape of both curves are in fairly good agreement. Some discrepancies might come from the misorientation of crystal axes, misalignment of crystal mounting in the magnetic field, and the complex many-ellipsoidal energy structure of bismuth. On the other hand, this anisotropic band structure may allow the strong coupling of the electrons with the shear mode. The experimental fact that the sharpness of the kink and the differential resistance beyond the kink field vary with the crystal orientation may indicate the directional dependency of the strength of the coupling between the electrons and the phonons. It is also conceivable that our new method may give a slightly higher value than the actual sound velocity.

In some crystal direction, for instance, around 100 degrees in Fig. 4, we can see the occurrence of a sinusoidal electrical oscillation of frequency $\sim 10^6$ cycles/sec when the applied field exceeds the kink field, whose frequency f does not depend on the external circuit but is determined by means of the simple formula, $f = s/d$, where d and s are the width of the bismuth specimen and the sound velocity in the direction perpendicular to the crossed electric and magnetic fields. This oscillation phenomenon may indicate an acoustic standing wave built up of frequencies that resonate corresponding to the size of the specimen; in other words, the generation of coherent phonons.

Moreover, we may have to think about the electron-hole recombination and generation velocities, v_r and v_g, on the bismuth surface, if these are not much larger than the velocity $v_x \sim 10^5$ cm/sec, which may give an accumulation of electrons and holes on the surface, because the recombination and generation processes in bulk may be fairly slow at low temperature. No positive evidence has been observed so far on this surface accumulation.

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TRANSITION RADIATION FROM METAL FILMS

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Because of the Coulomb force between charges, electrons in metals can participate in oscillations of a collective type, the so-called plasma oscillations. Both theory and experiment have shown that these plasma oscillations can be excited by high-energy charged particles passing through the metal. More recently it has been shown experimentally[1] that in thin metal films electrons can excite plasma oscillations which in turn emit a peak of electromagnetic radiation around the plasma frequency. This radiation from thin foils had first been predicted theoretically by Ferrell.[2] Subsequently an alternative theoretical treatment of this radiation was presented[3] which connected it with Russian work on transition radiation[4,5] and was more exact than the original theory of Ferrell. In view of the appearance of a recent paper[6] and the possibility that it may give the impression that Ferrell's mechanism for the peak in radiation differs from the interpretation of transition radiation, it was felt desirable to publish more details of the previous work[3] which emphasized that Ferrell's calculation and that of transition radiation give equivalent results for the peak in radiation. This is discussed further near the end of this paper.

It was first pointed out by Frank and Ginsburg[4] that a charged particle will emit electromagnetic radiation when passing through a boundary separating two different media even though the particle is moving at a constant velocity. The change in the electromagnetic fields surrounding the charged particle as it makes the transition from one medium to another with a different dielectric constant is the cause of the radiation, hence the name transition radiation.

In the more general case of two or more plane parallel boundaries separating two or more different media, several alternative methods have been presented for calculating the transition radiation.[5,7,8]

Although an expression for transition radiation from a slab has already been published,[5] there is apparently a typographical error in this result, and it appears worthwhile to outline the method used here[7] and restate the result.

The problem to be solved is the radiation from a particle of charge e and constant velocity v passing normally through one or more plane boundaries separating two or more different media. It will be assumed that the media in their interaction with electromagnetic fields can be represented by a frequency-dependent dielectric constant ϵ. This should be a satisfactory approximation for photons of energy less than 100 ev. Choosing the z direction along the particle path, the current produced by the particle is

$$\vec{j}(x, y, z, t) = \vec{k} e v \delta(x)\delta(y)\delta(z - vt), \qquad (1)$$

where $\vec{k}$ is a unit vector in the positive z direction. All quantities (field strengths, currents, etc.) are time Fourier-analyzed. The wave equation for the magnetic field is given by

$$\nabla^2 \vec{H}_\omega + \epsilon k_0{}^2 \vec{H}_\omega = -(4\pi/c)\nabla \times \vec{j}_\omega, \qquad (2)$$

where $k_0{}^2 = (\omega/c)^2$, and the subscript ω denotes the time Fourier component which varies as $e^{-i\omega t}$. Our problem is reduced to a scalar one by introducing a scalar function W_ω as follows:

$$\vec{H}_\omega = \vec{\nabla} \times \vec{k} W_\omega. \qquad (3)$$

Equation (2) can be satisfied if W_ω satisfies

$$\nabla^2 W_\omega + \epsilon k_0{}^2 W_\omega = -(4\pi/c) j_\omega. \qquad (4)$$

We can determine E_ω from Maxwell's equations, giving

$$\vec{E}_\omega = i k_0 [\vec{k} W_\omega + (\epsilon k_0{}^2)^{-1} (\vec{\nabla} \cdot \vec{k}) \vec{\nabla} W_\omega]. \qquad (5)$$

Thus a knowledge of W_ω from Eq. (4) permits a calculation of all of the electromagnetic fields.

In particular, the total energy radiated from the particle can be calculated by integrating Poynting's vector over the two planes $x = \pm x_0$, where x_0 is a constant. In terms of W_ω, the total radiated energy U becomes

$$U = 2\,\mathrm{Im}\int (c^2/\epsilon\omega)(\partial^2/\partial z^2 + \epsilon k_0{}^2) W_\omega (\partial W_\omega{}^*/\partial x)\, dz\, dy\, d\omega, \qquad (6)$$

where Im means imaginary part, $W_\omega{}^*$ is the complex conjugate of W_ω, and the integration with respect to z and y is over the $x = +x_0$ plane. By symmetry the integration over the $x = -x_0$ plane is the same as over the $x = x_0$ plane, giving a factor of 2 in Eq. (6). To facilitate the solution for W_ω, we use a method similar to the Weizsäcker-Williams treatment of scattering and Fourier analyze $W_\omega(x, y, z)$ with respect to x and y, obtaining in Eq. (4) for the Fourier component $W_K(z)$ an ordinary differential equation in z which can be solved by standard techniques.

It is found that the solutions for $W_K(z)$ can be divided into two parts, one part which varies as $\exp[i(\omega/v)z]$ and represents fields that move along in phase with the particle, and another part which varies as $\exp(-Kz)$ for $z > 0$ and represents fields which radiate from the boundaries. Denoting the coefficients of these two exponential terms by $\xi(K)$ and $\eta(K)$, respectively, it can be shown[7] that the expression for the transition radiation at frequency ω per unit frequency interval becomes

$$T_\omega = 4\pi^2 \omega^4 c^{-3} \epsilon_0{}^{3/2} \int \sin^2\theta \cos^2\theta\, |\eta(K)|^2 d\Omega. \qquad (7)$$

It is assumed that the dielectric constant in the region of observation ϵ_0 is real and positive so that the radiation can be transmitted to the point of observation. The angle θ is measured from the z axis and $d\Omega = \sin\theta d\theta d\phi$, ϕ being the angle measured in the x, y plane from the x direction. It is seen from (7) that the transition radiation per unit solid angle is

$$dT_\omega/d\Omega = 4\pi^2 \omega^4 c^{-3} \epsilon_0{}^{3/2} \sin^2\theta \cos^2\theta\, |\eta(K)|^2. \qquad (8)$$

For the case of interest, a slab of thickness τ and dielectric constant ϵ oriented perpendicular to the charged particle's path and surrounded on both sides by vacuum, $\eta(K)$ has the form

$$\frac{4\pi c^2}{e}\eta(K) = \frac{h}{K(K - i\alpha)} - \frac{1}{K(K + i\alpha)} + \frac{B e^{-i\alpha\tau}}{K(K + i\alpha)} + \frac{(1 - f)}{K'(1 + h'f)}\left[\frac{1 - e^{-(K' + i\alpha)\tau}}{K' + i\alpha} - \frac{h'(1 - e^{(K' - i\alpha)\tau})}{K' - i\alpha}\right], \qquad (9)$$

where

$\alpha = \omega/v$,

$h = (l^2 - 1)(1 - e^{-2K'\tau})[(l-1)^2 e^{-2K'\tau} - (l+1)^2]^{-1}$,

$l = K'/K\epsilon$,

$K' = k_0(\sin^2\theta - \epsilon)^{1/2}$; $\mathrm{Re}K' > 0$; $\mathrm{Im}K' < 0$,

$k_0 = \omega/c$,

$K = -ik_0 \cos\theta$,

$f = (1-l)(1+l)^{-1}$,

$h' = -fe^{-2K'\tau}$,

$B = 2e^{-K'\tau} l(1+h)(l+1)^{-1}(1+h')^{-1}$.

The expression for the transition radiation is given in Eq. (8), where $\eta(K)$ is given in Eq. (9). The resulting expression is very cumbersome and not very illuminating. In order to make the result more tractable, the nonrelativistic limit is taken and $|\epsilon|$ is assumed not very large. In this limit $|K'|$ and $|K| \ll \alpha$. Equation (9) becomes

$$\frac{4\pi c^2 \eta(K)}{e} = \frac{2(1-\epsilon)[(1-l)e^{-K'\tau} - (1+l)e^{K'\tau} + 2le^{-i\alpha\tau}]}{k_0\cos\theta\ \epsilon\alpha[(l-1)^2 e^{-K'\tau} - (l+1)^2 e^{K'\tau}]}. \tag{9'}$$

If in addition it is assumed that $|\epsilon| \ll 1$ and $\sin^2\theta \gg |\epsilon|$, the expression for the transition radiation becomes

$$\frac{dT_\omega}{d\Omega} = \frac{e^2 v^2 \sin^2\theta}{\pi^2 c^3}\left[\frac{(e^x \sinh x - e^x \cos\alpha\tau + 1)^2 + e^{2x}\sin^2\alpha\tau}{(2\epsilon_2 + e^x \sinh x \tan\theta)^2 + 4\epsilon_1^2}\right], \tag{10}$$

where $x = k_0\tau \sin\theta$, and ϵ_1 and ϵ_2 are the real and imaginary parts of ϵ. The expression in Eq. (10) has a sharp maximum at $\epsilon_1 = 0$ if $(2\epsilon_2 + e^{x_p}\sinh x_p \times \tan\theta)^2 \ll 1$, with a half-width of

$$\Delta\omega = (e^{x_p}\sinh x_p \tan\theta + 2\epsilon_2)[(d\epsilon_1/d\omega)_p]^{-1}, \tag{11}$$

where all quantities are evaluated at the plasma frequency ω_p where $\epsilon_1 = 0$ and $x_p = (\omega_p\tau/c)\sin\theta$. This frequency is just the bulk plasma frequency of the metal. Thus we expect to find a peak in the transition radiation at the plasma frequency for metal slabs satisfying the condition that $(2\epsilon_2 + e^{x_p}\sinh x_p \tan\theta)^2 \ll 1$. Except for the situation where $\tan\theta \approx 0$, this will only occur when x_p and ϵ_2 are much less than one or for thin, nonabsorbing films.

The total intensity in the peak per unit solid angle can be obtained by integrating (10) over frequency and obtaining

$$\frac{dT}{d\Omega} = \frac{e^2 v^2 \cos\theta \sin\theta}{2\pi c^3 (d\epsilon_1/d\omega)_p}\left[\frac{(e^{x_p}\sinh x_p - e^{x_p}\cos\alpha\tau + 1)^2 + e^{2x_p}\sin^2\alpha\tau}{e^{x_p}\sinh x_p + 2\epsilon_2/\tan\theta}\right], \tag{12}$$

where again all quantities are evaluated at the plasma frequency.

An interesting special case is that of a thin $(x_p \ll 1)$ ideal metal slab of free electrons whose dielectric constant $\epsilon_1 = 1 - (\omega_p/\omega)^2$ and $\epsilon_2 = 0$. In that case the half-width becomes, from (11),

$$(\Delta\omega/\omega_p) = (\omega_p\tau/2c)\sin\theta\tan\theta, \tag{11'}$$

and the total intensity in the peak per unit solid angle becomes, from (12),

$$\frac{dT}{d\Omega} = \frac{e^2 v\omega_p \cos\theta}{2\pi c^2}\,\frac{\sin^2(\omega_p\tau/2v)}{(\omega_p\tau/2v)}. \tag{12'}$$

The results in (11′) and (12′) agree with those of

Ferrell.[2]

In conclusion, it has been shown that Ferrell's[2] calculation of a peak of radiation at the plasma frequency from thin films can also be obtained from a calculation of transition radiation, and in fact they are two different ways to consider the same phenomenon. Since the transition radiation calculation includes all radiation from the film, it is more general. Ferrell's method only calculates the peak though it does so correctly and shows the physical mechanism causing the peak. Reference 6 also points out that the peak is predicted by transition radiation but then implies that Ferrell's mechanism is the wrong interpretation and plasma effects do not cause the peak in radiation. This apparently occurs because of a misinterpretation of Ferrell's result. The plasmon referred to by Ferrell is not the same as that referred to by reference 6. Ferrell's plasmon is not a bulk one with a charge distribution in the interior, as reference 6 seems to assume, but is solely a surface effect, the charges existing only on the surface.

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LIFETIME OF THE EXCITED F CENTER*

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Because of the strong absorption of the F band in alkali halide crystals, it has generally been assumed that the radiative lifetime of the excited state would be of the order of that for an allowed atomic transition, i.e., about 10^{-8} sec.[1] It is the purpose of this Letter to report direct measurements of this lifetime, showing that it is of the order of 10^{-6} sec. In addition, it will be shown how measurements of photoconductivity, luminescence, and lifetime may be used to obtain interesting information about the excited states of the F center.

The F band in a crystal such as KCl has been interpreted as a transition which carries the electron (occupying a negative-ion vacancy) from the ground state to a bound excited state lying about 0.2 ev below the bottom of the conduction band. At temperatures above about 125°K in the case of KCl most electrons so excited will be thermally ejected into the conduction band before they can return to the ground state by radiation. Assuming that the probability per second of thermal ionization may be expressed by the relation $1/\tau_i=(1/\tau_0)\times\exp(-\Delta E/kT)$, while the probability of radiative return to the ground state, $1/\tau_R$, is temperature independent, one has for the total probability $1/\tau=1/\tau_R+(1/\tau_0)\exp(-\Delta E/kT)$, so the expression for the mean lifetime of the state as a function of temperature is given by

$$\tau=\tau_R/[1+(\tau_R/\tau_0)e^{-\Delta E/kT}]. \tag{1}$$

In this equation, τ_R is the radiative lifetime, ΔE the activation energy for ionization, and $1/\tau_0$ the escape frequency.

Using the above relationships, one can write expressions for the fraction of excited F centers which decay by luminescence[1]:

$$\eta_L=1/[1+(\tau_R/\tau_0)e^{-\Delta E/kT}], \tag{2}$$

or by thermal ionization:

$$\eta_i=1/[1+(\tau_0/\tau_R)e^{+\Delta E/kT}]. \tag{3}$$

In this simple model $\eta_L+\eta_i=1$. Only one excited state is considered, and other processes, such as nonradiative recombination with the ground state,