TWO-MAGNON PROCESSES IN FERROMAGNETIC RELAXATION[*]

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The coupling between two magnons induced by local disturbances in the lattice such as pits left on the surface of the sample by the polishing process, porosity, polycrystalline grains, etc., causes the annihilation of one magnon and creation of another magnon of the same energy. The question of relaxation by these two-magnon processes is important in every type of ferromagnetic relaxation experiment of current interest: usual resonance with the rf field perpendicular to the dc field, Suhl main resonance breakdown, Suhl subsidiary resonance absorption, parallel pumping, Suhl suppression of the main resonance breakdown Morgenthaler's proposed method[1] of exciting magnons of arbitrary angle θ_k between wave vector $\vec{k}$ and applied field, etc. Recently reported experimental results by LeCraw and co-workers[2-6] at Bell Telephone Laboratories indicate that the two-magnon process induced by small pits left on the surface of a spherical sample of yttrium iron garnet (YIG) by the polishing process has very little effect on the instability threshold in parallel pumping[1] experiments. This is in contrast to the Sparks, Loudon, and Kittel pit scattering theory,[7] which predicts that the two-magnon relaxation frequency is often much larger than all other relaxation frequencies affecting the parallel pumping threshold.

The previous theory,[1] which assumes no scattering between degenerate magnons, predicts that the parallel pumping threshold is controlled by the relaxation frequency of the particular magnon which happens to go unstable first. We propose that the two-magnon scattering is usually so strong that all magnons having the same energy are coupled together so tightly that they must be considered as a unit in any process involving degenerate magnons. This is demonstrated most clearly by the parallel pumping experiment for the case in which every degenerate S magnon relaxes much more rapidly by the two-magnon pit processes than by other processes, including three-magnon and two-magnon–one-phonon processes, so that the occupation numbers of all degenerate S magnons are very nearly equal. Using the Sparks, Loudon, and Kittel pit model for two-magnon scattering [Eq. (31) of reference 7], we estimate that the S-S relaxation frequencies are sufficiently rapid to maintain the occupation numbers n equal when the potentially unstable magnons have frequencies well above the extrapolation of the $\theta_k=\pi/2$ curve of ω vs k to $k=0$.

The reason for the rapidity of the two-magnon relaxation process is simply that the coupling between two magnons induced by surface pits and volume porosity is extremely strong when the wave vectors of the two magnons differ by no more than ~(1/pit diameter), which has typically small values of 10^3 to 10^4 cm^{-1}. This strong, short-range coupling would give rise to very large uniform precession linewidths if it were not for the small density of S states [$\rho(k)\sim k^2$ for small k] at the low wave vectors $k\approx 10^3$-10^4 cm^{-1} into which the uniform precession scatters. On the other hand, the density of states is infinite for magnons having $\theta_k=\pi/2$. Hence these $\pi/2$ magnons, which would normally be the first to go unstable in the absence of two-magnon processes, relax extremely rapidly by the two-magnon process into neighboring degenerate magnons before they have time to relax by other processes, such as three-magnon processes, magnon-phonon processes, etc. The two-magnon process is indeed so rapid that it stops only after the occupation numbers of the neighboring magnons are equal to the occupation number of the $\pi/2$ magnons.

In the case of equal occupation numbers of all S magnons, instability occurs at a critical field h_c at which the rate of energy flow, const$\times hn\int dk\,\rho(k)\sin^2\theta$, into all of the S magnons from the parallel field, is equal to the net rate of dissipation, $(n-\bar{n})\times\int dk\,\rho(k)2\eta_k$, of energy from all of the S magnons, giving

$$n=\bar{n}/[1-(h/h_c)], \tag{1}$$

where

$$h_c=\frac{\omega_{\text{pump}}}{4\pi\gamma M}\frac{\langle 2\eta_k\rangle}{\langle\sin^2\theta_k\rangle};$$

h is the rf parallel field, n is the occupation number of the S magnons, $\bar{n}$ is the thermal equilibrium value of n, $\rho(k)$ is the density of S states, $2\eta_k$ is the relaxation frequency of the occupation number (or energy) of magnon $\vec{k}$, not including the two-magnon process, and $\langle x\rangle$ denotes average over degenerate S magnons, $\int dk\,\rho(k)x(k)/\int dk\,\rho(k)$. The energy equilibrium method of calculating the critical field was introduced by Kittel.[8] He found that when two-magnon processes are neglected, (1) is valid for each magnon $\vec{k}$ if the averages over S magnons in (1) are replaced by the values of $2\eta_k$ and $\sin^2\theta_k$ for the magnon of interest. This result was obtained by other means in reference 1. The present theory requires reinterpretation of the relaxation frequencies previously obtained[2-4] from parallel pumping experiments and applies to all the resonance experiments mentioned above.

When the pump frequency in the parallel pump experiments is sufficiently low so that none of the pumped magnons have $\theta_k = \pi/2$, the analysis is more complicated because the density of states near the potentially unstable (neglecting two-magnon processes) magnons is low. The exact wave number of these magnons is unknown; the small value of $\sim 10^4$ cm^{-1} is not unreasonable. It is therefore not unlikely that the two-magnon linewidth of these potentially unstable magnons will no longer be many orders of magnitude larger than the two-magnon linewidths of the uniform precession. Thus, in certain cases it may be possible to observe a dependence of critical field on surface roughness. It should be emphasized that the effect of surface roughness should show up first in intrinsically bad, but well-polished, samples, such as samples containing rare earth impurities. For when the direct relaxation of the potentially unstable magnon by, say, rare earth impurity becomes comparable with the two-magnon relaxation rate, the occupation numbers of neighboring magnons will no longer be equal to those of the potentially unstable magnons. The degenerate magnons are then in an intermediate state between complete coupling and no coupling of the degenerate magnons.

In occupation number transition probability calculations the question of phase effects always must be considered separately. If phase is preserved in the two-magnon scattering process no problem arises. If phase is completely destroyed in the two-magnon process, then one half of the scattered magnons will have the proper phase to accept energy from the rf field as assumed in deriving (1). The other half will be out of phase with the rf field and will be "pumped down" by the field. Thus, all the magnons that survive will have the proper phase. This is analogous to the original excitation of thermal magnons by parallel pumping where only the thermal magnons of the proper phase are excited by the field.

The two-magnon processes induced by arrangement disorder of magnetic ions on the different sites in spinel ferrites,[9] if this is indeed important,[10] are basically different from surface pit-induced two-magnon processes. The two-magnon processes induced by the small scattering centers in the spinel ferrites couple a given magnon to all degenerate magnons, not just to near neighbors in $\vec{k}$ space. The linewidths of magnons having $\theta_k = \pi/2$ are then of the same order of magnitude as the uniform precession linewidth. In a forthcoming paper[11] these ideas will be developed in greater detail and extended to other cases.

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[5] Particularly striking evidence of this is the report by R. C. LeCraw (private communication) that the critical field in these parallel pumping experiments is quite insensitive (~2-3 % change for best sample) to changes in the grit size used in polishing the sample; the corresponding change in linewidth can be as large as two orders of magnitude.

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[9] R. T. Denton and E. G. Spencer, Phoenix Magnetism Conference, 1961 (unpublished). This reference reports a difference in the parallel pumping threshold between the ordered and disordered phases of cobalt ferrite.

[10] C. Kittel, Proceedings of the International Conference on Magnetism and Crystallography, Kyoto, 1961 (unpublished).

[11] M. Sparks (to be published).