DECAY MODES OF THE η MESON*

Laurie M. Brown and Paul Singer
Northwestern University, Evanston, Illinois
(Received April 16, 1962)

Recent evidence has indicated that the 550-MeV, $T=0$ three-pion resonance[1] may be a pseudoscalar particle of positive G parity.[2,3] Assuming the correctness of these assignments (0^{-+}), decays via the strong interactions alone are essentially forbidden since the first strongly allowed final state, that of four pions, is energetically suppressed. We consider the following electromagnetically permitted decay final states:

$$\text{(a) } \pi^+ + \pi + \gamma, \quad \text{(b) } 3\pi, \quad \text{(c) } 2\gamma,$$

$$\text{(d) } (\pi^0) + e^+ + e^- \text{ or } (\pi^0) + \mu^+ + \mu^-,$$

and find their partial rates to be consistent with the rather meager experimental evidence now available, providing that process (b) is enhanced by a strong final-state interaction.

The 550-MeV resonance, called the η meson (and by some authors χ), was identified originally by its decay into $\pi^+ + \pi^- + \pi^0$ which has a rate proportional to α^2 ($\alpha = 1/137$) if η is a 0^{-+} state. The final state (c) occurs in the same electromagnetic order, while (a) is of order α. As we will show later, (d) is actually of order α^4. It would thus appear at first sight that (a) should be the predominant final state, while phase-space considerations indicate that (c) should predominate over (b) by a factor of several hundred. This would be in striking disagreement with the experimental indications[2] that

$$\Gamma_\eta(\pi^+\pi^-\gamma)/\Gamma_\eta \text{ (all modes)} < \tfrac{1}{20}$$

and that[4]

$$\Gamma_\eta \text{ (all neutral modes)}/\Gamma_\eta(\pi^+\pi^-\pi^0) \approx 3.$$

However, the presence of strong pion-pion interaction can completely change the above theoretical estimates.

Consider first the decay of η into three pions. The simplest allowed final state has all the pions in relative S states. Let us assume now that there is a strong interaction for two pions in a $T=0$, $J=0$ state. Or as an alternative simple model, let us assume the existence of a "particle"[5] having the spin and parity 0^+, and call it σ. Then the 3π decay of the η would proceed in two distinct steps: $\eta \to \sigma + \pi^0$, $\sigma \to \pi^+ + \pi^-$ or $2\pi^0$, where the first step is an electromagnetic decay, while the second occurs through a strong interaction. This mechanism would have a definite effect on the Dalitz plot of the η. For example, if σ were on its mass shell and had a zero width, one would expect a monoenergetic π^0. The Dalitz plot of the Berkeley group[2] does indeed show a marked asymmetry, most of the π^0's having momenta between 65 and 115 MeV/c. This fits a σ "particle" having a mass of about 370 MeV and a full width of about 50 MeV. If the decay is dominated by this kind of process, the ratio $\Gamma_\eta(3\pi^0)/\Gamma_\eta(\pi^+\pi^-\pi^0)$ is practically given by the decay ratio $\Gamma_\sigma(2\pi^0)/\Gamma_\sigma(\pi^+\pi^-)$ which has the value $\frac{1}{2}$.

For a crude estimate of the ratio $\Gamma_\eta(2\gamma)/\Gamma_\eta(3\pi)$, we note that the matrix elements for $\eta \to \sigma + \pi^0$ and $\eta \to 2\gamma$ are of equal electromagnetic order α, and using the fact that a 370-MeV σ "particle" is practically on its mass shell in this process, we calculate the ratio of the relevant two-body phase space. Thus we get

$$\frac{\Gamma_\eta(2\gamma)}{\Gamma_\eta(3\pi)} \approx \frac{\pi m_\eta^2/2}{(\pi/2m_\eta^4)[m_\eta^4 - (m_\sigma^2 - m_\pi^2)^2][(m_\eta^2 - m_\pi^2 - m_\sigma^2)^2 - 4m_\sigma^2 m_\pi^2]^{1/2}} = 3.3. \tag{1}$$

This would give a ratio of Γ_η (all neutrals)/$\Gamma_\eta(\pi^+\pi^-\pi^0) \approx 5$, which agrees roughly with experiment.

To compare processes (a) and (c), we use the "ρ-dominant model" of Gell-Mann, Sharp, and Wagner.[6] That is, we assume an electromagnetic vertex ("black box") in which $\eta \to \rho^0 + \gamma$, with a virtual ρ^0 which goes to a γ ($V_{\rho\gamma}$) for process (c) or alternatively to a charged pion pair ($V_{\rho\pi\pi}$) for process (a). We use the following gauge and Lorentz invariant expressions for the vertices:

$$\text{"black box,"}\quad K\epsilon_{\alpha\beta\gamma\delta}\epsilon_\alpha^{(\gamma)}k_\beta^{(\gamma)}p_\gamma^{(\eta)}\epsilon_\delta^{(\rho)}; \qquad (2)$$

$$V_{\rho\gamma},\qquad (em_\rho^{\ 2}/f_\rho)\epsilon_\mu^{(\rho)}\epsilon_\mu^{(\gamma)}; \qquad (3)$$

$$V_{\rho\pi\pi},\qquad f_{\rho\pi\pi}\epsilon_\mu^{(\rho)}(p_\mu^{(+)}-p_\mu^{(-)}). \qquad (4)$$

The p's are the momenta of the designated particles, the ϵ's are the polarization vectors, and $f_{\rho\pi\pi}$, f_ρ, and K are effective coupling constants. The widths of (a) and (c) come out to be

$$\Gamma_\eta(\pi^+\pi^-\gamma)=\frac{K^2(f_{\rho\pi\pi}^{\ 2}/4\pi)m_\pi^{\ 5}m_\eta^{\ 2}}{3\pi^2m_\rho^{\ 4}}\times 0.4, \qquad (5)$$

$$\Gamma_\eta(2\gamma)=\frac{K^2m_\eta^{\ 3}\alpha}{16\pi(f_\rho^{\ 2}/4\pi)}. \qquad (6)$$

In our notation, $f_{\rho\pi\pi}^{\ 2}/4\pi\simeq 2$, as determined from a ρ^0 width of 100 MeV, and $f_{\rho\pi\pi}\approx f_\rho$, assuming the ρ resonance dominates the isovector form factor of the nucleon.[6] For the constant K^2 we estimate on dimensional grounds a magnitude of $\alpha/m_\rho^{\ 2}$, again assuming that the relevant intermediate mass is that of the ρ meson. The resulting partial widths, computed from (5), (6), and (1) are

$$\Gamma_\eta(2\gamma)=160\ \text{eV};\quad \Gamma_\eta(\pi^+\pi^-\gamma)=20\ \text{eV};$$

$$\Gamma(3\pi)=50\ \text{eV}. \qquad (7)$$

Evidently the ratio of the partial widths for processes (a) and (c) is independent of the effective constant K^2, and thus agrees with the ratio previously given in reference 6. By an obvious analogy we can calculate the 2γ-decay width of the π^0 meson from formula (6) using the same value for K^2, since the relevant intermediate state is $\rho+\omega$. (Note that $m_\rho\approx m_\omega$.) One obtains $\Gamma_{\pi^0}(2\gamma)=2.6$ eV, in good agreement, fortuitously, with experiment.

Regarding final states (d) containing a lepton pair or a lepton pair and one or more neutral pions, the essential feature is that the charge conjugation number $C=1$, both for η and for π^0. Thus the lepton pair must have $C=1$, cannot be coupled to a single internal photon, and must be a process of order α^4. In fact, it is further suppressed[7] compared to $\eta\to 2\gamma$ by the factor $(m_{e,\,\mu}/m_\eta)^2$.

In summary, we conclude that $\Gamma_\eta(3\pi)$ may be enhanced by the presence of a strong two-pion interaction ("ABC particle"?) to dominance over $\Gamma_\eta(\pi^+\pi^-\gamma)$ and to a magnitude comparable to $\Gamma_\eta(2\gamma)$. This would imply, assuming other final-state interactions are less important, $\Gamma_\eta(3\pi^0)\simeq\frac{1}{2}\Gamma_\eta(\pi^+\pi^-\pi^0)$, so that the 2γ final state predominates among the neutral decay modes. The total width of the η is estimated to be about 250 eV, giving a lifetime of $\sim 3\times 10^{-18}$ sec.

---

*Research supported by the National Science Foundation.

[1] A. Pevsner et al., Phys. Rev. Letters 7, 421 (1961).

[2] P. Bastien et al., Phys. Rev. Letters 8, 114 (1962); 8, 302(E) (1962).

[3] L. M. Brown and P. Singer, Phys. Rev. Letters 8, 155 (1962).

[4] Note that $(3\pi^0/\pi^+\pi^-\pi^0)\leq\frac{3}{2}$ and the $\gamma+n\pi^0$ modes are forbidden by charge conjugation invariance.

[5] Evidence for strong two-pion interaction in the $T=0$ state with spin and parity 0^+ has been reported several times in the recent literature, the "mass" of the system lying between 300 and 400 MeV. See, for example, N. E. Booth, A. Abashian, and K. M. Crowe, Phys. Rev. Letters 7, 35 (1961); Yu. K. Akimov et al., Nuclear Phys. 30, 258 (1962); J. Button et al., Phys. Rev. (to be published); J. Schwartz, J. King, and R. Tripp, Bull. Am. Phys. Soc. 7, 282 (1962).

[6] M. Gell-Mann, D. Sharp, and W. G. Wagner, Phys. Rev. Letters 8, 261 (1962). We wish to thank these authors for pointing out a numerical error in a preliminary calculation.

[7] S. M. Berman and D. A. Geffen, Nuovo cimento 18, 1192 (1960).