line from the remaining spectrum. The relative width of the line at half intensity is $\Delta k/k \simeq 0.3$.

This fact is very important in single-pion photoproduction measurements, because the line falls in the neighborhood of the first resonance.

We also performed measurements with a collimator acceptance of 0.8 mrad; the line height is decreased by a small amount, but the beam intensity is increased to $\sim 7\times 10^9$ equivalent quanta/minute. As far as the intensity of the beam is concerned, it should be kept in mind that a peaked spectrum is equivalent to a normal bremsstrahlung spectrum which has the same magnitude as the peak at the energy of the peak.

Simultaneously with this work we have calculated the polarization of the photon lines.[10] By making use of the lattice of Fig. 1(a) we find for the line at $x = x_1$ in Fig. 1(b) a polarization $P = (I_\perp - I_\parallel)/(I_\perp + I_\parallel) = 33\,\%$, where $I_\perp$ and $I_\parallel$ are the bremsstrahlung intensity for photons polarized perpendicular and parallel to the plane $(\vec{p}_1, \vec{b}_1)$, respectively.

In the previous Letter[1] we gave results concerning first approximation calculations for a spectrum with $E_1 = 6$ GeV and a line at $k = 1$ GeV; the sum appearing in (4) was extended over the points of Fig. 1(a) enclosed in the circles only.

If the sum is extended over all the points, a line at $k = 1$ GeV, having a peak value equal to the preceding one, is obtained for $\theta = 0.88$ mrad and $\vec{p}_1$ in the plane of the axes (110), ($1\bar{1}0$).

The corresponding calculated value of the polarization is 34 %.

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S-WAVE $\bar{K}$-N SCATTERING AMPLITUDES

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The recent measurements of Tripp, Watson, and Ferro-Luzzi have provided a great deal of information concerning the S-wave $\bar{K}$-N scattering and absorption interactions in the range of lab-system K-momentum 300-500 MeV/c.[1] These authors state that the data are consistent with a smooth extrapolation of the first set of $\bar{K}$-N scattering amplitudes proposed recently by Humphrey and Ross[2] (we denote this set by HR-I). The purpose of this note is to point out that the data of Tripp, Watson, and Ferro-Luzzi are not consistent with the HR-I amplitudes, but rather suggest that the real part of the isotopic spin 0 scattering amplitude is appreciably negative at the $\bar{K}$-N threshold. Such a large negative $\mathrm{Re}A_0$ is consistent with the predictions of Schult and Capps[3,4] and with the assumption that the 1405-MeV Y_0^* is an S-wave resonance of the Dalitz-Tuan type.[5]

We denote the relative phase of the S-wave isotopic spin 0 and 1 $\bar{K}+N \rightarrow \pi + \Sigma$ amplitudes by $\phi_\gamma = \phi_0 - \phi_1$. The experimental data on the ratios of $\Sigma^+ + \pi^-$, $\Sigma^- + \pi^+$, and $\Sigma^0 + \pi^0$ states produced in K^--p collisions indicate that ϕ_γ is approximately equal to ±60°, ±90°, and ±110° at $\bar{K}$-p threshold, 175 MeV/c, and 400 MeV/c, respectively.[6,1] In order to explain the interference of the S-wave amplitudes with the 395-MeV/c $D_{3/2}$ resonance, Tripp, Watson, and Ferro-Luzzi are forced to assume that ϕ_γ is negative at this energy. Since no violent fluctuations in the Σ^-/Σ^+ ratio are observed between 175 MeV/c and 400 MeV/c, ϕ_γ must be negative below 175 MeV/c as well. However, the HR-I set is characterized by a positive value of this phase difference.[2]

We use the indices t and t' to refer to the $K^- + p$

and $\bar{K}^0+n$ threshold energies, respectively. Similarly, $\hbar k$ and $\hbar k'$ denote the magnitudes of the particle momenta in the center-of-mass system in the K^-+p and $\bar{K}^0+n$ states, respectively. In the zero-range model, the relative phase is given by

$$\phi_r = \arg\left(\frac{1}{A_0^{-1}-ik'}\right) - \arg\left(\frac{1}{A_1^{-1}-ik'}\right) + \phi_c, \quad (1)$$

where ϕ_c is an energy-independent constant, and the Aj denote the complex scattering lengths, i.e., $A_j = a_j + ib_j$.[7] At the energy of the $K^- - p$ threshold, k' is imaginary and is given approximately by $i0.3\ \text{F}^{-1}$. The experimental fact that at $K^- - p$ threshold the absorption is predominantly in the $I=0$ state leads to the condition, $\phi_r(t') - \phi_r(t) > 0$. Hence, if ϕ_r is negative, the experimental data imply

$$\phi_r(175\ \text{MeV}/c) - \phi_r(t') \lesssim -30°. \quad (2)$$

It is seen from Eq. (1) that this condition requires either a large positive a_1, a large negative a_0, or both.

Schult and Capps predicted the existence of an $I=0$ resonance below the $K^- - p$ threshold from the assumption that the large difference in the $\Sigma^- + \pi^+ / \Sigma^+ + \pi^-$ ratio, observed when stopped K^- are absorbed in hydrogen and in deuterium, results from the dependence of the production amplitudes on $\Sigma + \pi$ energy.[3] Evidence for such a Y_0^* has been found by Alston et al.,[8] but this resonance has not been established definitely, and no determination of spin or parity has been made. Recently, Schult and Capps showed that their hypothesis requires that ϕ_r be negative, and that $a_0 \lesssim -1.3$ F.[4] Hence, the new evidence concerning ϕ_r supports this hypothesis.

The second set of amplitudes proposed by Humphrey and Ross[2] (HR-II) is characterized by a negative ϕ_r and a negative a_0, i.e.,

$$A_0 \approx -0.6+i, \quad A_1 \approx 1.2+0.6i, \quad \phi_r(t) \approx -62°.$$

However, if this set is extrapolated to the momentum range 240-360 MeV/c, it is in contradiction with the K_2^0+p data.[9] The inconsistency occurs because of the large positive value of a_1 in HR-II. If a_1 is chosen smaller, however, an even greater negative value of a_0 is required by the condition, Eq. (2).

If one assumes a small positive effective range in the $I=0$ state, it is possible to construct a set of amplitudes that fits all the data reasonably well. In order to illustrate this, we list below a set of amplitudes chosen somewhat arbitrarily; no systematic attempt has been made to minimize the deviation from experiment or to find all possible solutions. We write

$$A_j^{-1} = A_j^{-1}(t) + \tfrac{1}{2} r_j k^2, \quad (3)$$

and choose the parameters

$$A_0(t) = (-1.3+0.9i)\ \text{F}, \quad r_0 = 0.62\ \text{F},$$
$$A_1(t) = (0.4+0.6i)\ \text{F}, \quad r_1 = 0,$$
$$\phi_r(t) = -60°, \quad \epsilon = 0.5, \quad (4)$$

where ϵ is the ratio of the $\pi+\Lambda$ production cross section to the total $I=1$, $\pi+Y$ production cross section. In Table I the predictions resulting from these amplitudes are compared with the 0-240

Table I. Cross sections and branching ratios for low-energy $K^- - p$ and $K_2^0 - p$ processes calculated from Eq. (4), compared with the predictions of HR-I and HR-II and with experiment. The $(I=0/I=1)$ ratio R is given by $R = 3\Sigma^0/(\Sigma^- + \Sigma^+ + \Lambda - 2\Sigma^0)$. The Σ^-/Σ^+ ratios are obtained by setting $\phi_r(t) = \pm 60°$ and $\epsilon = 0.5$. The last column refers to K_2^0+p reactions; the calculation of this branching ratio involves both $\bar{K}+N$ amplitudes, and is explained by Luers et al.[a] The values in parentheses are obtained if the effective range r_0 of Eq. (4) is set equal to zero.

	$K^- - p$ at rest		$K^- - p$ at 175 MeV/c				$K_1^0/(\Lambda + 2\Sigma^0)$
	R	Σ^-/Σ^+	σ_{el} (mb)	$\sigma(\bar{K}^0 n)$ (mb)	$\sigma(\Sigma^\mp \pi^\pm)$ (mb)	Σ^-/Σ^+	at 240 MeV/c
HR-I	4.8	2.1	61	17	44	0.96	0.19
HR-II	4.2	2.1	60	18	41	1.2	1.2
Eq. (4)	4.2	2.1	60 (54)	16 (17)	43 (40)	1.2 (1.4)	0.48
Experiment	4 ~ 8	~2.2	70 ~ 90	11 ~ 19	33 ~ 49	0.7 ~ 2	0.4 ~ 0.9

[a] See reference 9.

MeV/c, $K^- + p$ and $K_2{}^0 + p$ data, and with the predictions of the HR amplitudes.[7] The real part of A_1 has been chosen smaller than that of HR-II in order not to contradict the 240-360 MeV/c $K_2{}^0 + p$ data.[9] The amplitudes of Eq. (4) are also in rough agreement with the 400-MeV/c data of reference 1. (The value of A_0 at 400 MeV/c is very close to that of HR-I.) Of course, there is no reason to believe that the effective range corrections to A_1 and to ϕ_C [see Eq. (1)] are negligible; however, a small effective range correction to A_0 makes an appreciable effect because of the small value of $A_0{}^{-1}$.

We emphasize again that the amplitudes of Eq. (4) are quite arbitrary. The essential point of this note is that the requirement that ϕ_γ be negative, together with the assumption that the effective ranges are not large compared to 0.5 F, requires an appreciably negative value of $a_0(t)$ in order to fit the low-energy $K^- + p$ data and the $K_2{}^0 + p$ data.

Since the key to this argument is the statement that the 400-MeV/c data[1] requires ϕ_γ to be ≈-110° rather than ≈110°, we shall summarize briefly the features of the data that lead to this conclusion. We define $T(\Sigma^\mp + \pi^\pm) = \frac{1}{2}(T_0 \pm T_1)$, and $T(\Sigma^0 + \pi^0) = \frac{1}{2}T_0$.[10] A value $\phi_\gamma = \pm 110°$, together with the observed $|T_1/T_0|$ ratio, leads to the phase differences between charge states, $\phi(\Sigma^+) - \phi(\Sigma^0) = \phi(\Sigma^0) - \phi(\Sigma^-) \approx \pm 60°$, where the upper signs are to be taken together. The rapid variations in angular distributior and polarization near 400 MeV/c result from interference of the S-wave amplitudes with a resonating $D_{3/2}$ amplitude in the $I = 0$ state. The angular distribution and polarization data imply that this resonant amplitude is in phase with the S-wave $\Sigma^+ + \pi^-$ amplitude at a momentum of about 400 MeV/c. The assumption $\phi(\Sigma^+) - \phi(\Sigma^0) = \phi(\Sigma^0) - \phi(\Sigma^-) \approx -60°$ then leads to the conclusions that the polar-equatorial ratio for the $\Sigma^- + \pi^+$ events should be negative somewhat below 400 MeV/c and large and positive above this energy, and the corresponding ratio for the $\Sigma^0 + \pi^0$ events should reach its maximum somewhat above 400 MeV/c. Both these predictions are in excellent agreement with the data. On the other hand, the assumption $\phi(\Sigma^+) - \phi(\Sigma^0) = \phi(\Sigma^0) - \phi(\Sigma^-) \approx 60°$ leads to predicted polar-equatorial ratios for the $\Sigma^- + \pi^+$ and $\Sigma^0 + \pi^0$ events that are essentially "reflections" through 400 MeV/c of the observed ratios.

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