# EXPERIMENTAL EVIDENCE FOR A QUASI-MONOCHROMATIC BREMSSTRAHLUNG INTENSITY FROM THE FRASCATI 1-GeV ELECTRONSYNCHROTRON 

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In a previous Letter ${ }^{1}$ we reported results of calculations concerning the possibility of obtaining photon "lines" by allowing a high-energy electron beam to strike a single crystal.

In this Letter we give the experimental results concerning $150-$ and $300-\mathrm{MeV}$ photon "lines" obtained by the use of a diamond single crystal at room temperature in the vacuum chamber of the Frascati 1-GeV electronsynchrotron; the experimental results are also compared with the theoretical calculations.

We used approximately the same experimental arrangement described in other papers. ${ }^{2,3}$

The single crystal is a diamond having the shape of a $(10 \times 5 \times 2) \mathrm{mm}^{3}$ parallelopiped, the widest face of which has the Miller indexes 110; it is mounted in a remotely controlled goniometer placed in a straight section of the synchrotron. Rotation is possible around horizontal and vertical axes, both perpendicular to the $\gamma$-ray beam direction; the crystal axis ( $1 \overline{1} 0$ ) is placed along the former direction and the axis (001) along the latter, within $\pm 2^{\circ}$.

The goniometer was much improved with respect to the preceding one. ${ }^{3}$ It has a sensitivity of 0.1 mrad ; the angles are measured with a relative systematic error of $\pm 0.1 \%$. The electrons strike the diamond at small angles $\theta$ with the axis (110).

The collimation of the $\gamma$-ray beam was improved by adding two further collimators in order to reduce the background at low energy. The angle of acceptance was lowered to 0.3 mrad , in order to select photons emitted from electrons whose angle with the $\gamma$-ray beam axis, due to multiple scattering, is of the same order of magnitude. In this way, although the angle $\theta$ is well defined, the $\gamma$-ray beam is approximately integrated over the emission angle of the photons with respect to the electrons. The $\gamma$-ray beam intensity after collimation, as monitored by a Wilson ${ }^{4}$ quantameter, is $\sim 10^{9}$ equivalent quanta/minute.

The $\gamma$-ray beam strikes an aluminum converter $1.1 \times 10^{-2}$ radiation length in thickness; electron pairs are analyzed by means of a pair spectrometer. ${ }^{5}$
Symmetrical electron pairs of a definite energy
are detected by a system of scintillation counters similar to the one already used ${ }^{2}$; the energy spread of the photons accepted from the detection system is $\Delta k / k=8.5 \%$; the error in determining the central energy of the photons is ${ }^{5} \delta k / k= \pm 0.3 \%$.

We measure the number $N(k, \theta)$ of symmetrical pairs accepted by the detectors per fixed number of monitor units, as a function of the nominal energy $k$ of the photons, at a fixed value of $\theta$, subtracting for delayed coincidences ( $10 \%$ of prompt coincidences). Background is negligible (always of the order of $0.1 \%$ ).

We define experimental normalized bremsstrahlung intensity by the relation

$$
\begin{equation*}
J_{\mathrm{ex}}(k, \theta)=\frac{N(k, \theta) \sigma_{p}\left(k_{0}\right)}{N\left(k_{0}, \theta\right) \sigma_{p}(k)} f(k), \tag{1}
\end{equation*}
$$

where $k_{0} \approx 900 \mathrm{MeV}$ is the photon energy at which normalization is performed; with the monitor units fixed in the measurements ( $5 \times 10^{8}$ equivalent quanta), we obtain $N\left(k_{0}, \theta\right) \approx 2 \times 10^{3}$ for each $\theta$. In (1), $\sigma_{p}(k)$ is the symmetrical pair production cross section in aluminum, viz., the sum of the contribution due to nuclei ${ }^{6}$ and electrons. ${ }^{7} f(k)$ is a correction factor which takes into account the scintillator vertical counting losses, due to the electron multiple scattering in the converter. $f(k)$ was measured with high accuracy by compar ison with measurements performed with thin converters where there are no counting losses down to 50 MeV .

The experimental quantity (1) must be compared with the calculated quantity

$$
\begin{equation*}
J_{t h}(x, \theta)=I(x, \theta) / I\left(x_{0}, \theta\right) \tag{2}
\end{equation*}
$$

where $x=k / E_{1}$ is the fractional energy of the photons with respect to the energy $E_{1}=1 \mathrm{GeV}$ of the electrons and $x_{0}=k_{0} / E_{1} . I(x, \theta)$ is the bremsstrahlung intensity due only to the nuclei; it is the only contribution we consider, as we do not know yet the coherence effect of the electrons. $I(x, \theta)$ is given by the relation

$$
\begin{align*}
I(x, \theta)=[1 & +(1-x)]^{2}\left[\psi_{1}{ }^{c}(\delta)+\psi_{1}{ }^{0 *}(\theta, \delta)\right] \\
& -\frac{2}{3}(1-x)\left[{\psi_{2}}^{c}(\delta)+{\psi_{2}}^{0 *}(\theta, \delta)\right] \tag{3}
\end{align*}
$$

with the same notations as in Überall, ${ }^{8}$ where $\delta$ $=\left(m c^{2} / 2 E_{1}\right)(x / 1-x)$ is the minimum momentum transferred to the nucleus in units of $m c$ ( $m c^{2}$ is the electron rest mass).

The functions $\psi_{1,2}{ }^{0 *}(\theta, \delta)$ appearing in (3) correspond to Überall's functions $\psi_{1,2}{ }^{0}(\theta, \delta)$, and were calculated by us without his approximation of considering the reciprocal lattice planes as continuous. We have used the actual structure of the lattice.

These functions represent the contribution of the reciprocal lattice plane passing through the origin; the contribution of the other planes is less important and we have neglected it.

The functions $\psi_{1,2}{ }^{0 *}(\theta, \delta)$ were calculated assuming the primary electron momentum $\overrightarrow{\mathrm{p}}_{1}$ in the plane of two perpendicular reciprocal lattice axes $\overrightarrow{\mathrm{b}}_{1}, \overrightarrow{\mathrm{~b}}_{2}$, as in our experimental situation. We obtained:

$$
\begin{gather*}
\psi_{1}^{0 *}(\theta, \delta)=\frac{(2 \pi)^{2}}{\Delta} 4 \delta \sum_{\overrightarrow{\mathrm{g}}}|F|^{2} \frac{\exp -A g^{2}}{\left(\beta^{-2}+g^{2}\right)^{2}} \frac{g^{2}}{g_{2}^{2} \theta^{2}}, \\
\psi_{2}{ }^{0 *}(\theta, \delta)=\frac{(2 \pi)^{2}}{\Delta} 24 \delta^{2} \sum_{\overrightarrow{\mathrm{g}}}|F|^{2} \frac{\exp -A g^{2}}{\left(\beta^{-2}+g^{2}\right)^{2}} \frac{g^{2}}{g_{2}^{4} \theta^{4}}\left(g_{2} \theta-\delta\right) \tag{4}
\end{gather*}
$$

The symbols are the same as in Überall ${ }^{8}$; the sum is extended over the reciprocal lattice vectors $\overrightarrow{\mathrm{g}}$ lying in the $\overrightarrow{\mathrm{b}}_{2}, \overrightarrow{\mathrm{~b}}_{3}$ plane and satisfying the relation $g_{2}=\mathrm{g} \cdot \overrightarrow{\mathrm{b}}_{2} \geqslant \delta / \theta$.
$|F|$ is the Laue-Bragg structure factor. ${ }^{9}$ The structure of the reciprocal lattice plane, perpendicular to the axis (110) and passing through the origin, is given in Fig. 1(a); we get $|F|^{2}=8$ for the points enclosed in the circles and $|F|^{2}=4$ for the others.
The numerical values of (4) were calculated, using the IBM 1620 computer, from the following data: $A=126 ; a x_{c}=3.56 \AA .\left(a x_{c}\right.$ is the edge of the fundamental cube in units of the electron Compton wavelength $\lambda_{c}$.)
In the measurements we report $\vec{p}_{1}$ lies in the plane of the axes $\vec{b}_{1} \equiv(110)$ and $\vec{b}_{2} \equiv(1 \overline{1} 0)$; i.e., the diamond is rotated about the vertical axis $\overrightarrow{\mathrm{b}}_{3} \equiv(001)$.
If we assume the recoil momenta of the nuclei applied to the origin, their extremities belong to a region whose intersection with the lattice plane is the dashed region in Fig. 1(a); its boundary line is at a distance $\delta / \theta$ from the origin. The curve of Fig. 1(b) represents, as a function of $x$, the function (2) calculated in the preceding conditions for $\theta=\theta_{1}$ $=4.6 \mathrm{mrad}$; this angle is chosen in such a way that

FIG. 1. (a) Structure of the diamond reciprocal lattice plane through the origin and perpendicular to the axis (110); $a / 2 \pi$ is the edge of the fundamental cube in units of the Compton wavelength of the electron. For the points enclosed in the circles $|F|^{2}=8$; for the other points $|F|^{2}=4$ ( $F$ is the Laue-Bragg structure factor). (b) Normalized bremsstrahlung intensity of $E_{1}=1 \mathrm{GeV}$ electrons in a diamond single crystal at room temperature versus the fractional energy $x=k / E_{1}$ of the photon; the momentum $\vec{p}_{1}$ of the primary electron makes the angle $\theta$ with the crystal axis $\vec{b}_{1} \equiv(110)$ and lies in
 the plane of the axes $\vec{b}_{1}$ and $\overrightarrow{\mathrm{b}}_{2} \equiv(1 \overline{1} 0)$ 。 The solid curve represents the calculated quantity (2) and the dots represent the measured quantity (1), for $\theta=\theta_{1}=4.6 \pm 0.1 \mathrm{mrad}$. The statistical error is indicated for some points.
the boundary of the dashed region is in the position of Fig. 1(a) for $x=x_{1}=0.15$, where the curve of Fig. 1 (b) has the first discontinuity. If $x$ is increased, the boundary line moves to the right; we get the second discontinuity of the spectrum when the boundary reaches the dot-dash line, and similarly for the other peaks.

The solid curve of Fig. 2(a) represents formula (2) calculated for $\theta=\theta_{2}=11.3 \mathrm{mrad}$; this angle is chosen in such a way that the first discontinuity falls at $x=x_{2}=0.3$.
In Fig. 1(b) the experimental results defined by (1), for $\theta=\theta_{1}$, are also represented; in Fig. 2(a) the dots represent the data for $\theta=\theta_{2}$ and the triangles the data for $\theta=\theta_{3}=22.9 \mathrm{mrad}$; in this case the first peak falls at $x=x_{3}=0.465$. The corresponding theoretical curve is not drawn, in order
to avoid confusion.
The statistical error changes along the spectrum and is at most (for $x \approx 1$ ) $\Delta J_{\mathrm{ex}} / J_{\mathrm{ex}}= \pm 3 \%$; it is represented for some points in the figures.

The results are uncorrected for the energy spread of the photons and for the angular divergence of the primary electrons; nevertheless, the agreement between measurements and calculations is satisfactory, both for the energies of the discontinuities, and for the heights of the lines.
The measurement of Fig. 2(a) for $\theta=\theta_{3}$ was performed in order to obtain a spectrum equal as much as possible to the one measured for $\theta=\theta_{2}$, but without the peak at $x=x_{2}$. In Fig. 2(b) the difference $J\left(x, \theta_{2}\right)-J\left(x, \theta_{3}\right)$ is quoted; this result shows that one can actually discriminate, by a difference method, the contribution of a photon


FIG. 2. (a) The same as Fig。1(b). The solid curve and the dots are obtained for $\theta=\theta_{2}=11.3 \pm 0.1 \mathrm{mrad}$ and the triangles are obtained for $\theta=\theta_{3}=22.9 \pm 0.1$ mrad . (b) Difference of the bremsstrahlung intensities of Fig. 2(a)。
line from the remaining spectrum．The relative width of the line at half intensity is $\Delta k / k \simeq 0.3$ ．
This fact is very important in single－pion photo－ production measurements，because the line falls in the neighborhood of the first resonance．
We also performed measurements with a colli－ mator acceptance of 0.8 mrad ；the line height is decreased by a small amount，but the beam inten－ sity is increased to $\sim 7 \times 10^{9}$ equivalent quanta／ minute．As far as the intensity of the beam is concerned，it should be kept in mind that a peaked spectrum is equivalent to a normal bremsstrahlung spectrum which has the same magnitude as the peak at the energy of the peak．

Simultaneously with this work we have calculated the polarization of the photon lines．${ }^{10}$ By making use of the lattice of Fig．1（a）we find for the line at $x=x_{1}$ in Fig．1（b）a polarization $P=\left(I_{\perp}-I_{\|}\right) /$ $\left(I_{\perp}+I_{\|}\right)=33 \%$ ，where $I_{\perp}$ and $I_{\|}$are the brems－ strahlung intensity for photons polarized perpen－ dicular and parallel to the plane（ $\vec{p}_{1}, \vec{b}_{1}$ ），respec－ tively．
In the previous Letter ${ }^{1}$ we gave results concern－ ing first approximation calculations for a spec－ trum with $E_{1}=6 \mathrm{GeV}$ and a line at $k=1 \mathrm{GeV}$ ；the sum appearing in（4）was extended over the points of Fig．1（a）enclosed in the circles only．

If the sum is extended over all the points，a line at $k=1 \mathrm{GeV}$ ，having a peak value equal to the pre－ ceding one，is obtained for $\theta=0.88 \mathrm{mrad}$ and $\overrightarrow{\mathrm{p}}_{1}$ in the plane of the axes（110），（1 $\overline{1} 0)$ ．

The corresponding calculated value of the polar－ ization is $34 \%$ ．

[^0]$S$－WAVE $\bar{K}-N$ SCATTERING AMPLITUDES

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The recent measurements of Tripp，Watson， and Ferro－Luzzi have provided a great deal of information concerning the $S$－wave $\bar{K}-N$ scatter－ ing and absorption interactions in the range of lab－system $K$－momentum $300-500 \mathrm{MeV} / c .{ }^{1}$ These authors state that the data are consistent with a smooth extrapolation of the first set of $\bar{K}-N$ scat－ tering amplitudes proposed recently by Humphrey and Ross ${ }^{2}$（we denote this set by HR－I）．The pur－ pose of this note is to point out that the data of Tripp，Watson，and Ferro－Luzzi are not consist－ ent with the HR－I amplitudes，but rather suggest that the real part of the isotopic spin 0 scattering amplitude is appreciably negative at the $\bar{K}-N$ threshold．Such a large negative $\operatorname{Re} A_{0}$ is consist－ ent with the predictions of Schult and Capps ${ }^{3,4}$ and with the assumption that the $1405-\mathrm{MeV} Y_{0}{ }^{*}$ is an
$S$－wave resonance of the Dalitz－Tuan type．${ }^{5}$
We denote the relative phase of the $S$－wave iso－ topic spin 0 and $1 \bar{K}+N \rightarrow \pi+\Sigma$ amplitudes by $\phi_{r}$ $=\phi_{0}-\phi_{1}$ ．The experimental data on the ratios of $\Sigma^{+}+\pi^{-}, \Sigma^{-}+\pi^{+}$，and $\Sigma^{0}+\pi^{0}$ states produced in $K^{-}-$ $p$ collisions indicate that $\phi_{r}$ is approximately equal to $\pm 60^{\circ}, \pm 90^{\circ}$ ，and $\pm 110^{\circ}$ at $\bar{K}-p$ threshold， $175 \mathrm{MeV} / c$ ， and $400 \mathrm{MeV} / c$ ，respectively．${ }^{6,1}$ In order to explain the interference of the $S$－wave amplitudes with the $395-\mathrm{MeV} / \mathrm{c} D_{3 / 2}$ resonance，Tripp，Watson，and Ferro－Luzzi are forced to assume that $\phi_{r}$ is neg－ ative at this energy．Since no violent fluctuations in the $\Sigma^{-} / \Sigma^{+}$ratio are observed between $175 \mathrm{MeV} /$ $c$ and $400 \mathrm{MeV} / c, \phi_{r}$ must be negative below 175 $\mathrm{MeV} / c$ as well．However，the HR－I set is charac－ terized by a positive value of this phase difference．${ }^{2}$

We use the indices $t$ and $t^{\prime}$ to refer to the $K^{-}+p$


[^0]:    ${ }^{1}$ G。Barbiellini，G。Bologna，G。Diambrini，and G．P。 Murtas，Phys。Rev。Letters 8， 112 （1962）。
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    ${ }^{9} \mathrm{~J}$ 。 M ．Bijvoet，N。H．Kolkmeyer，and C．H．Mac－ Gillavry，X－Ray Analysis of Crystals（Butterworths Scientific Publications，Ltd．，London，1951），pp． 64 and 239 。
    ${ }^{10}$ After the publication of our previous Letter（refer－ ence 1），the calculation of the bremsstrahlung intensity and of the polarization was performed independently also by Überall［H．Überall（private communication）］。

