$\Lambda\pi^-)/\Gamma(Y_1^{*-}\to\Sigma^0\pi^-+\Sigma^-\pi^0)\simeq 5$ while $\Gamma(Y_1^{*0}\to\Lambda+\pi^0)/\Gamma(Y_1^{*0}\to\Sigma^+\pi^-+\Sigma^-\pi^++\Sigma^0\pi^0)\simeq 1.3$ in disagreement with the requirements of charge independence. See also references 3 and 7. The present data are not sufficient to rule out $I=1$ for the resonant state. The absence of a strong effect in the $\Sigma^-\pi^0K^+$ and $\Sigma^0\pi^-K^+$ events could be due to destructive interference between the production amplitudes from the initial $I=\frac{1}{2}$ and $I=\frac{3}{2}$ states of the π^-+p system. If this is so, the resonant state must be produced in π^++p interactions in the same energy range. The absence of any effect would then ensure the assignment $I=0$.

[12]The possibility for resonances of this type was first pointed out by R. Dalitz and S. F. Tuan, Phys. Rev. Letters 2, 425 (1959). The model has been discussed further by R. Dalitz, Phys. Rev. Letters 6, 239 (1961); and Revs. Modern Phys. 33, 471 (1961).

[13]R. L. Shult and R. H. Capps, Nuovo cimento 13, 416 (1962), have emphasized the importance of the Y_0^* in the low-energy K^--d absorption reactions.

[14]The enhancement for an S-wave final state is proportional to $(\sin\delta e^{i\delta}/q)^2$. To obtain the correct energy dependence we used $\tan\delta=q[\gamma-\beta^2\kappa/(1+\kappa\alpha)]$. A satisfactory fit to the data is obtained with $\gamma=(248\text{ MeV})^{-1}$, $\alpha=(110\text{ MeV})^{-1}$, and $\beta=(175\text{ MeV})^{-1}$. For the $I=0$ KN system, these values yield a complex scattering length $a+ib=-1.23+i0.75$ F, in reasonable agreement with the (b-) solutions of reference 12, or the type-II solution of William E. Humphrey, Lawrence Radiation Laboratory Report UCRL-9752, 1961 (unpublished); and Ronald R. Ross, Lawrence Radiation Laboratory Report UCRL-9749, 1961 (unpublished).

[15]The peak is composed predominantly of $\Sigma^-\pi^+K^0$ events. Whether this is due to an interference with the K^* background or a nonresonant $I=1$ $\Sigma\pi$ amplitude cannot be determined from the present data.

[16]The narrow width reported in reference 2 has been interpreted as evidence for the existence of a centrifugal barrier in the resonant state of the $K\pi$ system. In addition, M. A. Baqi Bég and P. C. DeCelles [Phys. Rev. Letters 6, 145 (1961)] have favored the assignment $J=I$ by assuming a simple relation between the production cross section and decay width given in reference 2. In view of the width observed in this experiment these arguments are no longer compelling.

[17]M. Gell-Mann, in California Institute of Technology Synchrotron Laboratory Report CTSL-20, 1961 (unpublished) and Phys. Rev. 125, 1067 (1962), has discussed the possibility for the existence of such a particle with $I=\frac{1}{2}$, called the M meson. In this scheme the expected mass is given to lowest order by $m_M=\frac{1}{4}(3m_\omega+m_\rho)=780$ MeV.

[18]Y. Ne'emen, Nuclear Phys. 26, 222 (1961).

[19]The same calculation yields $\Gamma=42$ MeV for the K^* at 885 MeV, so that $J=1$ for this state cannot be ruled out by this argument.

EXPERIMENTAL STUDIES OF THE FORM FACTORS IN $K_{\mu 3}^+$ AND K_{e3}^+ DECAY*

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Three-body leptonic decays of K^+ mesons ($K_{\mu3}^+\to\mu^++\pi^0+\nu$, $K_{e3}^+\to e^++\pi^0+\nu$) provide a fruitful field for the study of strangeness-nonconserving weak interactions. Previous work in this area includes determinations of rates,[1] studies of pion energy spectra and angular correlations in K_{e3}^+ decay,[2] and investigations of the muon energy spectrum in $K_{\mu3}^+$ decay.[3-5] In this Letter we present the results of a detailed analysis of 76 $K_{\mu3}^+$ events observed in a 12-inch xenon bubble chamber.[6] In particular, we show that within the framework of the usual V-A Fermi interaction, the muon and electron are coupled identically in three-body leptonic K^+ decay. In addition, we place limits on the magnitudes and possible energy dependence of the "form factors" inherent in the decay process and compare these results to those recently obtained from studies of the $K_{\mu3}^+$-muon energy spectrum.[5]

A number of authors[7,8] have shown that under the assumption of vector coupling, the following distribution function holds for either K_{e3}^+ or $K_{\mu3}^+$ decay for the pion momentum and pion-neutrino angular correlation:

$$F(P,\theta)dPd\cos\theta=\frac{P^2(W^2-P^2-m_L^2)^2}{E\ (W+P\cos\theta)^4}\left[P^2\sin^2\theta f_V^2+\frac{m_L^2}{M_K^2}(M_Kf_V+(W+P\cos\theta)g_V)^2\right]dPd\cos\theta. \quad (1)$$

Here P is the pion momentum and E is the total pion energy; θ is the angle between the pion and the neutrino; M_K is the K mass; $W=M_K-E$; m_π is the pion mass; m_L is the lepton (μ or e) mass; and f_V, g_V are functions ("form factors") of $q^2=M_K^2+m_\pi^2-2M_KE$, the square of the invariant four-momentum transfer. We assume time-reversal invariance and hence take the form factors to be real. Experimental studies have confirmed the validity of Eq. (1) for the description of the K_{e3}^+ mode.[2] However, it should be noted that for this decay mode, the term containing g_V is practically zero, since the ratio of the electron mass to that of the K meson is very small; consequently, information concerning this term must come exclusively from the investigation of the $K_{\mu3}^+$ decay mode.

The $K_{\mu3}^+$ events under consideration are a specially selected group that fulfill the following requirements: (1) The K^+ decay occurs in a given fiducial volume; (2) the μ^+ stops in the chamber with a range greater than 1 cm; and (3) both γ rays from the π^0 convert into electron pairs within a given fiducial volume. These requirements insure that (a) $K_{\mu3}^+$ events are separated from the much more numerous $K_{\pi2}^+$ decays on the scanning table, and (b) the measurements of the muon energy and direction and gamma-ray directions provide enough information for a complete reconstruction of the event. This statement has one qualification: The reconstruction is double-valued, and the correct solution is chosen from a comparison between the predicted gamma-ray energies and the observed ionizing path lengths of the electron pairs. In general, the two solutions predict widely different energies for the gamma rays, and even crude experimental information on this point can define the correct solution.

The above criteria for selecting the $K_{\mu3}^+$ sample leave one important source of contamination, namely τ' ($\tau'\to\pi^++2\pi^0$) decays in which only two out of the four gamma rays convert in the chamber. Some of these events can be reconstructed as fake $K_{\mu3}^+$ decays with secondaries of range less than 8.2 cm (the maximum range of the π^+ in τ' decay). These τ' events cannot be reliably distinguished from $K_{\mu3}^+$ decays by observation of the characteristic π-μ-e decay chain, since the range of the μ from the π-μ decay is only 1.3 mm. To examine the effect of this contamination, we have put a large sample of τ' decays generated by a Monte Carlo procedure through the $K_{\mu3}^+$ reconstruction calculation. The result is that virtually all τ' events can be eliminated by suitably restricting the π^0-decay configurations accepted for events in the ambiguous (τ' or $K_{\mu3}^+$) category. We estimate that our final sample of 76 events contains about three τ''s.

For the purposes of our analysis it is convenient to rewrite Eq. (1) with new variables P and E_μ, where E_μ is the total muon energy:

$$F(P,E_\mu)dPdE_\mu$$
$$=\frac{2P}{E}\left\{f_V^2[-4E_\mu^2+4WE_\mu-(W^2-P^2-m_L^2)]+4f_Vg_V(W-E_\mu)\frac{m_L^2}{M}+\frac{m_L^2}{M^2}g_V^2(W^2-P^2-m_L^2)\right\}dPdE_\mu. \quad (2)$$

For each event, the $K_{\mu3}^+$ reconstruction calculation yields an appropriate P value with an average error of 8 MeV/c, and the E_μ value comes directly from the muon range measurement. For comparison with our observed (P,E_μ) distribution, the function $F(P,E_\mu)$ is multiplied by the known probability of observing in our chamber a stopping muon of whatever range corresponds to E_μ. We can roughly describe this probability by saying that it is high for kinetic energies up to 60 MeV, drops to 50% at 70 MeV, and is negligibly small above 100 MeV. It should be noted that this restriction only limits the range of probable observation of q^2/m_π^2 to $0.9\le q^2/m_\pi^2\le 6.8$ instead of the total range $0.6\le q^2/m_\pi^2\le 7.1$; i.e., the behavior of the form factors f_V and g_V can be studied over practically their whole range of variation.

To make our analysis definite, we have taken for f_V and g_V the first two terms of a series expansion in q^2:

$$f_V=A(1+\lambda q^2/m_\pi^2)\text{ for }|\lambda|\le 0.1, \quad (3a)$$

$$g_V=B(1+\lambda' q^2/m_\pi^2)\text{ for }|\lambda'|\le 0.1, \quad (3b)$$

where A, B, λ, and λ' are taken to be real constants. The above restrictions on $|\lambda|$ and $|\lambda'|$, which permit variations in f_V and g_V of nearly a factor of two over the range of q^2, seem reasonable because:

1. Our determinations of λ from K_{e3}^+ decay are fully compatible with these restrictions.

2. Most of the various models proposed for the calculation of f_V and g_V[9-11] give rise to forms which can very well be represented by the expansion in Eq. (3) with values of λ, λ' that never go beyond 0.07.

The parameters A, B, λ, λ' need not be the same for $K_{\mu3}^+$ and K_{e3}^+ decay if the muon and electron are not coupled identically. With sufficient data, one can obtain, in principle, A_e, λ_e from a study of K_{e3}^+ decay, and A_μ, λ_μ, B_μ, λ_μ' from work on $K_{\mu3}^+$ decay. As noted earlier, B_e and λ_e' are unobservable. We can then check universality by verifying the equality of A_e, λ_e with A_μ, λ_μ. Our $K_{\mu3}^+$ data are not adequate to carry out this program in full, and we are unable to determine λ_μ and λ_μ' with good accuracy. However, we still make a significant test of μ-e universality by taking $\lambda_\mu \equiv \lambda_e \equiv \lambda$ and permitting both λ and λ' to assume various values in the range given in Eq. (3). For each set λ, λ', we use our $K_{\mu3}^+$ and K_{e3}^+ data to determine A_e, A_μ, and B_μ. The ratio A_μ/A_e so obtained is nearly independent of λ, λ' and has the value 1.07 ± 0.18. In other words, if we assume the same pion energy dependence for f_V^e and f_V^μ, we find that their ratio agrees with unity within the 18% experimental error.

We now assume universality and combine all our results to obtain the best possible information on f_V and g_V. From the measured total K_{e3}^+ decay rate,[1] we calculate A as a function of λ, which we plot in Fig. 1 in the form of a band whose limits correspond to ± 1 standard deviation in the K_{e3}^+ rate. The ordinates in Fig. 1 are normalized so that when Eq. (1) is integrated over P and $\cos\theta$, one obtains the K_{e3}^+ rate in $\sec^{-1}$. Using the K_{e3}^+ pion momentum spectrum, we then determine the value of λ and obtain 0.036 ± 0.045.[12] The central value and the one-standard-deviation limits on λ are shown as vertical lines in Fig. 1.

We now take as exact the measured values of the K_{e3}^+ rate and λ (i.e., $A = 7.0 \times 10^{-2}$ MeV^{-2} sec$^{-1/2}$; $\lambda = 0.036$) and calculate B as a function of λ', using only the measured $K_{\mu3}^+$ rate. The result is double-valued and is plotted in Fig. 2 as two bands whose limits correspond to ± 1 standard deviation in the $K_{\mu3}^+$ rate. If, instead, we combine the above A, λ with only our (P, E_μ) distribution, we obtain the single, somewhat broader band also displayed in Fig. 2. It is clear that of the two solutions permitted by the rate data, only the upper one in Fig. 2 is consistent with the remainder of the information, the lower one being

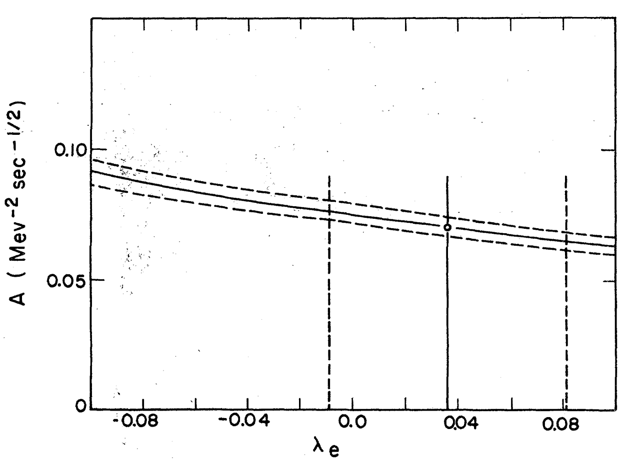

FIG. 1. Relation between A and λ. Horizontal solid and dashed curves are for nominal K_{e3}^+ rate and ± 1 standard deviation variation in rate. Vertical solid and dashed curves are for nominal λ and ± 1 standard deviation variation in λ, as determined experimentally from the K_{e3}^+ π^0 momentum spectrum.

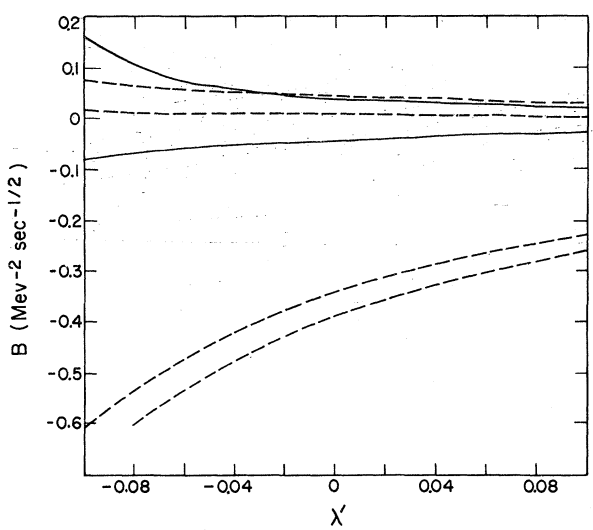

FIG. 2. Relation between B and λ' as determined from $K_{\mu3}^+$ data. Dashed curves outline region allowed by one standard deviation variation in $K_{\mu3}^+$ rate. Solid curves outline region allowed (at one standard deviation level) by $K_{\mu3}^+$ (P, E_μ) distribution.

ruled out with a confidence of better than 99.9%. This result is in disagreement with the conclusion of Dobbs et al.,[5] based on their study of the $K_{\mu 3}^+$ muon energy spectrum, that the lower solution in Fig. 2 is favored.[13]

If we combine all our $K_{\mu 3}^+$ information, our best estimate of B is 1.6×10^{-2} MeV^{-2} sec$^{-1/2}$. The uncertainties in this number arise from the following causes:

(a) the statistical errors in the $K_{\mu 3}^+$ data, including both the rate and the (P,E_μ) distribution, which contribute $\pm2.4\times10^{-2}$ MeV^{-2} sec$^{-1/2}$.

(b) the impossibility of determining from our data the value of λ'. The rather weak dependence of B on λ', shown by the central curve of Fig. 3, leads to an uncertainty in B which is small relative to the error discussed in (a).

(c) the errors in A and λ arising from the limited statistics of the K_{e3}^+ sample. The effects on B arising from variations of one standard deviation in the K_{e3}^+ rate and in λ are shown by the various dot-dash curves in Fig. 3.

From the analysis presented here, we conclude that:

1. Within the 18% experimental uncertainty, the muon and electron couplings in the three-body K^+ leptonic decays have the same strength.

2. For the ratio g_V/f_V, we have 0.4 ± 0.3 if the form factors have no q^2 dependence.[14] With the q^2 dependence assumed in this paper, this ratio is only slightly modified.

3. One can learn little about the q^2 dependence of g_V without very large statistics because, as implied by Conclusion (2) above, the form factor f_V determines the main observable features of $K_{\mu 3}^+$ decay.

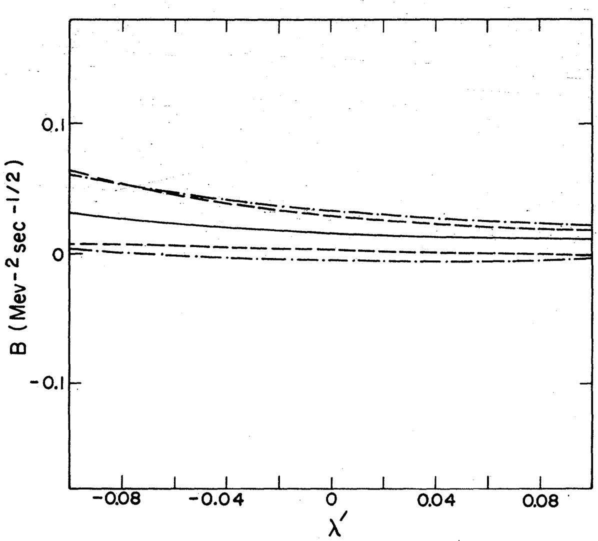

FIG. 3. Relation between B and λ' as determined from all the data. The solid curve represents the best value of B vs λ'. The dashed curve shows the effect of varying the K_{e3}^+ rate by one standard deviation; the dot-dash curve shows the effect of varying λ by one standard deviation.

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[11]N. Brene, L. Egardt, and B. Qvist, Nuclear Phys. 22, 553 (1961).

[12]This result is based on a sample of 217 K_{e3}^+ events, which includes 175 presented in reference 2, and is analyzed in a similar, though slightly more sophisticated, way.

[13]It is worth noting that the conclusions of Dobbs et al. are based on the μ^+ energy spectrum, whereas ours come principally from the π^0 energy spectrum. They are incompatible in the sense that it is not possible to fit Eq. (1) with the observed K_{e3}^+ and $K_{\mu 3}^+$ rates, the muon data of Dobbs et al., and our (P,E_μ) data. It is conceivable that the two results are reconcilable if one abandons the theoretical framework on which Eq. (1) is based. In view of the agreement between Eq. (1) and the K_{e3}^+ data, we consider this last possibility as the least likely explanation of the disagreement.

[14]In the models proposed in references 9, 10, and 11, the predicted ratio g_V/f_V is somewhat smaller (i.e., more negative) than our result. This difference would be removed if the ratio of $K_{\mu 3}^+$ to K_{e3}^+ rates were on the low side of our measured value of 0.96 ± 0.15.