PHYSICAL REVIEW LETTERS

VOLUME 8 JUNE 1, 1962 NUMBER 11

COLLISION DAMPING OF PLASMA OSCILLATIONS

D. F. DuBois, V. Gilinsky, and M. G. Kivelson
The Rand Corporation, Santa Monica, California
(Received March 5, 1962; revised manuscript received April 23, 1962)

It is well known that the Vlasov equation, i.e., the "collisionless" Boltzmann equation with a self-consistent electric field, predicts a damping of plasma oscillations[1] (Landau damping). In quantum mechanical language, Landau damping corresponds to plasmon decay into a one-particle final state. We have calculated the damping rate to a two-particle final state (collision damping) and we find that this higher order process is the main damping mechanism for all but very short wavelengths.

Two cases are considered in detail: an electron-ion plasma and an electron gas with a uniform positive background. We present results for the collision damping rate $\gamma_c(k)$ divided by the classical electron plasma frequency

$$\Omega_p = (4\pi e^2 n/m)^{1/2};$$

(a) electron-ion plasma,

$$\frac{\gamma_c(k)}{\Omega_p} = \frac{1}{6\sqrt{2}\,\pi^{3/2}} \frac{k_D^3}{n} \ln\left[\left(\frac{k_T}{k_D}\right) 4e^{-C}\right]; \qquad (1)$$

(b) electron gas,

$$\frac{\gamma_c(k)}{\Omega_p} = \frac{2}{15\,\pi^{3/2}} \left(\frac{k}{k_D}\right)^2 \frac{k_D^3}{n} \ln\left[\left(\frac{k_T}{k_D}\right) 4e^{-C}\right]. \qquad (2)$$

The constants are the Debye wave number $k_D = (4\pi e^2 n\beta)^{1/2}$, $\beta = 1/kT$, the thermal de Broglie wave number $k_T = (m/\beta\hbar^2)^{1/2}$, the density n, the electron mass m, and Euler's constant $C = 0.56$. These results are to be compared with the Landau-damping rate which for both systems is [neglecting terms $O(m/M)$ where M is the ion mass]

$$\frac{\gamma_L(k)}{\Omega_p} = \left(1+\Lambda\right)\left(\frac{\pi}{2}\right)^{1/2}\left(\frac{k_D}{k}\right)^3 \exp\left[-\tfrac{1}{2}\left(\frac{k_D}{k}\right)^2\right], \qquad (3)$$

where Λ is of higher order in k_D^3/n. Both Eqs. (1) and (2) give collision damping rates considerably larger than Eq. (3) when $k \ll k_D$. It is clear then that one cannot treat the damping of plasmons in the self-consistent field approximation as in the work of Klimontovich and Silin,[2] or in the equivalent random phase approximation of Pines and Schrieffer,[3] since these approximations fail to include the collision effects. The damping term in the kinetic equations for the plasmon distribution is incorrect in both papers. The electron-ion plasma provides the most striking example: In the limit of infinite wavelength the Landau damping vanishes exponentially but the collision damping remains constant. Even for the electron gas for which the collision damping actually vanishes in the long wavelength limit, the ratio γ_c/γ_L for $k = 0.1\,k_D$ is about 10^{13}, for $n = 10^{16}$ cm^{-3} and $T = 10^6$ °K.

The vanishing of the damping in the long-wavelength limit for the electron gas is related to the fact that electron-electron scattering does not affect the long-wavelength conductivity since current and momentum are proportional. A similar calculation at $T = 0$ and high densities (which then must include exchange effects) has shown that γ_c/Ω_p is also of order k^2 for long wavelengths.[4]

The divergence of the logarithms of Eqs. (1) and (2) in the $\hbar \to 0$ limit is a familiar effect.[5,6] Except for a factor in the argument of the logarithm, Eq.

(1) can be obtained from elementary considerations. In a completely classical calculation the Coulomb divergence is corrected by cutting off integrals at a maximum impact parameter k_D^{-1} and a suitable minimum impact $r_{\min}$. For low temperatures, $r_{\min}$ is of the order of the distance at which the kinetic energy of relative motion of the particles is equal to the potential energy of their interaction. At high temperatures, when this distance becomes less than the de Broglie wavelength of the electrons, it is usual to take $r_{\min}=k_T^{-1}$. Our calculation deals with the high-temperature region.

We can give here only an outline of our calculational methods which will be discussed in detail in a later communication. Starting with the recently developed techniques of quantum statistical perturbation theory in which the conductivity, expressed as a trace, is represented by a series of closed Feynman diagrams,[6,7] we have reduced the calculation to a consideration of open diagrams. The absorption rate is very simply related to the conductivity,

$$\gamma(k)=4\pi\,\mathrm{Im}\sigma(k,\omega).$$

The diagrams are conveniently ordered according to the number of particle-hole pairs (each pair representing a single-particle excitation from the equilibrium medium) in the final state as in Fig. 1. The matrix elements for all contributions to a given process are then squared and averaged over final states in essentially the same manner as in calculating a transition probability. The Landau damping, which is the decay of the collective state into a single-particle excitation, is given by Fig. 1(a), while higher order diagrams such as 1(b) give rise to the correction factor Λ in Eq. (3).

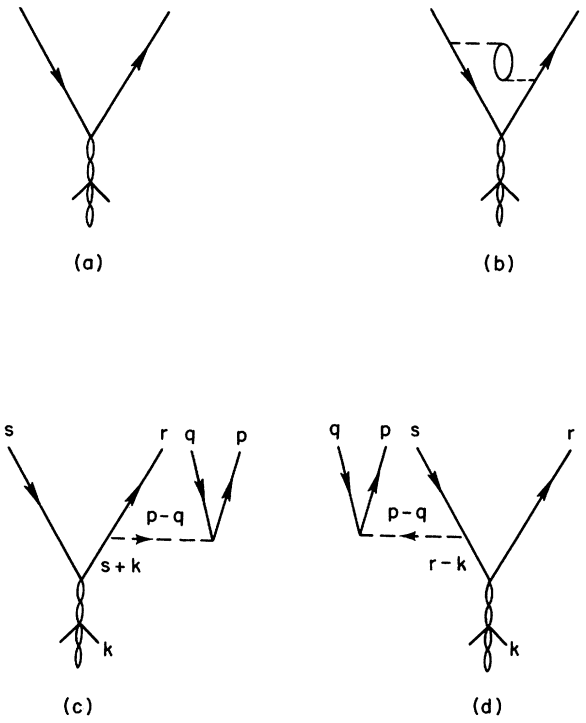

FIG. 1. (a) Landau-damping diagram—decay into a single-particle excitation; (b) higher order correction to Landau damping; (c) and (d) lowest order collision damping processes.

Figures 1(c) and 1(d) represent the decay into a single-particle excitation which then scatters with another particle to form a final state of two single-particle excitations, i.e., collision damping. In addition to Figs. 1(c) and 1(d), two diagrams arise in which the labels of both final pairs are interchanged. The contribution from these four diagrams is

$$\mathrm{Im}\sigma_c(k,\omega)=\tfrac{1}{2}e^2\frac{\omega}{k^2}\int\frac{d^3p}{(2\pi)^3}\int\frac{d^3q}{(2\pi)^3}\int\frac{d^3r}{(2\pi)^3}\int\frac{d^3s}{(2\pi)^3}(2\pi)^3\delta^3(\vec{k}-\vec{p}+\vec{q}-\vec{r}+\vec{s})2\pi\delta(\omega-\epsilon_p+\epsilon_q-\epsilon_r+\epsilon_s)$$

$$\times[f(\epsilon_r)-f(\epsilon_s)][f(\epsilon_p)-f(\epsilon_q)][g(\epsilon_p-\epsilon_q)-g(\epsilon_s-\epsilon_r)]\left|\frac{4\pi e^2}{|\vec{r}-\vec{s}|^2}\left[\frac{1}{\omega-(\epsilon_{q+k}-\epsilon_q)}-\frac{1}{\omega-(\epsilon_p-\epsilon_{p-k})}\right]\right.$$

$$\left.+\frac{4\pi e^2}{|\vec{p}-\vec{q}|^2}\left[\frac{1}{\omega-(\epsilon_{s+k}-\epsilon_s)}-\frac{1}{\omega-(\epsilon_r-\epsilon_{r-k})}\right]\right|^2, \qquad (4)$$

where $f(x)=[1+e^{\beta x}]^{-1}$, $g(x)=[1-e^{\beta x}]^{-1}$, $\epsilon_p=p^2/2m-\mu_e$ for electrons and $\epsilon_p=p^2/2M-\mu_i$ for ions, and μ is the chemical potential, the subscripts referring to the species. This result must be divided by 2 for identical particles. When the masses are equal, as in the electron gas, there is a can-

cellation between the two terms within the absolute value sign for small k which explains why Eq. (2) vanishes as $k \to 0$. For electron-ion scattering the first term in the absolute value is of order m/M compared with the second term and the result does not vanish as $k \to 0$.

The integrals are finite without including the screening of the interparticle interaction, since conservation of energy and momentum prohibits momentum transfers in scattering which are less than $\hbar k_D$. The effect of screening is to change the argument of the logarithm in Eqs. (1) and (2) by a factor of the order of one. Corrections of higher order in (k_D^3/n) arise from diagrams not considered here.

We are happy to acknowledge the assistance of Dr. William Karzas of The Rand Corporation.

[1]L. D. Landau, J. Phys. (U.S.S.R.) 10, 25 (1946).
[2]Y. L. Klimontovich and V. P. Silin, Uspekhi Fiz. Nauk 70, 247 (1960) [translation: Soviet Phys. - Uspekhi 3, 84 (1960)].
[3]D. Pines and J. R. Schrieffer, Phys. Rev. 125, 804 (1962).
[4]D. F. DuBois, Ann. Phys. (New York) 8, 24 (1959).
[5]L. Spitzer, Physics of Fully Ionized Gases (Interscience Publishers, Inc., New York, 1956), p. 73.
[6]V. I. Perel and G. M. Eliashberg, J. Exptl. Theoret. Phys. (U.S.S.R.) 41, 886 (1961) [translation: Soviet Phys. - JETP 14, 633 (1962)].
[7]A. A. Abrikosov, L. P. Gor'kov, and I. E. Dzyaloshinskii, J. Exptl. Theoret. Phys. (U.S.S.R.) 36, 900 (1959) [translation: Soviet Phys. - JETP 9, 636 (1959)].

NEW TECHNIQUE FOR CALCULATING CORRELATIONS FOR VACANCY DIFFUSION*

H. B. Huntington and P. B. Ghate
Rensselaer Polytechnic Institute, Troy, New York
(Received April 20, 1962)

It was pointed out by Bardeen and Herring[1] that theoretical treatments for diffusion should include a correction or correlation factor in those cases where the motion of the diffusing entity was not completely random. Subsequently there have been many detailed analyses[2-4] of correlation particularly as it enters into vacancy diffusion. The formulation of LeClaire and Lidiard[2] gave closed form results for those cases where all the jump possibilities were crystallographically equivalent, and the correlation between diffusing atom and vacancy that had once parted company was neglected. Solutions obviating the above approximation were developed by Compaan and Haven[3] using electrical analogs. Schoen[5] and Mullen[6] have been particularly concerned with the relation between the correlation factor and the variation of diffusivity with isotopic mass. The extension of correlation considerations to anisotropic diffusion has been considered by Manning[7] and then treated in detail by Mullen.[6]

It is the purpose of this Letter to point out a new and simplified technique for calculating the correlation factor in the case of vacancy diffusion which appears to have many advantages. In the LeClaire and Lidiard approximation, the method not only gives quite simply the usual closed form expressions for impurity diffusion in cubic lattices, etc., but also gives coupled equations for correlation factors in cases of anisotropic diffusion. We start with the basic expressions[6] for the correlation factor for $\vec{\lambda}_{i,j}$, which is the jth component of the ith jump for the diffusing atom, written as a vector,

$$f_{i,j} = 1 + 2 \sum_{n=1}^{\infty} \frac{(\vec{\lambda}_{i,j}) \cdot (\vec{\lambda})_{n(i)}}{(\lambda_{i,j})^2}, \tag{1}$$

where the subscript $n(i)$ outside the parentheses numbers the jumps of the diffusing atom which follow the ith-type jump in question. Let us introduce the vector

$$\vec{S}_i = \sum (\vec{\lambda})_{n(i)}, \tag{2}$$

which can be thought of as representing the average final displacement of the impurity at its present site as a result of exchanges with the vacancy in question. In general this vector is closely antiparallel to the last jump that the impurity atom made and represents the extent to which subsequent correlated jumps tend to reduce that displacement. One rewrites

$$f_{i,j} = 1 + \vec{S}_i \cdot \vec{j}/(\lambda_{i,j}), \tag{1a}$$