is the cause of the particular interest in experiments on measuring the polarization at high energies.

(4) Spin correlations between the different particles in nucleon-nucleon and nucleon-antinucleon scattering are absent.

(5) The spin flip of polarized particles or targets at fixed t is the same for the cases of π-N, K-N, N-N, and N-$\bar{N}$ scattering. (Including Λ-hyperon-nucleon scattering, one can show that the spin flip of target nucleons ought to be the same.) A test of this result should be one of the most decisive tests of the whole conception of moving poles.

In conclusion we should like to thank V. B. Berestetsky, I. I. Lewintow, L. B. Okun', and I. M. Shmushkevitch for useful discussions.

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DALITZ PLOT FOR THE DECAY $\eta \to \pi^+\pi^-\pi^0$

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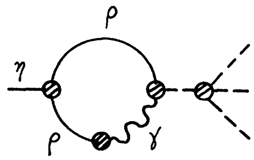
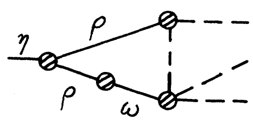
This note suggests a rough combined test of the decay mechanism for the η meson proposed by Gell-Mann[1] and by Gell-Mann, Sharp, and Walker,[2] and of the adequacy of the pion pole approximation in $K \to 3\pi$ decay proposed by Bég and DeCelles.[3]

In references 1 and 2 it is assumed, as now appears to be well substantiated,[4] that the η meson has the same quantum numbers as the π^0, except that it is an isotopic singlet. Therefore decay into two pions is forbidden by reflection invariance, and decay into three pions can proceed only by the interposition of two virtual electromagnetic interactions in which the isospin changes from 0 to 1 and the G parity from +1 to -1. (Decay into four pions, though allowed by the quantum numbers, is impossible energetically.) Hence the 3π modes compete on a comparable footing with various radiative modes. It is then proposed[1,2] that the η decays predominantly by a primary strong dissociation into two virtual vector particles, which for definiteness we take to be ρ's. This is an intrinsically reasonable model even without reliance on the resonant nature of the ρ; if we picture the η as a bound state of four pions, the simplest coupling scheme that can realize its quantum numbers does indeed correspond to two $I=1$, $J=1$ pairings in a P state relative to each other. Qualitatively, the 2γ and $\gamma\pi^+\pi^-$ modes are ascribed to the subsequent electromagnetic processes: both ρ's $\to\gamma$ or one $\rho\to\gamma$ and the other $\to\pi^+\pi^-$. However, the only definitively established charged mode, $\pi^+\pi^-\pi^0$, is not catered for with equal immediacy. On this model, the simplest diagrams for the 3π modes are those shown in Fig. 1. For the following argument it is immaterial what succession of vector propagators is substituted for the internal line along which the electromagnetic coupling takes effect. Thus in Fig. 1(b) the ρ-ω line with its black box, which is of order e^2, could be replaced by a ρ-γ or a γ line, and in Fig. 1(a) the ρ-γ line could be replaced by a γ line.

This model clearly makes it impossible to form a reliable estimate of the ratio of the 3π to the other modes, since only the former in-



(a)



(b)

FIG. 1. The two simplest diagrams for $\eta \to 3\pi$ on the model making $\eta \to 2\rho$ the primary process. Dashed lines represent pions.

volve (cutoff-dependent) closed loops. In particular, the neutral:charged ratio [presumed to be essentially the ($3\pi^0$ and 2γ):($\pi^+\pi^-\pi^0$) ratio] is not open to rational interpretation without considerable special pleading.

The purpose of the present note is to show that nevertheless one can subject the model to a rough experimental check by exploiting the Dalitz plot for $\eta \to \pi^+\pi^-\pi^0$. First we show that in the light of the data, the mechanism of Fig. 1(b) cannot be dominant. To see this, we neglect the structure due to the denominators of the internal lines, and take into account only the minimal structure introduced by the momentum dependence that the individual vertices must have by virtue of Lorentz and gauge invariance. The actual amplitude will then equal this minimally structured expression multiplied by some invariant form factor. It turns out that the invariant amplitude corresponding to this diagram, when properly symmetrized, is proportional to

$$E_1 = [2(\eta\cdot\pi^0)(\pi^+\pi^-) - (\eta\cdot\pi^+)(\pi^-\cdot\pi^0) - (\eta\cdot\pi^-)(\pi^+\cdot\pi^0)]. \quad (1)$$

In (1), the letters on the right denote the four-momenta of the corresponding particles. Below, M_η and μ are the η and π masses. Expressed in the η rest frame in terms of the coordinates of the Dalitz plot, T = kinetic energy of π^0, δT = energy difference between $\pi^\pm$, $|E_1|^2$ becomes

$$|E_1|^2 = (M_\eta/2)^2[(M_\eta - 3\mu - 3T)(M_\eta\mu + \mu^2 + M_\eta T) + M_\eta(\delta T)^2]^2. \quad (2)$$

This, i.e., the density of the Dalitz plot, vanishes at the symmetric point where the three pion energies are all equal; even after integration over δT, it leads to a distribution of events which dips practically to zero around $T \approx 0.35\,\mu$, and peaks sharply both below and above this region, in clear contradiction to the data.[4]

Therefore we go on to ask whether the other alternative mechanism, that of Fig. 1(a), gives a more sensible prediction regarding the Dalitz plot. Note that here the dominance of the single-pion intermediate state, which is now under examination, is a highly specific consequence of the model which makes $\eta \to 2\rho$ into the primary process. No other simple model that we have investigated[5] makes this state important. The relevance of this remark will become clearer in our last paragraph.

At this point we recall the suggestion of Bég and DeCelles[3] that the single-pion intermediate state dominates likewise in $K \to 3\pi$ (τ and τ') decays. Although this state is reached by very different mechanisms in η and K decay, we see that the structure (momentum and isospin dependence) of both is given by the same function of the covariant Mandelstam-type variables and of the mass M of the virtual pion (i.e., the mass of the decaying particle). This function is in principle obtainable from the $\pi\pi$ scattering amplitude by analytic continuation not only in the Mandelstam variables, but also, necessarily, in one of the external masses. In practice, continuation in the external mass is a hazardous undertaking, because it is known[6,7] not only that the amplitude has a singularity in M at the external instability point $M = 3\mu$, but also that its analytic properties as a function of the Mandelstam variables are very different in the stable and unstable regions. Therefore it is difficult to make watertight arguments from $\pi\pi$ scattering to the structure of these 3π amplitudes, even where such inferences have a certain theoretic[6,3] or empirical[3] plausibility.

It appears quite reasonable, however, that once the external mass is well onto its branch cut, the dependence on it becomes fairly weak. Thus we would argue with some confidence that as a first approximation one can neglect the change in the structure of the amplitude as M varies from the K to the η mass (3.54μ to 4μ) even in the otherwise unfavorable contingency that the change from $M = \mu$ to $M > 3\mu$ is considerable. The precise meaning of this hypothesis will be made clearer below. Then the η, τ, and τ'

amplitudes are just different isotopic projections of the same function, apart from a constant factor depending on the mechanism whereby the single-pion state is reached, which will be different for K and η. We write this function in the standard way[8] as

$$A(s_a, s_b, s_c)\delta_{\rho\alpha}\delta_{\beta\gamma} + B(s_a, s_b, s_c)\delta_{\rho\beta}\delta_{\alpha\gamma} + C(s_a, s_b, s_c)\delta_{\rho\gamma}\delta_{\alpha\beta}. \quad (3)$$

In (3), ρ is the isotopic index of the virtual intermediate pion, k_a, k_b, and k_c are the four-momenta of the final-state pions, and α, β, γ their isotopic indices; and $s_a = (k_b + k_c)^2$, etc. Then

$$s_a + s_b + s_c = M^2 + 3\mu^2, \quad (4)$$

and, as argued above, all further dependence on M is neglected. By this we mean that the coefficients of the three Mandelstam s variables in A, B, and C are taken as independent of M [in particular the constant λ introduced in Eq. (9)], except insofar as the constraint (4) may be applied. This is to be true at least as M varies from M_K to M_η. A, B, and C have the symmetry properties:

$$A(s_a, s_b, s_c) = A(s_a, s_c, s_b) = B(s_b, s_a, s_c) = C(s_c, s_b, s_a). \quad (5)$$

In particular $A = B = C$ when $s_a = s_b = s_c = s_0 = (M^2 + 3\mu^2)/3$. For τ and τ' decay we identify k_c as the momentum of the unlike pion; for $\eta \to \pi^+\pi^-\pi^0$ we let k_c be the π^0 momentum. Then the invariant amplitudes are

$$E_\eta = f_\eta \cdot C, \quad (6)$$

$$E_\tau = f_K \cdot (A + B), \quad (7)$$

$$E_{\tau'} = f_K \cdot C. \quad (8)$$

Here, f_η and f_K are constants.

To a good first approximation, confirmed both by experiment[9,10] and by calculation,[11] it is possible in the physical region to take A, B, and C as linear functions each of only a single variable:

$$C = D[1 + \lambda(s_c - s_0)]. \quad (9)$$

In (9), D and λ are constants; for a first orientation it is legitimate to take λ as real.[11] Setting $\mu = 1$ ($M \approx 4$), one has from the K data[9,10] the approximate value $\lambda = 0.185$. For $\eta \to \pi^+\pi^-\pi^0$ this leads to the following prediction in the η rest frame:

$$E_\eta = D\{1 + \lambda[2M_\eta^{\ 2}/3 - 2M_\eta(1 + T)]\} = D[1 + (8\lambda/3)(1 - 3T)], \approx \text{constant} \times (1 - T). \quad (10)$$

We see that in this linear approximation E_η is the same function of $(\pi^+ + \pi^-)^2$ as is $E_{\tau'}$ of $(\pi_1^{\ 0} + \pi_2^{\ 0})^2$. A direct comparison of these amplitudes would serve as a test of the dominance of the single-pion intermediate state in each, subject only to the uncertainty introduced by the continuation in M as discussed above.

Preliminary data[4] on the $\eta \to \pi^+\pi^-\pi^0$ Dalitz plot do indicate an appreciable thinning for high T, though actually they are more suggestive of approximate constancy for low T followed by a fairly rapid falloff. [Such behavior could result from an admixture with appropriate phase of some of the amplitude (1) to the dominant amplitude (10).]

Lastly we underline that even a qualitative indication of an appreciable falloff for increasing T is quite an achievement for a theoretical model, because the final-state interactions by themselves (i.e., without the influence of the single intermediate pion) would lead to precisely the opposite prediction, if we accept the current notions[12,13] that the $I = 0$ $\pi\pi$ scattering length is large and attractive while the $I = 2$ scattering length is small and probably repulsive. Such an interaction would tend to bunch the charged pions together, giving the π^0 more than its statistical share of recoil momentum.

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ζ-η DEGENERACY*

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We would like to point out that a particle decaying into pions may decay, with the aid of electromagnetic interactions, principally into two pions in its charged state and into three pions in its neutral state:

$$\zeta^{\pm} \rightarrow 2\pi,$$
$$\zeta^{0} \rightarrow 3\pi.$$

We suggest that the η^0,[1] previously identified as a $T=0$ particle, be identified as the ζ^0, the neutral component of the ζ.[2] What is required[3] is, at least, that the ζ, which is a $T=1$ triplet in the presence of strong interactions, have odd spin and parity with even charge parity for the ζ^0:

$$C\zeta^0 = \zeta^0, \quad (J, P, \text{odd}), \tag{1}$$

or even spin and parity (with $J \geqslant 2$) and odd charge parity:

$$C\zeta^0 = -\zeta^0, \quad (J \geqslant 2, P, \text{even}). \tag{2}$$

The charge parity eigenvalue is a constant of the motion in the presence of electromagnetic interactions, while according to the generalized Pauli principle for a 2-pion system

$$C = (-1)^J.$$

Thus ζ^0 decay into 2π is forbidden. This charge conjugation argument does not apply to a charged system.

According to the relation $G = C(-1)^T$ for a neutral system, the ζ, of whatever charge, has, in the presence of strong interactions,

$$G = \pm 1 \text{ for } J\begin{pmatrix}\text{even}\\ \text{odd}\end{pmatrix},$$

and in the presence of strong interactions the ζ could thus be described by a model of 3 or 4 pions in the J odd or even cases, respectively.[4] There are a variety of configurations one could adopt for these 3- and 4-pion systems so that detailed calculations would be very speculative.

Let us concentrate on the perhaps most interesting case (1) with $J^P = 1^-$. In the presence of electromagnetic interactions we can state that the neutral ζ cannot decay into $\pi + \gamma$:

$$\zeta^0 \rightarrow \pi^0 + \gamma \text{ forbidden},$$

because the photon has odd charge parity. Also $\zeta^0 \rightarrow 2\gamma$ is forbidden. Experiments[1] have indicated that the branching ratio in η^0 decay, $R \equiv$ all neutral/$\pi^+\pi^-\pi^0$, may be larger than $\frac{3}{2}$, the maximum allowed for $(3\pi^0)/(\pi^+\pi^-\pi^0)$ for $T=1$. We have to assume here that $R \approx \frac{3}{2}$, or that $\pi^0 + 2\gamma$ decay is competitive with $\pi^+\pi^-\pi^0$ decay. Neither possibility can be ruled out at present. Meanwhile the