mobility,[2,3] is 3.5×10^{12} cm^{-3} sec^{-1} in agreement with the theoretical estimate quoted. Because this calculation should not be directly applied to an organic crystal, and because Northrop and Simpson's determination of the rate of carrier production is subject to considerable uncertainty, our result should be regarded as a demonstration of the plausibility of the mechanism whereby charge carriers are produced from exciton-exciton interactions in a molecular crystal. Further details of this investigation and the development of techniques suitable to the proper study of organic crystals will be presented in a separate publication.

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SPIN STRUCTURE OF THE MESON-NUCLEON AND NUCLEON-NUCLEON SCATTERING AMPLITUDES AT HIGH ENERGIES

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It has been recently shown[1] that generalized unitarity relations obtained by one of us (V.G.)[2] lead to the existence of relations between the amplitudes of the various processes at high energies when asymptotically these processes are caused by a single Regge pole.[3,4]

In this note we shall consider the vacuum pole[4] in meson-nucleon and nucleon-nucleon scattering amplitudes in an attempt to analyze their spin structure and the relation between them.

Let us write a general expression for the amplitudes of meson-nucleon scattering G and nucleon-nucleon scattering H[5]:

$$G=a+b\hat{q},\quad q=\tfrac{1}{2}(k_1+k_1'),\tag{1}$$

$$\begin{aligned}H=&H_1(s,t)+H_2(s,t)(\gamma^{(1)}p+\gamma^{(2)}k)\\&+H_3(s,t)(\gamma^{(1)}p)(\gamma^{(2)}k)+H_4(s,t)(\gamma_5^{(1)}\gamma^{(1)}p)(\gamma_5^{(2)}\gamma^{(2)}k)\\&+H_5(s,t)\gamma_5^{(1)}\gamma_5^{(2)},\\&p=\tfrac{1}{2}(p_2+p_2'),\quad k=\tfrac{1}{2}(p_1+p_1').\end{aligned}\tag{2}$$

Here k_1, k_1'; p_1, p_2; p_1', p_2' are initial and final pion and nucleon momenta, respectively. As far as the vacuum pole, which is responsible for the amplitudes at high energies and low t, is of interest to us, these amplitudes are unit matrices in isobaric space. It can be seen that terms containing $\gamma_5[H_4(s,t),H_5(s,t)]$ cannot be generated by the vacuum pole and are therefore asymptotically small. Thus at high s and low t, the nucleon-nucleon amplitude contains only three invariant functions. As the vacuum pole appears only in the part of the amplitude symmetric under permutation of s and u,[4] the same expressions describe the scattering of antiparticles by nucleons. As in the case of spinless particles, let us consider the partial-wave expansion of amplitudes in the annihilation channel.

For the meson-meson scattering amplitude we have the usual expression

$$f(s,t)=\sum_j(2j+1)f_j(t)P_j(Z_f).\tag{3}$$

To expand two-meson annihilation and nucleon-nucleon scattering amplitudes, we shall use the helical amplitudes[6]

$$\begin{aligned}&\langle|G|\lambda',\lambda\rangle=\sum_j(2j+1)g_j(\lambda',\lambda)d_{\lambda-\lambda',0}^{\,j}(Z_g),\\&\langle\lambda_2',\lambda_2|H|\lambda_1',\lambda_1\rangle\\&=\sum_j(2j+1)\langle\lambda_2',\lambda_2|H^j|\lambda_1',\lambda_1\rangle d_{\lambda_1-\lambda_1',\lambda_2-\lambda_2'}^{\,j}(Z_h),\end{aligned}\tag{4}$$

Z_f, Z_g, Z_h being cosines of scattering angles in the annihilation channel for the corresponding processes. For the partial waves thus introduced, the usual unitarity condition holds in the interval $4\mu^2 < t < 16\mu^2$ (here μ is the pion mass).

$$\frac{1}{2i}(f_j - f_j^*) = \frac{k}{\omega} f_j f_j^*,$$

$$\frac{1}{2i}[g_j(\lambda', \lambda) - g_j^*(\lambda', \lambda)] = \frac{k}{\omega} g_j^*(\lambda', \lambda) f_j,$$

$$\frac{1}{2i}[\langle \lambda_2', \lambda_2 | H^j | \lambda_1', \lambda_1 \rangle - \langle \lambda_2', \lambda_2 | H^j | \lambda_1', \lambda_1 \rangle^*]$$

$$= \frac{k}{\omega} g_j^*(\lambda_2', \lambda_2) g_j(\lambda_1', \lambda_1). \quad (5)$$

The first equation in (5) is valid if the scattered meson is a pion. With the use of the particular shape of the helical amplitudes, $\langle | G | \lambda', \lambda \rangle$ and $\langle \lambda_2', \lambda_2 | H | \lambda_1', \lambda_1 \rangle$, in terms of invariant functions with known analytical properties, it may be shown that helical partial waves are analytic functions of j obeying generalized unitarity conditions which can be obtained from (5) substituting f_j^* for $f_{j^*}^*$, g_j^* for $g_{j^*}^*$, and $\langle \lambda_2', \lambda_2 | H^j | \lambda_1', \lambda_1 \rangle^*$ for $\langle \lambda_2', \lambda_2 | H^{j^*} | \lambda_1', \lambda_1 \rangle^*$.[2] As a consequence, all the helical amplitudes have the vacuum pole and their residues at this pole are bounded by the relations[1]

$$r(\lambda_2', \lambda_2 | \lambda_1', \lambda_1) r_f = r(\lambda_2', \lambda_2) r(\lambda_1', \lambda_1). \quad (6)$$

Using the analyticity of partial amplitudes as functions of j and transforming the sum on j into an integral similarly to the case of spinless particles, it is easy to conclude that our amplitudes at high s and t are defined by the vacuum pole.

Using the relation between residues (6) and taking into account the explicit form of $d_{\mu\nu}^j(Z)$ at high Z,[6] one can easily be convinced of the fact that the nucleon-antinucleon scattering matrix $\langle \lambda_2', \lambda_2 | H | \lambda_1', \lambda_1 \rangle$, multiplied by the meson-meson scattering amplitude f, is the direct product of two-meson annihilation matrices $\langle | G | \lambda_1', \lambda_1 \rangle$ and $\langle | G | \lambda_2', \lambda_2 \rangle$. Such a result is natural if the nucleon-antinucleon amplitude itself in the form (2) is proportional to the direct product of spin matrices of the two-meson annihilation, and as the set of the helical amplitudes determines invariant functions $H_i(s, t)$ uniquely, one can write, treating now the scattering channel:

$$[H_1 + H_2(\gamma^{(1)}p + \gamma^{(2)}k) + H_3(\gamma^{(1)}p)(\gamma^{(2)}k)]f$$

$$= (a + b\gamma^{(1)}p)(a + b\gamma^{(2)}k). \quad (7)$$

If we consider kaon-nucleon scattering, the nucleon-nucleon scattering matrix can be expressed in terms of the kaon-nucleon matrix and kaon-nucleon amplitude in the same way:

$$[H_1 + H_2(\gamma^{(1)}p + \gamma^{(2)}k) + H_3(\gamma^{(1)}p)(\gamma^{(2)}k)]f_K$$

$$= (a_K + b_K\gamma^{(1)}p)(a_K + b_K\gamma^{(2)}k). \quad (8)$$

The expressions for scattering amplitudes $f_{\pi\pi}$, f_{KK}, $f_{\pi N}$, f_{KN}, and f_{NN} corresponding to the contribution of the vacuum pole and taking into account the relations (7) and (8) may be written in the following form[7]:

$$f_{\pi\pi} = \Gamma_\pi^2(t) D(s), \quad f_{KK} = \Gamma_K^2(t) D(s),$$

$$f_{\pi N} = \Gamma_\pi(t)\left[\Gamma_N^{(1)}(t) + \Gamma_N^{(2)}(t)\frac{2m\hat{q}}{s}\right]D(s),$$

$$f_{KN} = \Gamma_K(t)\left[\Gamma_N^{(1)}(t) + \Gamma_N^{(2)}(t)\frac{2m\hat{q}}{s}\right]D(s), \quad (9)$$

$$f_{NN}$$

$$= \left[\Gamma_N^{(1)}(t) + \Gamma_N^{(2)}(t)\frac{2m}{s}\gamma^{(1)}p\right]\left[\Gamma_N^{(1)}(t) + \Gamma_N^{(2)}(t)\frac{2m}{s}\gamma^{(2)}k\right]D(s),$$

where

$$D(s) = s^{l(t)}\left[1 + e^{i\pi l(t)}\right].$$

These expressions remind one of those obtained if we consider Feynman graphs corresponding to the one-meson exchange with meson Green's function $D(s)$ and other functions in (9) being put in correspondence to vertices.

Let us note some consequences of (9).

(1) At $t = 0$, the relation between the total cross sections as in[1]: $\sigma_{\pi\pi}\sigma_{NN} = \sigma_{\pi N}^2$, $\sigma_{KK}\sigma_{NN} = \sigma_{KN}^2$, follows from (9).

(2) Exactly the same relations hold for the spin-average elastic scattering differential cross sections.

(3) Using the property that the absorptive parts of all the invariant functions appearing in (1) and (2) are real, one can show that all the products $\Gamma_i(t)\Gamma_j(t)$ appearing in (9) are real. The consequence is that the polarization in meson-nucleon scattering vanishes at high energies. If we take into consideration other poles at the same time with the vacuum pole, we should find that the interference between the vacuum pole and the nearest other pole gives rise to polarization. This fact

is the cause of the particular interest in experiments on measuring the polarization at high energies.

(4) Spin correlations between the different particles in nucleon-nucleon and nucleon-antinucleon scattering are absent.

(5) The spin flip of polarized particles or targets at fixed t is the same for the cases of π-N, K-N, N-N, and N-$\bar{N}$ scattering. (Including Λ-hyperon-nucleon scattering, one can show that the spin flip of target nucleons ought to be the same.) A test of this result should be one of the most decisive tests of the whole conception of moving poles.

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DALITZ PLOT FOR THE DECAY $\eta \to \pi^+\pi^-\pi^0$

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This note suggests a rough combined test of the decay mechanism for the η meson proposed by Gell-Mann[1] and by Gell-Mann, Sharp, and Walker,[2] and of the adequacy of the pion pole approximation in $K \to 3\pi$ decay proposed by Bég and DeCelles.[3]

In references 1 and 2 it is assumed, as now appears to be well substantiated,[4] that the η meson has the same quantum numbers as the π^0, except that it is an isotopic singlet. Therefore decay into two pions is forbidden by reflection invariance, and decay into three pions can proceed only by the interposition of two virtual electromagnetic interactions in which the isospin changes from 0 to 1 and the G parity from +1 to -1. (Decay into four pions, though allowed by the quantum numbers, is impossible energetically.) Hence the 3π modes compete on a comparable footing with various radiative modes. It is then proposed[1,2] that the η decays predominantly by a primary strong dissociation into two virtual vector particles, which for definiteness we take to be ρ's. This is an intrinsically reasonable model even without reliance on the resonant nature of the ρ; if we picture the η as a bound state of four pions, the simplest coupling scheme that can realize its quantum numbers does indeed correspond to two $I=1$, $J=1$ pairings in a P state relative to each other. Qualitatively, the 2γ and $\gamma\pi^+\pi^-$ modes are ascribed to the subsequent electromagnetic processes: both ρ's $\to \gamma$ or one $\rho \to \gamma$ and the other $\to \pi^+\pi^-$. However, the only definitively established charged mode, $\pi^+\pi^-\pi^0$, is not catered for with equal immediacy. On this model, the simplest diagrams for the 3π modes are those shown in Fig. 1. For the following argument it is immaterial what succession of vector propagators is substituted for the internal line along which the electromagnetic coupling takes effect. Thus in Fig. 1(b) the ρ-ω line with its black box, which is of order e^2, could be replaced by a ρ-γ or a γ line, and in Fig. 1(a) the ρ-γ line could be replaced by a γ line.

This model clearly makes it impossible to form a reliable estimate of the ratio of the 3π to the other modes, since only the former in-