where

$$s=(C_{11}-C_{44})/(C_{12}+C_{44}),\quad t=(C_{12}-C_{44})/C_{44}.$$

N is the total number of atoms, V is the volume, ρ is the density, and the remaining symbols have their usual meanings. $f(s,t)$ is presented in tabular form. From reference 2 one can obtain

$$\frac{d\ln\Theta_0}{d\ln V}=\frac{1}{2}\frac{d\ln C_{44}}{d\ln V}+\frac{1}{6}+\frac{1}{3}\frac{d\ln f(s,t)}{d\ln V},$$

which can be evaluated using the tables and the known values of C_{ij} and $d\ln C_{ij}/d\ln V$ for Ge[6] and Si.[7] Term by term the results are, for Ge and Si,

Ge: $d\ln\Theta_0/d\ln V=-\gamma_0=-0.751+0.167+0.092=-0.492;$

Si: $-\gamma_0=-0.490+0.167+0.073=-0.250.$

Note that the third term, involving the interpolation in the table and arising from the change of elastic anisotropy and Poisson ratios with volume, is a relatively small correction for these materials. The values of γ_0 so obtained are indicated by $\times$'s on the $T/\Theta_\infty=0$ ordinate of Fig. 1. A possible interpolation of the data is indicated by a dotted line, whence the γ of Ge does exhibit the same behavior as that of Si and InSb. It seems probable that Gibbons' extrapolation given by the dashed line is incorrect in the case of Ge and that the similarity of behavior of Ga, Si, and InSb is preserved.

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[3]J. C. Phillips, Phys. Rev. 113, 147 (1958).

[4]F. W. Sheard, Phil. Mag. 3, 1381 (1958).

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NEW PHENOMENON IN MAGNETORESISTANCE OF BISMUTH AT LOW TEMPERATURE

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In the course of studying the galvanomagnetic properties at low temperatures in pure bismuth at a magnetic field above several kilo-oersteds, we have found a strong nonlinear conduction behavior; that is, a sharp change of slope in the current-voltage curve at a certain high electric field, which we call the kink field, as illustrated in Fig. 1. Each trace was taken at constant transverse magnetic field ($B\parallel$ trigonal).

Several specimens of approximate cross section 1 mm^2 and length from 0.5 to 5 mm were carefully cut from pure bismuth single crystals grown by the Czochralski technique.[1] In the present experiment, the direction of current flow was chosen parallel to the bisector direction between the binary and the bisectrix axes and the temperature was around 2°K.

The solid curve in Fig. 2 indicates the well-known Shubnikov–de Haas oscillatory behavior of our specimen in the transverse magnetoresistance with period $\Delta(1/B)=e\hbar/E_f m^* c\sim1.5\times10^{-5}$ oersted^{-1}, where E_f is the Fermi energy and the magnetic field B is parallel to the trigonal axis. The lower curve in Fig. 2 shows the differential magnetoresistance derived from the straight line beyond the kink field in the current-voltage curve. The background of the latter curve is fairly independent of the magnetic field, contrary to the strong magnetic field dependency of the former curve, though the latter curve also seems to show some oscillatory effect. It should be noted

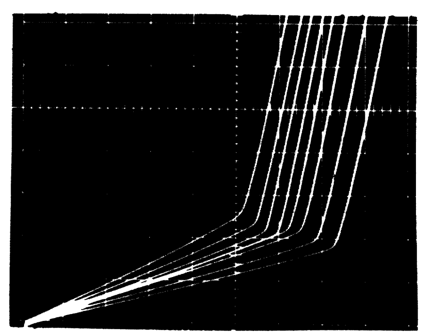

FIG. 1. Current-voltage curves showing the kink. Each curve corresponds to 14, 15, 16, 17, 18, 19, 20, and 21 kilo-oersteds from left to right. The abscissa and the ordinate are 0.2 volt/div and 100 ma/div, respectively. ($B\parallel$ the trigonal axis.)

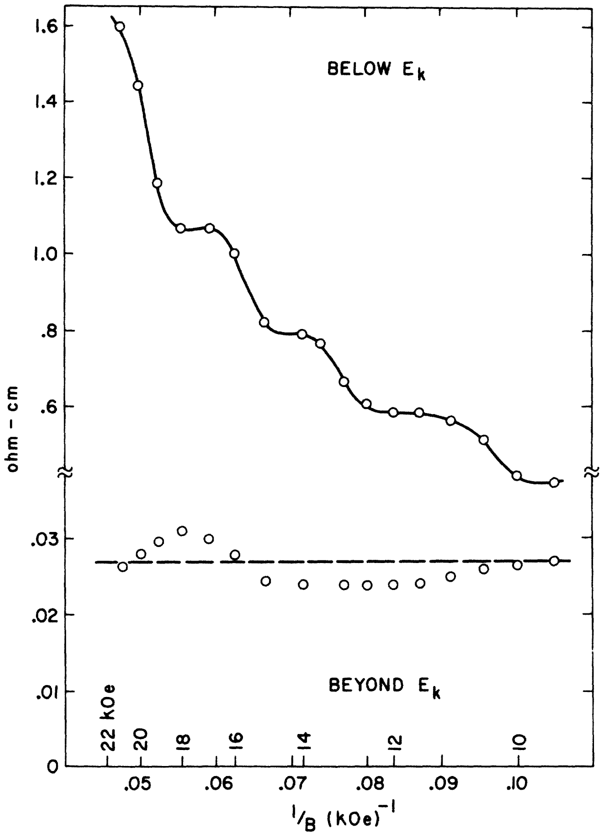

FIG. 2. The upper and lower figures show the ordinary transverse magnetoresistance and the differential magnetoresistance beyond the kink field plotted against the inverse of the magnetic field, respectively. ($B \parallel$ the trigonal axis.)

that the magnetoresistance drops from several times to more than fifty times after the onset of the kink. Figure 3 illustrates the magnetic field dependency of the kink field $E_{\rm kink}$, which gives a very simple relation, $E_{\rm kink} = \alpha B$, where the constant α is $\sim 10^{-3}$ volt/cm oersted over the whole range of the applied magnetic field, no matter what the length of the specimen is. However, it has been seen that both the sharpness of the kink and the kink field strength are fairly sensitive to the crystal orientation, when the transverse magnetic field is rotated from the trigonal axis toward the other direction.

The motion of a charged particle of mass m^* and velocity v in a strong magnetic field $B = B_z$ and an electric field $E = E_y$, perpendicular to each other, is classically given by a cyclotron rotation of angular frequency $\omega = eB_z/m^*c$ and radius $r = m^*cv/eB_z$ and a motion of velocity $v_x = cE_y/B_z$ in the x direction, which is independent of both the particle's mass and its velocity, as well as of the sign of its charge. In our case, this velocity v_x has turned out to be approximately 10^5 cm/sec at the kink electric field, no matter what the magnetic field is. This numerical value, moreover, seems fairly comparable to the sound velocity in bismuth.[2] This fact may suggest that a strong electron-phonon interaction occurs when the velocity v_x reaches this critical value. The wave function $\psi_{k_x n k_z}$ for a charged particle at such crossed fields, B_z and E_y, as described

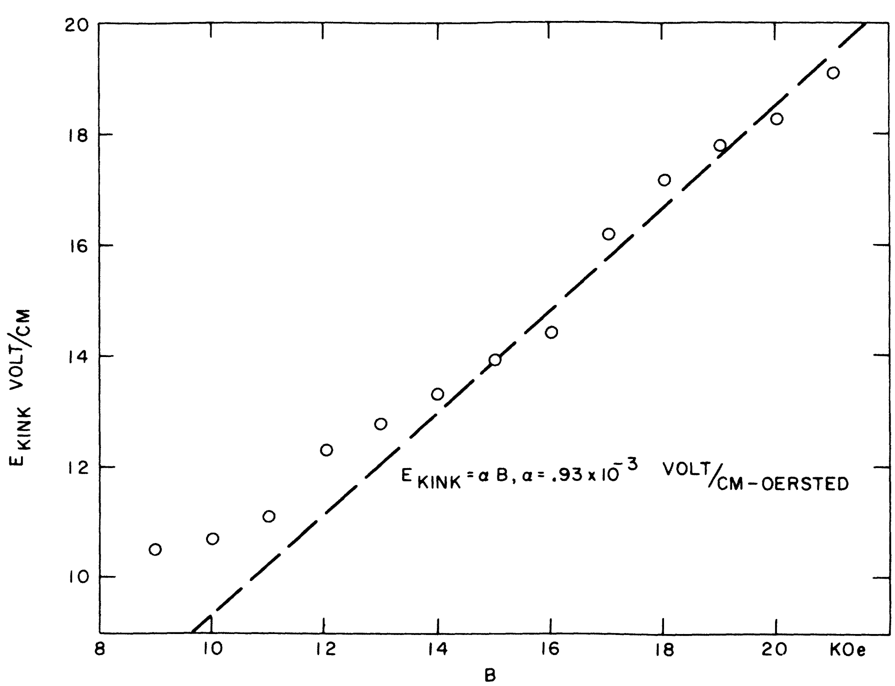

FIG. 3. The relation between the kink field and the magnetic field. ($B \parallel$ the trigonal axis.)

above, is given by

$$\psi_{k_x n k_z} = \frac{\exp i(k_x x + k_z z)}{(L_x L_z)^{1/2}} \phi_n\left(y - \frac{\hbar c k_x}{eB_z} + \frac{E_y mc^2}{eB_z^{\,2}}\right),$$

where $\vec{k} = (k_x, k_y, k_z)$ is the electron wave vector. The central coordinate y_0 of the cyclotron motion is

$$y_0 = \hbar c k_x / eB_z.$$

The current, that is the transport of carriers in the direction of the electric field E_y, is brought about only by scattering for such a case as the strong transverse magnetic field.[3] In our bismuth, there is an evidence of considerable contribution of the lattice scattering even below 4.2°K. Therefore from momentum conservation the change of the electron wave vector $(k_x' - k_x)$, due to the change of the central coordinate y_0 by one scattering, should be equal to the wave vector, q_x, of a phonon emitted; and also the energy, which the electron can get from the electric field and can give to the phonon, should be greater than the phonon energy $\hbar\omega(q_x)$ of such a wave vector, q_x. This single-phonon emission condition is written as

$$eE_y \delta y_0 = eE_y \frac{\hbar c}{eB_z}(k_x' - k_x) \geq \omega(q_x).$$

Therefore,

$$\frac{cE_y}{B_z} \geq \frac{\omega(q_x)}{k_x' - k_x} = \frac{\omega(q_x)}{q_x} = s_x.$$

The maximum energy which an electron can get in one scattering process may be estimated as $eE_y \delta y_0 = 2erE_y \sim 2 \times 10^{-4}$ ev, if a circular orbit is assumed rather than an elongated ellipse. Therefore, because only a linear portion in the dispersion curve near $q_x = 0$ is involved, the right-hand side s_x of the above inequality is exactly equal to the sound velocity in the x direction. If this sort of consideration is correct, the crystal orientation dependency of the quantity cE_{kink}/B may represent the orientation dependency of the sound velocity of some mode. The lower curve in Fig. 4 shows the experimental value $v = cE_{\text{kink}}/B$ as a function of rotation angle of transverse magnetic field (total 180 degrees rotation), keeping the electric field direction constant, though E_k can hardly be well defined at some directions because of the broad curvature. The upper curves in Fig. 4 show the sound velocities of the two shear modes calculated from the six independent elastic constants determined by Eckstein et al.[2] for the directions corresponding to the present experiment. The direction at 0 and 180 degrees and that at 90 degrees on the abscissa are the trigonal axis and the 15 degrees off direction to-

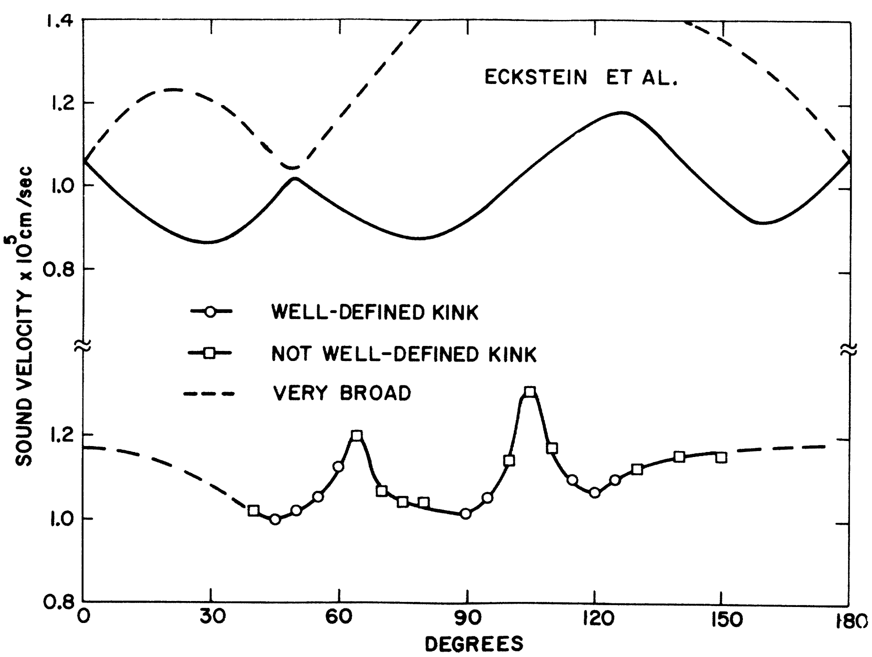

FIG. 4. The lower curve shows the experimental values $v = cE_{\text{kink}}/B$ as a function of rotation angle of the transverse magnetic field, keeping the electric field direction constant, though E_{kink} can hardly be well defined at such directions as indicated. The upper curves in Fig. 4 show the sound velocities of the two shear modes calculated from the elastic constants of Eckstein et al. The trigonal axis: 0 and 180 degrees. The bisector direction between both the binary and bisectrix axes: 90 degrees.

ward the bisectrix from the binary axis, respectively. If we take into account that our experimental values may always correspond to the lower one of two shear modes indicated with a solid curve in the figure, the absolute values of the velocity and the general shape of both curves are in fairly good agreement. Some discrepancies might come from the misorientation of crystal axes, misalignment of crystal mounting in the magnetic field, and the complex many-ellipsoidal energy structure of bismuth. On the other hand, this anisotropic band structure may allow the strong coupling of the electrons with the shear mode. The experimental fact that the sharpness of the kink and the differential resistance beyond the kink field vary with the crystal orientation may indicate the directional dependency of the strength of the coupling between the electrons and the phonons. It is also conceivable that our new method may give a slightly higher value than the actual sound velocity.

In some crystal direction, for instance, around 100 degrees in Fig. 4, we can see the occurrence of a sinusoidal electrical oscillation of frequency $\sim 10^6$ cycles/sec when the applied field exceeds the kink field, whose frequency f does not depend on the external circuit but is determined by means of the simple formula, $f = s/d$, where d and s are the width of the bismuth specimen and the sound velocity in the direction perpendicular to the crossed electric and magnetic fields. This oscillation phenomenon may indicate an acoustic standing wave built up of frequencies that resonate corresponding to the size of the specimen; in other words, the generation of coherent phonons.

Moreover, we may have to think about the electron-hole recombination and generation velocities, v_r and v_g, on the bismuth surface, if these are not much larger than the velocity $v_x \sim 10^5$ cm/sec, which may give an accumulation of electrons and holes on the surface, because the recombination and generation processes in bulk may be fairly slow at low temperature. No positive evidence has been observed so far on this surface accumulation.

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TRANSITION RADIATION FROM METAL FILMS

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Because of the Coulomb force between charges, electrons in metals can participate in oscillations of a collective type, the so-called plasma oscillations. Both theory and experiment have shown that these plasma oscillations can be excited by high-energy charged particles passing through the metal. More recently it has been shown experimentally[1] that in thin metal films electrons can excite plasma oscillations which in turn emit a peak of electromagnetic radiation around the plasma frequency. This radiation from thin foils had first been predicted theoretically by Ferrell.[2] Subsequently an alternative theoretical treatment of this radiation was presented[3] which connected it with Russian work on transition radiation[4,5] and was more exact than the original theory of Ferrell. In view of the appearance of a recent paper[6] and the possibility that it may give the impression that Ferrell's mechanism for the peak in radiation differs from the interpretation of transition radiation, it was felt desirable to publish more details of the previous work[3] which emphasized that Ferrell's calculation and that of transition radiation give equivalent results for the peak in radiation. This is discussed further near the end of this paper.

It was first pointed out by Frank and Ginsburg[4] that a charged particle will emit electromagnetic radiation when passing through a boundary separating two different media even though the particle is moving at a constant velocity. The change in the electromagnetic fields surrounding the charged particle as it makes the transition from one medium to another with a different dielectric

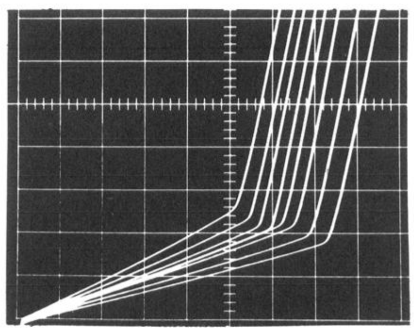

FIG. 1. Current-voltage curves showing the kink. Each curve corresponds to 14, 15, 16, 17, 18, 19, 20, and 21 kilo-oersteds from left to right. The abscissa and the ordinate are 0.2 volt/div and 100 ma/div, respectively. ($B \parallel$ the trigonal axis.)