the tensile strength of the target windows.

The polarized protons were elastically scattered from twelve nuclides comprising four pairs of isotopes and three pairs of isobars. The scattered protons were detected by nuclear emulsions and the angular dependence of the left-right asymmetry was determined for each nuclide. From these measurements was deduced the angular dependence of the polarization that would obtain in the scattering of unpolarized protons under identical conditions. The results are displayed in Figs. 1 and 2.

It is observed (Fig. 1) that the largest differences in polarization occur for the isotopes of H, He, and Li, where we know that the addition of a neutron to the lighter isotope produces a large change in the surface structure. On the other hand, we observe no detectable difference between the mirror nuclei H^3 and He^3 (Fig. 2), although the elastic scattering cross sections for these nuclides differ by a factor of almost two at some angles (implying a very significant difference in the central potentials), and the variation in the symmetry parameter $(N-Z)/A$ is a maximum for this pair of isobars.

C^{12} and C^{13} show very similar polarization patterns, in keeping with the hypothesis that C^{13} consists of a neutron outside a core of C^{12}, which is not much perturbed by the extra neutron. The isobaric pairs Ar^{40}, Ca^{40} and Fe^{58}, Ni^{58} also show very small polarization differences, with $Ar^{40}-Ca^{40}$ indicating the greater difference. This is presumably related to the fact that Ca^{40} is a doubly magic nucleus and that substitution of two neutrons for two protons produces a significant perturbation in the nuclear surface.

It is concluded that polarization measurements may provide a sensitive probe for investigating the surface structure of nuclei.

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NEUTRON FORM FACTORS AND NUCLEON STRUCTURE*

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Last year new information on the Dirac and Pauli form factors of the neutron was reported.[1-3] A theoretical model based on the existence of two resonances involving pions was proposed by Bergia, Stanghellini, Fubini, and Villi[4]; which was consistent with the Clementel-Villi expressions for the form factors. From a different point of view, an attempt to fit the experimental neutron and proton form factor material in terms of Yukawa clouds with different ranges and delta functions was made by Hofstadter and Herman.[2] As is well known, the Clementel-Villi model is equivalent to the latter interpretation. Therefore the two models are identical in all practical aspects. An important feature of each model was that it gave a neutron rms electric radius of zero and values of the proton rms radii and neutron magnetic radius in agreement with experiment. The Cornell group[5] has confirmed the form factor data and the model of nucleon structure as given in references 1 and 2.

We now wish to present a concise version of our recent results. In summary we find that while the idea of the nucleon models proposed above[1,2,4] appears to be quite satisfactory, the numerical values of the parameters involved require certain changes. Thus the numerical values of the parameters we are now reporting differ from those of the previous Stanford results, and noting the above-mentioned agreement of

Stanford and Cornell results, the new values also differ from the Cornell results.

The differences between the older and the present results are ascribed to the following sources: (a) more precise and more abundant Stanford electron scattering measurements on hydrogen[6] and deuterium[7]; (b) an improved theory for inelastic electron scattering by the deuteron, given by Durand.[8]

The new cross sections (item a) agree very well with the older experimental data.[1,9,10] Because they are more numerous these new cross sections yield much more information concerning the quasi-elastic scattering peak of the deuteron as a function of scattering angle (60, 75, 90, 120, and 135 degrees) than the older data. The new data include values of the four-momentum transfer, q^2, lying between 3.0 and 22 F^{-2}. ($1\ F \equiv 10^{-13}$ cm.)

The theory mentioned under item (b) gives a formula for the quasi-elastic peak cross section which is in good agreement with the impulse approximation results of Goldberg.[11] The main effect of using the Durand-Goldberg formula for the peak of the deuteron inelastic cross section curve, instead of using the result of the Jankus theory,[12] as modified in reference 10, is to depress the results for F_{1n} toward lower values.[8] Whereas the earlier indications obtained from using the modified Jankus theory gave positive values of F_{1n}, we now find numerical values of F_{1n} close to zero or perhaps slightly negative throughout the region of q^2 under consideration.[13] The new values of F_{2n} are slightly lower than reported before but are still higher than those of F_{2p}.

Since a full account of the experiments will be given in a subsequent article, we merely show in Fig. 1 typical results of the actual measurements. In this figure we have plotted the ratio of the elastic scattering cross section of the proton to the cross section at the peak of the inelastic deuteron spectrum for different scattering angles as a function of q^2. The experimental points have been corrected for radiative effects[14] but not for the influence of the interaction in the final state. The straight lines in Fig. 1 were obtained by weighted least-squares fitting of the data. Smoothed ratio values obtained in this way were used in the subsequent analysis.

Neutron cross sections have been obtained from the combination of these ratios and the absolute proton cross sections by using the appropriate formula given by Durand.[8] For the absolute proton cross sections we have used two separate sets of form factors: (1) the central values for F_{1p} and F_{2p} as published in reference 6, which are the solid lines in the upper part of Fig. 2, and (2) the central values (dashed lines in Fig. 2) which follow from a kind of analysis somewhat different from that discussed in reference 6. This analysis will be described subsequently. Because of the correlation of errors which is qualitatively indicated by the direction of the arrows in the top part of Fig. 2, the absolute cross sections calculated from the two sets of proton form factors are not significantly different, and, of course, both sets agree with experiment within experimental errors.

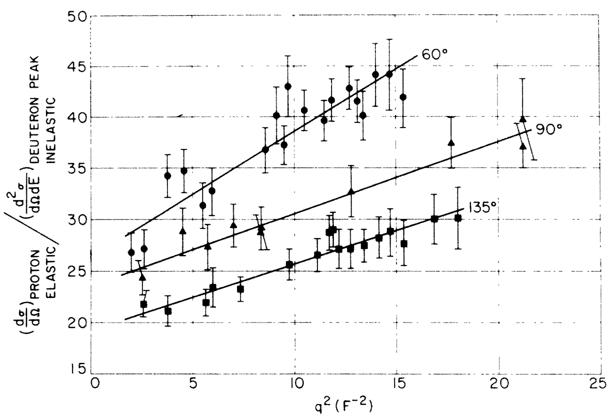

FIG. 1. This figure shows examples of the experimental ratios of the differential proton cross section to the doubly-differential peak deuteron cross section at various angles, as a function of q^2. The straight lines were obtained by weighted least-squares fittings of the data. Measurements have also been made at 75 and 120 degrees (see reference 7) but are not shown here.

Considering the present Stanford information on proton cross sections it is difficult to prefer one set of central values over the other. More precise determinations of the absolute proton cross sections are now being made. However, for the present we have carried along both sets of central values for F_{1p} and F_{2p}, together with their individual error assignments.

The results for the neutron form factors have been plotted in the lower part of Fig. 2. (Note the comment about the other solutions in footnote 13.) The vertical widths of the bands are associated mainly with the errors in the proton form factors and to a lesser extent with the experimental errors of the ratio measurements. The arrows in the neutron bands correspond, therefore, qualitatively to the correlated errors in the proton form factors.

It can be seen that at, say, $q^2 \geqslant 8\ \mathrm{F}^{-2}$, F_{1n} tends to be slightly negative, while at lower q^2 values this form factor appears to have positive values. In the low-q^2 region we have also plotted the results of Glendenning and Kramer[15] on the analysis of the elastic deuteron scattering results of Friedman, Kendall, and Gram.[16] The same trend was previously indicated by Schiff[17] who analyzed the elastic deuteron results of McIntyre.[18] Because we have neglected the corrections for the final-state interaction in our inelastic data, we feel that our present positive values for F_{1n} at low q^2 are not inconsistent with the Glendenning results.[15] We should note that the final-state interaction has very little influence on the form factors of the neutron at values of $q^2 \geqslant 8\ \mathrm{F}^{-2}$. Therefore the dotted curves have been drawn so as to combine the results of the present work in the higher q^2 region with the data from the elastic scattering from the deuteron. The corresponding changes in the neutron magnetic form factor, F_{2n}, are also indicated by dotted curves.

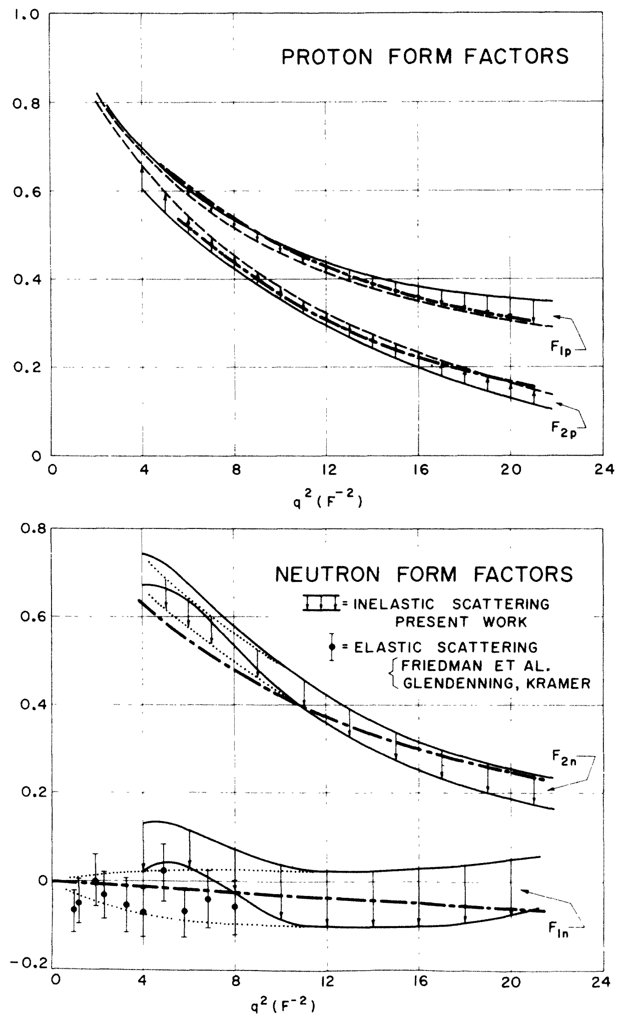

FIG. 2. The upper part of this figure shows the proton form factors as functions of q^2. The solid lines refer to the central values in reference 6; the dashed lines to the central values which follow from a slightly different analysis of the same data (see text). The arrows indicate the correlation of errors, as explained in reference 6. The lower part of the figure shows the neutron form factor results for F_{1n} and F_{2n}. Note also the comment about the other solutions in footnote 13. The neutron form factors are expected to lie in the bands indicated. The points with error bars for F_{1n} refer to a recent analysis made by Glendenning and Kramer.[15] The dotted curves are drawn as a compromise between the trend indicated by the Glendenning points and the trend of the inelastic data of the present work in the higher q^2 region. The heavy dashed-dotted curves are the results of the theoretical fit described in the text.

Experimental values of the isotopic form factors can now be obtained from proton and neutron form factors using the definitions and normalizations of reference 2. In an attempt to fit those experimental form factors we have used the theoretical form of form factors as proposed by Bergia et al.[4] and Hofstadter and Herman[2]:

$$F_{1S} = \frac{s_1}{1 + 2.04\, q^2/M_S^{\,2}} + 1 - s_1,$$

$$F_{2S} = \frac{s_2}{1 + 2.04\, q^2/M_S^{\,2}} + 1 - s_2,$$

$$F_{1V} = \frac{v_1}{1 + 2.04\, q^2/M_V^{\,2}} + 1 - v_1,$$

$$F_{2V} = \frac{v_2}{1 + 2.04\, q^2/M_V^{\,2}} + 1 - v_2. \qquad (1)$$

These form factors are based on dispersion theory and strong pion-pion interactions and have the well-known Clementel-Villi form. In Eq. (1) q^2 is given in F^{-2} and $M_S^{\,2}$ and $M_V^{\,2}$ are given in units of $m_\pi^{\,2}$. It can be shown that the following relations between the parameters apply:

$$s_1/M_S^{\,2} = v_1/M_V^{\,2} = a_{1p}^{\,2}/12.24,$$

$$v_2/M_V^{\,2} = 0.0406(a_{2p}^{\,2} + a_{2n}^{\,2}),$$

$$s_2/M_S^{\,2} = -1.22\, a_{2p}^{\,2} + 1.30\, a_{2n}^{\,2}, \qquad (2)$$

where a_{1p}, a_{2p}, and a_{2n} are the rms radii of the proton charge and proton and neutron magnetic moment distributions, respectively.

Before giving the results of the fitting procedure we wish to make the following remarks: (a) If only one two-pion state and one three-pion state are responsible for the behavior of the vector and scalar form factors, respectively, then the form of Eq. (1) is justified approximately. In this event we should use the numerical values 28 and 32 for M_V^2 and M_S^2, respectively, because these values are well established (ρ meson and ω meson).[19] (b) If there is more than one two-pion state and/or more than one three-pion state (perhaps the ζ meson and η meson[20]; see, however, Bastien et al.[21]), then the general form of the isotopic form factors should contain explicit contributions from each of the appropriate resonances. It can be shown, in that case, however, that the simple form of Eq. (1) is still a good first approximation and the fit to the data will then yield "effective" mass values.

The result of the fitting procedure is indicated in Table I, where the numerical values of the parameters with their error assignments are given. The best fit to all the form factors is obtained for a value of $a_{1p} = 0.79$ F, which is in excellent agreement with the precise determination of this quantity by Lehmann, Taylor, and Wilson[22] ($a_{1p} = 0.785 \pm 0.04$ F) and also with earlier determinations,[6,23] and a value of $a_{2p}^2 + a_{2n}^2 = 1.50$ F^2, also in agreement with earlier work.[6,10] These results correspond to values of $M_V^2 = 18\, m_\pi^2$ and $M_S^2 = 23 \times m_\pi^2$. The fits to the experimental material using these parameters are shown in Fig. 2 (dashed-dotted lines).

It may be of interest to note that a good fit to F_{1V} can be obtained with the ρ-particle mass ($M_V^2 = 28\, m_\pi^2$), but only if $a_{1p} \sim 0.69$ F. For this low value of the rms radius of the charge distribution of the proton, a fit to F_{1S} can be obtained by using a value of $M_S^2 \sim 40\, m_\pi^2$, but the fit is poor.

With respect to the numerical values given in the table we may draw the following conclusions: (1) The constants $(1 - s_1,\ 1 - v_1,\ 1 - v_2)$ take on rather small values; this fact supports the usefulness of the analysis we have made. (2) It does not seem possible to fit the data by using only the known ρ meson and ω meson. Our data indicate that both for the isovector and isoscalar form factor, at least one other heavy meson is needed. The required masses would have to be smaller than $28\, m_\pi^2$ and $32\, m_\pi^2$, respectively. (See also Fubini[24] for relevant remarks.) (3) Since F_{2S} is related to the difference of the nearly equal anomalous magnetic moments of the nucleons, it is very difficult to measure this quantity accurately and the only conclusion one can draw is that it is of the order of unity or slightly greater than unity. (4) If M_S^2 is larger than M_V^2, which is favored by our data, then the outer region of the neutron exhibits a slight negative charge density, but of course the accurate behavior at small q^2 must still be determined. (5) The experimental data indicate that F_{1n} is approximately zero. From Table I it can be seen that a good fit for this condition corresponds to $M_S^2 = M_V^2 \sim (19 \pm 3) \times m_\pi^2$ for $a_{1p} = 0.79 \mp 0.03$ F. The theoretical significance of $F_{1n} \equiv 0$ would be very great.

As already indicated, the results reported here do not agree with the data of the Cornell group.[5]

Table I. Parameter fits to the isotopic form factors.

Best fit	Typical other fits	
$a_{1p} = 0.79$ F	$a_{1p} = 0.75$ F	$a_{1p} = 0.83$ F
$a_{2p}^2 + a_{2n}^2 = 1.50$ F^2		
$M_V^2 = (18 \pm 2) m_\pi^2$	$M_V^2 = (21 \pm 3) m_\pi^2$	$M_V^2 = (15 \pm 2) m_\pi^2$
$V_1 = 0.92 \pm 0.10$		
$M_S^2 = (23 \pm 3) m_\pi^2$	$M_S^2 = (29 \pm 3) m_\pi^2$	$M_S^2 = (23 \pm 3) m_\pi^2$
$s_1 = 1.17 \pm 0.15$		
$v_2 = 1.10$		
$s_2 = -0.5$		

We believe this is mainly due to small systematic differences in the absolute proton cross sections. Work is now in progress at Stanford to redetermine absolute proton and neutron data, using improved techniques. We hope that this proposed work will clarify the present situation.

We wish to thank Dr. Sol Krasner and Mr. Carol Crannell for their help with numerous calculations. We also wish to thank other members of the electron-scattering group at Stanford for their assistance with the experiments. We wish to acknowledge with thanks the many discussions we have had with Dr. B. Bosco regarding the final-state interactions in the deuteron.

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†On leave from Institute of Nuclear Research, Amsterdam, Holland.

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[7]A preliminary version of the present data was given at the Aix-en-Provence meeting in September, 1961 [Proceedings of the Aix-en-Provence International Conference on Elementary Particles, 1961 (C.E.N. Saclay, France, 1961), Vol. 1, p. 121].

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[13]There are two possible sets of solutions for F_{1n} (and F_{2n}) which led in the earlier work to positive and negative values. In the present work the solution plotted in Fig. 2 corresponds to the older positive solution. The second solution now gives even more negative values for F_{1n} than the values of Fig. 2. We will present the second solution in a forthcoming article. At the present time we consider only the first solution and we refer the reader to references 2 and 7 for the reasons why we have taken the present solution more seriously.

[14]The radiative corrections to the cross section of the elastic proton peak and to the height of the inelastic deuteron peak have been calculated with the formulas of Sobottka.[9] Uncertainties in these corrections will probably cancel in forming the ratios of Fig. 1 of the present paper. We have investigated the influence of the Tsai radiative correction [Phys. Rev. 122, 1898 (1961)] on the absolute cross section of reference 6 in place of that given by Sobottka.[9] If Tsai's radiative correction had been applied, the absolute cross sections given in reference 6 would probably have been a few percent higher than reported. Since the differences between Tsai and Sobottka results are almost independent of scattering angle, for constant q^2, the numerical values of the form factors are insensitive to whichever of the two corrections is chosen for the radiative effects.

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