REMARKS ON WEAK-INTERACTION FORM FACTORS*

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We present in this note a discussion of the Goldberger-Treiman (G-T) relations and suggest a model for the axial-vector form factor in strangeness-conserving baryon-lepton weak interactions.

On the basis of the space inversion invariance, isospace rotation invariance, and G invariance of the strong interaction, we know that the virtual transitions associated with the axial-vector form factor $F_A(q^2)$, the induced pseudoscalar form factor $F_P(q^2)$, and the induced pseudotensor form factor $F_T(q^2)$, involve

$$B+\bar{B}'\to[(3\pi),\ (5\pi),\cdots]_{J=1^+,\,T=1,\,G=-1}\to e+\bar{\nu};\quad F_A(q^2)$$
$$B+\bar{B}'\to[\pi,\ (3\pi),\cdots]_{J=0^-,\,T=1,\,G=-1}\to e+\bar{\nu};\quad F_P(q^2)$$
$$B+\bar{B}'\to[(4\pi),\ (6\pi),\cdots]_{J=1^+,\,T=1,\,G=+1}\to e+\bar{\nu};\quad F_T(q^2) \tag{1}$$

where B, B' are the initial and final baryons, and where we have explicitly included only the lowest mass intermediate states. It is thus clear that $F_A(q^2)$, for example, will depend on the characteristics of any intermediate $J=1^+$, $T=1$, $G=-1$ three-pion state, and, if this state should be resonant, only on its mass and effective coupling strength to B, B' and to e, $\bar{\nu}$. In fact, any such resonant state will play a role in determining $F_A(q^2)$ which is entirely analogous to the role played by the $J=0^-$, $T=1$, $G=-1$ one-pion state in determining $F_P(q^2)$ or to the role played by the $J=1^-$, $T=1$, $G=+1$ two-pion state (ρ meson) in determining the isovector electromagnetic form factors $F_{\text{el}}^{\text{isov}}(q^2)$, $F_{\text{mag}}^{\text{isov}}(q^2)$, or equivalently, on the basis of the conserved vector current theory,[1] the polar-vector and weak-magnetism form factors $F_V(q^2)$, $F_M(q^2)$.

Quantitatively, we define $F_A(q^2)$, $F_P(q^2)$, and $F_T(q^2)$ by

$$\langle B';p_f,s_f|A_\lambda|B;p_i,s_i\rangle=\bar{u}_{B'}(p_f,s_f)[\gamma_\lambda F_A(q^2)+iq_\lambda F_P(q^2)+i\sigma_{\lambda K}q_K F_T(q^2)]\gamma_5 u_B(p_i,s_i), \tag{2}$$

so that

$$\langle B';p_f,s_f|\partial A_\lambda/\partial x_\lambda|B;p_i,s_i\rangle=\bar{u}_{B'}(p_f,s_f)\gamma_5 u_B(p_i,s_i)[(m_B+m_{B'})F_A(q^2)+q^2F_P(q^2)], \tag{3}$$

where $q_\lambda\equiv(p_f-p_i)_\lambda$, A_λ is the axial-vector weak current which we assume to be odd under G,[2] and $|B\rangle$, $|B'\rangle$ are baryon states of equal parity and strangeness. Then, remembering the one-pion pole term[3] in $F_P(q^2)$, and assuming that the contributing $J=0^-$, $T=1$, $G=-1$ three-pion state is resonant ("β meson") with mass m_β, we have

$$F_P(q^2)=\frac{c_\pi(B,B')}{q^2+m_\pi^{\ 2}}+\frac{c_\beta(B,B')}{q^2+m_\beta^{\ 2}}-\frac{1}{\pi}\int_{(5m_\pi)^2}^\infty\frac{\text{Im}F_P(-\xi)d\xi}{\xi+q^2+i\epsilon}, \tag{4}$$

where

$$c_\pi(B,B')\equiv-a_\pi g_{\pi BB'},$$
$$c_\beta(B,B')\equiv-a_\beta g_{\beta BB'},$$

a_π gives the pion decay matrix element

$$[\langle\text{vac}|A_\lambda|\pi\rangle=-\sigma_{\lambda 4}m_\pi(2m_\pi)^{-1}a_\pi],$$

$g_{\pi BB'}$ is the strong-interaction coupling constant associated with $B\to B'+\pi$, ($g_{\pi np}=\sqrt{2}\,g_{\pi pp}$), while a_β, $g_{\beta BB'}$ are the corresponding quantities for the "β meson." Equation (4) predicts that $F_P(\infty)$

$=0$, consistent with the presumed "wholly strong-interaction induced" character of $F_P(q^2)$.

Similarly, supposing that the $J=1^+$, $T=1$, $G=-1$ three-pion state contributing to $F_A(q^2)$ is resonant ("α meson") with mass m_α,

$$F_A(q^2) = F_A(\infty) + \frac{c_\alpha(B,B')m_\alpha}{q^2+m_\alpha^{\ 2}} - \frac{1}{\pi}\int_{(5m_\pi)^2}^{\infty} \frac{\mathrm{Im}\,F_A(-\xi)d\xi}{\xi+q^2+i\epsilon},$$

$$c_\alpha(B,B') \equiv -a_\alpha g_{\alpha BB'}, \tag{5}$$

where a_α, $g_{\alpha BB'}$ are "α-meson" quantities analogous to a_β, $g_{\beta BB'}$ for the "β meson" and to a_π, $g_{\pi BB'}$ for the pion.

We now discuss the G-T relations. On the basis of Eqs. (3)-(5), we can write

$$(m_B+m_{B'})F_A(q^2)+q^2F_P(q^2) = \frac{m_\pi^{\ 2}c_\pi(B,B')}{q^2+m_\pi^{\ 2}}[1+\psi(q^2)], \tag{6}$$

where $\psi(q^2)$ is analytic for q^2 close to $-(m_\pi)^2$ and has a zero for $q^2=-(m_\pi)^2$. We also assume asymptotic conservation of A_λ,[4] so that, from Eq. (3),

$$\lim_{q^2\to\infty}[(m_B+m_{B'})F_A(q^2)+q^2F_P(q^2)]=0. \tag{7}$$

Equations (6) and (7) immediately yield:

$$(m_B+m_{B'})F_A(0) = -c_\pi(B,B')[1+\psi(0)], \tag{8}$$

$$\psi(\infty)<\infty;$$

and if $\psi(0)\ll 1$, i.e., if $(m_B+m_{B'})F_A(q^2)+q^2F_P(q^2)$ for $q^2\approx 0$ is dominated by the one-pion pole at $q^2=-(m_\pi)^2$, we obtain the G-T relation:

$$F_A(0) \cong -c_\pi(B,B')/(m_B+m_{B'})$$

$$\equiv a_\pi g_{\pi BB'}/(m_B+m_{B'}). \tag{9}$$

We must, however, emphasize that it is easy to exhibit "reasonable" expressions for $F_A(q^2)$, $F_P(q^2)$ which satisfy Eq. (7) but for which $\psi(0)\ll 1$ only if Eq. (9) is imposed externally; in such cases the "proof" of Eq. (9), on the basis of Eq. (8) and the supposition that $\psi(0)\ll 1$, clearly involves circular reasoning unless further physical assumptions are made. Thus, let us suppose that $F_A(q^2)$, $F_P(q^2)$ in Eqs. (5), (4) are well approximated by

$$F_A(q^2) = \frac{c_\alpha(B,B')m_\alpha}{q^2+m_\alpha^{\ 2}}+F_A(\infty),$$

$$F_P(q^2) = \frac{c_\pi(B,B')}{q^2+m_\pi^{\ 2}}+\frac{c_\beta(B,B')}{q^2+m_\beta^{\ 2}}, \tag{10}$$

with Eq. (7) demanding the additional relationship:

$$(m_B+m_{B'})F_A(\infty)+c_\pi(B,B')+c_\beta(B,B')=0. \tag{11}$$

Equations (8), (10), and (11) then show that $\psi(0)=0$ only if

$$\left(\frac{m_B+m_{B'}}{m_\alpha}\right)c_\alpha(B,B')=c_\beta(B,B'): \tag{12}$$

which, however, is nothing else than an a posteriori imposition of Eq. (9), there being no known a priori relation between $c_\beta(B,B')$ and $c_\alpha(B,B')$. It is clear that asymptotic axial-vector weak current conservation can establish only one relation among the various constants in $F_A(q^2)$, $F_P(q^2)$, and this single relation is in general insufficient to determine $\psi(0)$ as necessarily $\ll 1$.

We therefore adopt the position that while empirically the G-T relation is indeed valid when $|B\rangle=|n\rangle$, $|B'\rangle=|p\rangle$, its validity for other $|B\rangle$, $|B'\rangle$ remains to be demonstrated, and predictions from Eq. (9) such as

$$\frac{[F_A(0)]_{\Sigma\Lambda}}{[F_A(0)]_{np}} \cong \frac{g_{\pi\Sigma\Lambda}}{g_{\pi np}}\left(\frac{m_n+m_p}{m_\Sigma+m_\Lambda}\right) \tag{13}$$

must be treated with appropriate reserve; in particular, one might expect on the basis of certain hyperon compound models that $[\psi(q^2)]_{\Sigma\Lambda}$ varies appreciably as q^2 increases from $-(m_\pi)^2$ to $-m_\pi(m_\Lambda+m_\pi-m_\Sigma)$, so that $[\psi(-m_\pi^{\ 2})]_{\Sigma\Lambda}=0$ by no means implies that $[\psi(0)]_{\Sigma\Lambda}\ll 1$. On the other hand, if for example $F_P(q^2)$ is dominated by π and "β" pole terms, and if either $m_\pi^{\ 2}\ll m_\beta^{\ 2}$ or $F_A(\infty)=0$, then

$$[F_P(0)]_{\Sigma\Lambda}/[F_P(0)]_{np} \cong (1/\sqrt{2})\,g_{\pi\Sigma\Lambda}/g_{\pi np}. \tag{14}$$

From a general point of view we consider Eq. (14) as somewhat more reasonable than Eq. (13) since both $[F_P(0)]_{\Sigma\Lambda}$ and $[F_P(0)]_{np}$ are wholly induced by the strong interactions and might therefore well be comparable, if the corresponding g's are equal. In contrast, for instance in a theory where A_λ contains no primitive $\Sigma \to \Lambda$ contributions,[5] $[F_A(0)]_{\Sigma\Lambda}$ and the difference between $[F_A(0)]_{np}$ and 1 ($\cong 0.2$) are induced by the strong interactions and we might therefore expect the ratio

$$\{[F_A(0)]_{\Sigma\Lambda}/([F_A(0)]_{np}-1)\}$$

rather than the ratio

$$\{[F_A(0)]_{\Sigma\Lambda}/[F_A(0)]_{np}\}$$

to be approximately equal to $g_{\pi\Sigma\Lambda}/g_{\pi np}$.[6]

In conclusion, we present an estimate of the mass m_α of the "α meson" which we shall suppose contributes predominantly to $F_A(q^2)$ for $0 \leq q^2 \lesssim (2m_p)^2$. We use for this purpose a relation proposed by Gell-Mann and Zachariasen,[7]

$$\frac{[F_A(0)]_{np}}{[F_V(0)]_{np}} = \lim_{q^2\to\infty}\left\{\frac{[F_V(q^2)]_{np}}{[F_A(q^2)]_{np}}\right\}, \tag{15}$$

and assume that the limit on the right side of Eq. (17) is already approximately attained for $q^2 \approx (2m_p)^2$ and so can be roughly estimated by extrapolation from data presently available at smaller q^2. We also take[8]

$$\begin{aligned}[F_V(q^2)]_{np} &= [F_{\mathrm{el}}^{\mathrm{isov}}(q^2)]_{np} \\ &= \frac{c_\rho(n,p)m_\rho}{q^2+m_\rho^2} + \frac{c_\delta(n,p)m_\delta}{q^2+m_\delta^2} \\ &= \frac{[c_\rho(n,p)/m_\rho]m_\rho^2}{q^2+m_\rho^2} + \frac{[1-c_\rho(n,p)/m_\rho]m_\delta^2}{q^2+m_\delta^2},\end{aligned} \tag{16}$$

where, up to now, only the limit $m_\delta^2 \to \infty$ has been considered and where on the basis of a fit to the electron scattering data[9] $[c_\rho(n,p)/m_\rho] \cong 1.3$. ($m_\rho = 5.2m_\pi$ is the mass of the ρ meson.) However, if $m_\delta^2 \to \infty$, $[F_V(\infty)]_{np}$ is negative ($\cong -0.3$), and, to avoid this undesirable behavior for $[F_V(q^2)]_{np}$,[10] we instead suppose that the second term in Eq. (16) represents the effect of a resonant $J=1^-$, $T=1$, $G=+1$ four-pion state ("δ meson") whose mass we assume to be $m_\delta = 7.4m_\pi$;[11] for such an m_δ the best fit value of $[c_\rho(n,p)/m_\rho] \cong 1.6$. Since from Eq. (16) we now have $[F_V(\infty)]_{np}=0$, Eq. (15) will imply that $[F_A(\infty)]_{np}=0$, so that every $[F(q^2)]_{np}$ obeys an unsubtracted dispersion relation and nucleons behave as "all cloud and no core" in weak interactions.

From Eqs. (10), (16), and (15), and remembering that, empirically, $[F_A(0)]_{np} = [c_\alpha(n,p)/m_\alpha] \cong 1.2$, we then obtain

$$m_\alpha = \frac{m_\rho}{1.2}\left[\frac{c_\rho(n,p)}{m_\rho} + \left(1 - \frac{c_\rho(n,p)}{m_\rho}\right)\frac{m_\delta^2}{m_\rho^2}\right]^{1/2} = 2.8m_\pi. \tag{17}$$

On the other hand, in view of the uncertainty regarding the very existence of the "δ meson," we may adopt a form for

$$[F_V(q^2)]_{np} = [F_{\mathrm{el}}^{\mathrm{isov}}(q^2)]_{np}$$

which theoretically is as simple as possible and empirically is not too inaccurate, viz.:

$$[F_V(q^2)]_{np} = \frac{[c_\rho(n,p)/m_\rho]m_\rho^2}{q^2+m_\rho^2}, \quad \frac{c_\rho(n,p)}{m_\rho} = 1. \tag{18}$$

We then get, using the first equality in Eq. (17),

$$m_\alpha = m_p/1.2 = 4.3m_\pi. \tag{19}$$

Equations (17) and (19) give a very rough indication that any experimental search for the "α meson" might well begin in the mass region: 400 Mev $< m_\alpha <$ 600 Mev.

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[1]R. P. Feynman and M. Gell-Mann, Phys. Rev. 109, 193 (1958).

[2]S. Weinberg, Phys. Rev. 112, 1375 (1958).

[3]M. L. Goldberger and S. B. Treiman, Phys. Rev. 110, 1178 (1958); 111, 354 (1958).

[4]Y. Nambu, Phys. Rev. Letters 4, 380 (1960); J. Bernstein, S. Fubini, M. Gell-Mann, and W. Thirring, Nuovo cimento 17, 757 (1960); M. Gell-Mann and M. Lévy, Nuovo cimento 16, 705 (1960).

[5]Such a primitive $\Sigma \to \Lambda$ contribution in A_λ is absent in the Sakata model, or in the "minimal weak-coupling" model [see J. Dreitlein and H. Primakoff, Phys. Rev. 125, 1671 (1962)].

[6]It is also worth mentioning that a recent study of $K_{\mu 3}^+$ decay by J. M. Dobbs, K. Lande, A. K. Mann, K. Reibel, F. J. Sciulli, H. Uto, D. H. White, and K. K. Young [Phys. Rev. Letters 8, 295 (1962)] has shown that $\xi \equiv [F_-(-m_\mu^2/F_+(-m_\mu^2)] \cong -9$, where the form

factors, $F_{\pm}(q_-^2)$, determine the decay matrix element $M \sim F_+(q_-^2)(q_+)_\lambda + F_-(q_-^2)(q_-)_\lambda$ with $(q_\pm)_\lambda \equiv (p_K \pm p_\pi)_\lambda$. This experimental value for ξ is inconsistent with a "G-T type" prediction for ξ, viz.: $\xi \cong (m_K^2 - m_\pi^2)/m_{K^*}^2 = 0.3$ or $\xi \cong (m_K^2 - m_\pi^2)/(m_{K^*}^2 + m_K^2 - m_\pi^2) = 0.2$ for spins of 0 or 1, respectively, associated with the assumed dominant K^* pole. [See, in connection with the spin 0 case, the work of J. Bernstein and S. Weinberg, Phys. Rev. Letters 5, 481 (1960), whose argument we have generalized a little; the spin 1 case is treated analogously.]

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[9]F. Bumiller, M. Croissiaux, E. Dally, and R. Hofstadter, Phys. Rev. 124, 1623 (1961); R. Hofstadter, C. deVries, and R. Herman, Phys. Rev. Letters 6, 790 (1961); R. M. Littauer, H. F. Schopper, and R. R. Wilson, Phys. Rev. Letters 7, 141 (1961).

[10]Negative values for $[F_V(\infty)]_{np}$ are ruled out by the fact that $[F_V(\infty)]_{np} = Z_{1V} = Z_2$ with $0 \leq Z_2 < 1$, where $(Z_{1V})^{-1}$ is the renormalization factor for the nucleon weak polar vector or electromagnetic isovector vertex $\gamma_\lambda \tau$ and Z_2 is the nucleon propagator renormalization factor. The first equality follows from arguments of G. Källén [Handbuch der Physik, edited by S. Flügge (Springer-Verlag, Berlin, 1958), Vol. 5, Part 1, pp. 358-363]; the second equality is an immediate consequence of Ward's identity; the limits on Z_2 are due to H. Lehmann [Nuovo cimento 11, 342 (1954)].

[11]We understand that some very preliminary data may indicate the existence of a resonant four-pion state with mass = 1.04 Bev = 7.4 m_π.

ERRATA

NEUTRAL DECAY MODES AND THE SPIN AND PARITY OF THE η MESON. Laurie M. Brown and Paul Singer [Phys. Rev. Letters 8, 155 (1962)].

In describing our calculation of the $\pi^0+\gamma$ decay of a vector meson, we remarked that an omitted diagram in which a photon emerges directly from the decay vertex together with a π^0 and a closed pion loop gives formally a quadratic divergence. However, closer examination shows that this term actually vanishes by symmetry after integration over the pion loop momentum. Thus in the approximation used, that of a structureless three-pion vertex of the type

$$V_\mu \epsilon^{\mu\nu\sigma\rho} p_\nu^{(+)} p_\sigma^{(-)} p_\rho^{(0)},$$

with photon interactions generated by gauge invariance, there is no ambiguity in our calculation as presented in the Letter. This remark explains also the gauge invariance of our result as presented previously, and, of course, strengthens our conclusion that the spin, parity, and G-parity assignment 0^{-+} is theoretically favored for the η meson, in agreement with recent experimental observations of the Berkeley group.

INTRINSIC STRUCTURE AND LOW-ENERGY $\pi\pi$ SCATTERING IN $K^\pm \to 3\pi$ DECAY. G. Barton and C. Kacser [Phys. Rev. Letters 8, 226 (1962)].

Due to two algebraic errors the integral $F_{1,1}$ and the quadratic coefficients were given incorrectly. The correct equations are:

$$F_{1,1} = \frac{a_1}{2\pi}\left\{\frac{2-k^2}{2\omega^4} + \frac{3k}{2\omega^5}\ln(\omega - k)\right\} = 0.0445\,a_1,$$

$$C_1 = (16\,\epsilon^2)[J_2 + H_1] = [0.23\,a_2 + 0.039\,a_1],$$

$$C_1' = (32\,\epsilon^2/3)[J_0 - J_2 - 3H_1] = [0.093\,a_0 - 0.16\,a_2 - 0.078\,a_1],$$

$$C_2 = (2\epsilon^2/3)[2J_0 + J_2 - 3H_1] = [0.012\,a_0 + 0.0097\,a_2 - 0.0049\,a_1],$$

$$C_2' = (4\epsilon^2)[J_2 + H_1] = [0.058\,a_2 + 0.0098\,a_1].$$