

This model of the K^+ decay may clarify the long-standing problem of why the ratio for $K^+ \rightarrow 2\pi$ versus $K_1^0 \rightarrow 2\pi$ is equal to 1/500 rather than $\alpha^2 \sim 1/20\,000$. The point is that our sequence gives a decay rate proportional to $(m_K - m_\zeta)^{-2}$. Let us compare this situation with the K_1^0 decay. Since this decay is allowed by $\Delta T = \frac{1}{2}$, the sequence $K_1^0 \rightarrow \zeta^0 \rightarrow 2\pi$ is unlikely to play an important role.¹² There is no known intermediate state compatible with $\Delta T = \frac{1}{2}$ which can give rise to as small an energy denominator. Therefore this mass effect may produce the necessary enhancement of the K^+ rate even in the absence of strong π - π forces at the K^+ -rest energy. However, such arguments would be invalid if close-lying $T=0$, 0^+ particles were to exist.

We further note that if the sequence Eq. (6) indeed dominates the K^+ decay, it follows that the $2\pi\gamma/2\pi$ branching ratio of K^+ would equal that of ζ^+ . Also the $2\pi\gamma$ spectrum would be the same in the two cases. Measurements of this could provide a test of our model of the K^+ decays.

In conclusion we reiterate that the relatively straightforward measurement of the various 2π rates of the ζ 's is sufficient to determine the isotopic spin character of ζ decay and the charge conjugation quantum number of ζ^0 .

We would like to thank Dr. J. Sanford, Dr. E. Pickup, and Dr. B. Sechi Zorn for discussing

with us the experimental material, and N. P. Chang and T. Yao for pointing out an error in an earlier version of this note.

*Alfred P. Sloan Foundation Fellow.

¹R. Barloutaud, J. Heughebaert, A. Leveque, J. Meyer, and R. Omnes, Phys. Rev. Letters **8**, 32 (1962).

²A. Erwin, R. March, W. Walker, and E. West, Phys. Rev. Letters **6**, 628 (1961).

³C. Peck, L. Jones, and M. Perl, University of Michigan Technical Report No. 4, 1962 (unpublished).

⁴B. Sechi Zorn, Phys. Rev. Letters **8**, 282 (1962).

⁵ Γ always denotes the rate.

⁶M. Nauenberg and A. Pais, Phys. Rev. Letters **8**, 82 (1962).

⁷G. Feinberg, Phys. Rev. Letters **8**, 151 (1962).

⁸R. Peierls and S. Treiman, this issue [Phys. Rev. Letters **8**, 339 (1962)].

⁹See a similar calculation by R. Good, Phys. Rev. **113**, 352 (1959), for $K \rightarrow 2\pi\gamma$. In the present case we omit the inner bremsstrahlung as this can be important only if 2π is the dominant mode.

¹⁰P. Bastien *et al.*, Phys. Rev. Letters **8**, 114, 302(E) (1962).

¹¹A possible explanation of this effect in terms of a light 2π resonant state with $T=0$ was pointed out to us by Dr. M. Nauenberg.

¹²A similar argument shows that if the η^0 decay is T -forbidden, the one- η^0 intermediate state will not play an important role in the K_2^0 mesonic decay. However, it is possible that the decays $K_1^0 \rightarrow 2\gamma$, $K_2^0 \rightarrow 2\gamma$ could go through intermediate states of a single ζ^0 , η^0 , respectively.

COMPLEX ANGULAR MOMENTA AND THE RELATION BETWEEN THE CROSS SECTIONS OF VARIOUS PROCESSES AT HIGH ENERGIES

V. N. Gribov

Physico-Technical Institute, Leningrad, U.S.S.R.

and

I. Ya. Pomeranchuk

Institute of Theoretical and Experimental Physics, Moscow, U.S.S.R.

(Received March 19, 1962)

Recently a connection has been found between the analytical properties of the amplitudes as functions of the angular momenta, and their asymptotic behavior at high energies.^{1,2} It is assumed that the asymptotic behavior of the scattering amplitudes of any particles in the diffraction region is determined by the moving pole $j(t)$ of a partial wave in the annihilation channel.³⁻⁶ Several important properties of strong interactions at high energies (s) follow from this assumption. In particular, the amplitude of the

elastic scattering of strongly interacting particles must have the form $f(t)s^{j(t)}$ (s and t are the usual Mandelstam variables). The total cross section is constant if $j(0)$ assumes the maximum possible value, equal to 1.⁷ The elastic scattering cross section must tend slowly to zero (as $1/\ln s$). The diffraction cone must narrow with increasing energy; this behavior corresponds to the scattering on a system whose transparency and radius increase with the energy.⁸ Such behavior seems to be in agreement with the experimental data

recently obtained.⁸ Up to now²⁻⁶ only the properties of $j(t)$ were discussed. It was emphasized, in particular, that $j(t)$ is the same for the various reactions. We show that the unitarity conditions on complex j , obtained by one of us (V.G.),² lead to a great number of relations for the functions $f(t)$, corresponding to the various reactions.

Particular cases of these relations are simple connections between total interaction cross sections for various particles at high energies (s). For example,

$$\sigma_{AB}/\sigma_{AC} = \sigma_{DB}/\sigma_{DC},$$

where σ_{AB} is the total interaction cross section for the particles A and B at the energy $s \rightarrow \infty$.

Hence, for instance, we have

$$\sigma_{\pi N}^2 = \sigma_{\pi\pi}\sigma_{NN}, \quad \sigma_{KN}^2 = \sigma_{KK}\sigma_{NN},$$

$$\sigma_{\Lambda N}^2 = \sigma_{NN}\sigma_{\Lambda\Lambda}, \quad \sigma_{K\Lambda}^2 = \sigma_{KK}\sigma_{\Lambda\Lambda}, \quad (1)$$

$$\sigma_{\Lambda N}/\sigma_{NN} = \sigma_{\pi\Lambda}/\sigma_{\pi N} = \sigma_{K\Lambda}/\sigma_{KK}. \quad (2)$$

Thus, using the values $\sigma_{NN} \approx 40$ mb, $\sigma_{\pi N} \approx 25$ mb, $\sigma_{KN} \approx 22$ mb, we obtain $\sigma_{\pi\pi} \approx 16$ mb, $\sigma_{KK} \approx 12$ mb.

To derive these relations, let us write the unitarity conditions for the amplitudes of the partial waves in the annihilation (t) channel $4\mu^2 < t < 16\mu^2$, where μ is the mass of the π meson. The choice of this interval and the exclusive choice of the π -meson states are due to the fact that we can write the exact unitarity condition only in that region.

For simplicity let us consider the spinless particles, π and K mesons. Let f_j , g_j , and h_j be the amplitudes of the partial waves of the reactions $\pi + \pi \rightarrow \pi + \pi$, $K + \bar{K} \rightarrow \pi + \pi$, and $K + \bar{K} \rightarrow K + \bar{K}$, in a given isotopic spin state [to each value of the isotopic spin corresponds a particular $j(t)$]. Then the unitarity condition has the form²

$$\begin{aligned} (1/2i)(f_j - f_j^*) &= (k/\omega)f_j f_j^*, \\ (1/2i)(g_j - g_j^*) &= (k/\omega)g_j f_j^*, \\ (1/2i)(h_j - h_j^*) &= (k/\omega)g_j g_j^*, \end{aligned} \quad (3)$$

where k and ω are the momentum and energy of the π meson:

$$\omega = \frac{1}{2}t^{1/2}, \quad k = \frac{1}{2}(t - 4\mu^2)^{1/2}.$$

If we express f_j , g_j , h_j through f_j^* , g_j^* , h_j^* :

$$\begin{aligned} f_j &= \frac{f_j^*}{1 - 2i(k/\omega)f_j^*}, \\ g_j &= \frac{g_j^*}{1 - 2i(k/\omega)f_j^*}, \\ h_j &= h_j^* + 2i \frac{k}{\omega} \frac{(g_j^*)^2}{1 - 2i(k/\omega)f_j^*}, \end{aligned} \quad (4)$$

then it can be seen that all the amplitudes have a pole at a $j(t)$ for which $(k/\omega)f_j^* = (1/2i)$. For j close to $j(t)$ we have

$$f_j^* = (\omega/2ik)\{1 - (1/\beta)[j - j(t)]\}. \quad (5)$$

Substituting (5) into (4), we obtain

$$f_j = \frac{\beta(\omega/2ik)}{j - j(t)}, \quad g_j = \frac{\beta g_j^*(t)}{j - j(t)}, \quad h_j = 2i(k/\omega)\beta \frac{(g_j^*(t))^2}{j - j(t)}. \quad (6)$$

Hence it follows that the residues of these amplitudes, $r_{\pi\pi}$, $r_{\pi K}$, and r_{KK} , satisfy the simple relation

$$r_{\pi K}^2(t) = r_{\pi\pi}(t)r_{KK}(t). \quad (7)$$

This relation corresponds precisely to the connection between the probabilities of the various processes in the case where they pass through one Breit-Wigner level.

Since $r_{\pi\pi}$, r_{KK} , and $r_{\pi K}$ are analytic functions of t , this relation is valid at any t . The total cross section at high energies is determined by the pole in the isotopic spin zero state and is connected with $r(0)$ by the relations

$$\begin{aligned} \sigma_{\pi\pi} &= 12\pi^2(1/\mu^2)r_{\pi\pi}(0), \\ \sigma_{\pi K} &= 12\pi^2(1/m\mu)r_{\pi K}(0), \\ \sigma_{KK} &= 12\pi^2(1/m^2)r_{KK}(0), \end{aligned} \quad (8)$$

where m is the K -meson mass. Hence

$$\sigma_{\pi K}^2 = \sigma_{\pi\pi}\sigma_{KK}. \quad (9)$$

If we considered the spinless particles π , A , B , and C , we should obtain in just the same way a relation of the form

$$r_{\pi\pi}/r_{\pi A} = r_{B\pi}/r_{BA} = r_{C\pi}/r_{CA}, \quad (10)$$

and others. Thus relations of the type (2) follow.

The nucleon spin, as detailed analysis shows, does not change the relation between the total cross sections obtained for spinless particles. Unfortunately, one cannot now compare the relation given above with experiment due to the instability of all the strongly interacting particles other than the nucleon.

If we include, however, the photon,⁹ we obtain, in the same manner, the relation between the total cross sections of the photon-nucleon, the nucleon-nucleon, and the photon-photon interactions:

$$\sigma_{\gamma N} = \sigma_{\gamma\gamma} \sigma_{NN} \quad (11)$$

$\sigma_{\gamma\gamma}$ may be found from the experimental study of the interaction of the photon with the Coulomb field of nuclei. The latter, for small nuclear recoil, is separable from the purely nuclear interaction.¹⁰

Let us list a number of other consequences following from the same unitarity relations:

(1) We have considered up to now the processes whose asymptotic form is determined by the pole having (according to the terminology of Chew and Frautschi) the vacuum quantum numbers. If we consider the processes whose asymptotic form is determined by the other poles, we can obtain a large number of relations between their amplitudes using the unitarity condition: for example, the relations between the values of the amplitudes of the processes $\pi^- + p \rightarrow \pi^0 + n$, $\gamma + N \rightarrow \pi + N$, and $\gamma + \pi \rightarrow 2\pi$ (the latter is observed from the process $\pi \rightarrow \pi + \pi$ on the Coulomb field of nuclei).

(2) The spin structures of the amplitudes for various processes are closely connected with each other. For example, the spin correlation experiments for nucleon-nucleon, π -nucleon, and K -nucleon scattering are determined by a single parameter.

(3) Interesting questions arise if we extend the relations mentioned above to the nuclei.¹¹ A number of observable relations arise here, whose regions of validity are not, however, absolutely clear, since the nuclear processes have anomalous thresholds.

In conclusion we should like to point out that according to (8) the residue in the pole of the partial wave for π - π scattering is

$$r_{\pi\pi} \sim 1/12\pi^2 \approx 10^{-2},$$

since

$$\sigma_{\pi\pi} \sim 1/\mu^2.$$

The authors feel that the smallness of the terms appearing here is connected with a slow change of the position of the vacuum pole $j(t)$, since the imaginary part of $j(t)$ [$j''(t)$] in the region $t > 4\mu^2$ is proportional to $r_{\pi\pi}(t)$ if $j''(t)$ is small.¹² [$j(t)$ changes essentially when t changes to a value of the order $m^2 \approx 40\mu^2$.^{5,8}]

We should like to express our gratitude to Ya. Azimov, V. V. Anisovitch, I. T. Dyatlov, B. L. Ioffe, I. Yu. Kobsarev, L. B. Okun', and A. P. Rudik for numerous interesting discussions. We are indebted to S. C. Frautschi, M. Gell-Mann, F. Zachariasen, G. F. Chew, and S. Mandelstam for the preprints of their papers.

¹T. Regge, *Nuovo cimento* **14**, 951 (1959); **18**, 947 (1960).

²V. N. Gribov, *J. Exptl. Theoret. Phys. (U.S.S.R.)* **41**, 1962 (1961) [translation: *Soviet Phys. - JETP* (to be published)].

³V. N. Gribov, *J. Exptl. Theoret. Phys. (U.S.S.R.)* **41**, 667 (1961) [translation: *Soviet Phys. - JETP* **14**, 478 (1962)].

⁴G. F. Chew and S. C. Frautschi, *Phys. Rev. Letters* **7**, 394 (1961).

⁵S. C. Frautschi, M. Gell-Mann, and F. Zachariasen (to be published).

⁶G. Domokosh, Dubna report (to be published).

⁷M. Froissart, *Phys. Rev.* **123**, 1053 (1961).

⁸Yu. D. Bayukov, G. A. Leksin, and Ya. Ya. Shalimov, *J. Exptl. Theoret. Phys. (U.S.S.R.)* **41**, 1025 (1961) [translation: *Soviet Phys. - JETP* (to be published)]; G. Cocconi, A. N. Diddens, E. Lillethun, G. Manning, A. E. Taylor, T. G. Walker, and A. M. Wetherell, *Phys. Rev. Letters* **7**, 450 (1961).

⁹The authors are indebted to L. B. Okun' for pointing out this application to the photon interactions.

¹⁰O. Piccioni, *Proceedings of the Ninth Annual International Conference on High-Energy Physics, Kiev, 1959* (Academy of Sciences, Moscow, 1960); I. Ya. Pomeranchuk and I. M. Shmushkevitch, *Nuclear Phys.* **23**, 452 (1961).

¹¹The authors are indebted to V. V. Anisovitch and B. L. Ioffe who attracted their attention to these problems and their specific properties.

¹²V. N. Gribov and I. Ya. Pomeranchuk, *J. Exptl. Theoret. Phys. (U.S.S.R.)* (to be published).