METHOD FOR MEASURING THE DECAY WIDTH OF THE ω MESON*

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We propose a method for measuring the (partial) width Γ of the decay

$$\omega^0 \to \pi^+ + \pi^- + \pi^0, \tag{1}$$

using the fact that the cross section for

$$\pi^- + \pi^0 \to \omega^0 + \pi^- \tag{2}$$

is related to Γ in a fairly model-independent way. The cross section for (2) turns out to be of the order of a millibarn for a width Γ of a few hundred kev at about 200 Mev (c.m.) above the threshold. The well-known Chew-Low extrapolation method[1] may be used to deduce the desired cross section for (2) from a study of the reaction

$$\pi^- + p \to \pi^- + \omega^0 + p. \tag{3}$$

We discuss the processes (1) and (2) under two extreme dynamical assumptions. First we assume that the ω-3π vertex is zero-ranged in the sense that it is given by the centrifugal barrier alone as follows:

$$\left(f_{\omega 3\pi}/m^3\right)\epsilon_{\mu\nu\lambda\sigma}\epsilon_\mu^{(\omega)}k_\nu^{(+)}k_\lambda^{(-)}k_\sigma^{(0)}, \tag{4}$$

where $k_\nu^{(+)}$, $k_\lambda^{(-)}$, and $k_\sigma^{(0)}$ refer to the pion four-momenta, and $\epsilon_\mu^{(\omega)}$ stands for the polarization vector of the ω meson. The factor m^3 is inserted to make the coupling constant $f_{\omega 3\pi}$ dimensionless. The decay width Γ can be calculated to be[2]

$$\Gamma = \left(\frac{f_{\omega 3\pi}^2}{4\pi}\right)\left(\frac{(m_\omega - 3m_\pi)^4 m_\pi^2 m_\omega U(m_\omega)}{2^6\times 3^3\times 3^{1/2}\pi m^6}\right), \tag{5}$$

where $U(m_\omega)$ is a relativistic correction factor which approaches unity as $m_\omega \to 3m_\pi$. For $m_\omega = 787$ Mev, we find numerically that $U \approx 1.6$. The differential cross section and the total cross section for (2) are given by

$$d\sigma/d\Omega = (f_{\omega 3\pi}^2/4\pi)p_i p_f^3 \sin^2\theta/16\pi m^6, \tag{6}$$

$$\sigma = (f_{\omega 3\pi}^2/4\pi)p_i p_f^3/6m^6, \tag{7}$$

where p_i (p_f) is the momentum of one of the incident (outgoing) particles in the center-of-mass system. For a Γ of 400 kev, we obtain a cross section of 4.0 mb at a total c.m. energy of 1.1 Bev (173 Mev above the $\omega\pi$ threshold). For a total c.m. energy $\gtrsim 1.2$ Bev, the predicted cross section with $\Gamma = 400$ kev becomes comparable to the p-wave unitarity limit for inelastic processes, $3\pi\lambda\!\!\!^{-2}$, so that the zero-range model must break down.

The most general expression for the ω-3π vertex with all particles on the mass shell is of the form

$$\epsilon_{\mu\nu\lambda\sigma}\epsilon_\mu^{(\omega)}k_\nu^{(+)}k_\lambda^{(-)}k_\sigma^{(0)}F(x,y),$$

where x and y are two independent invariant scalars that can be constructed from the external four-momenta. In the zero-range model considered in the preceding paragraph, $F(x,y)$ has been taken to be constant. In contrast, Gell-Mann, Sharp, and Wagner[3] suggest a model of ω decay in which the dispersion representation for $F(x,y)$ is assumed to be dominated by ρ-meson intermediate states. We call this the ρ-dominance model since we can visualize the decay interaction as proceeding via

$$\omega \to \rho + \pi, \tag{8}$$

followed by

$$\rho \to 2\pi. \tag{9}$$

Factors that enter in a diagram at the vertices (8) and (9) are, respectively,

$$\left(f_{\omega\rho\pi}/m\right)\epsilon_{\mu\nu\lambda\sigma}\epsilon_\mu^{(\omega)}k_\nu^{(\omega)}\epsilon_\lambda^{(\rho)}k_\sigma^{(\rho)} \tag{10}$$

and

$$f_{\rho\pi\pi}\epsilon_\mu^{(\rho)}\left(k_\mu^{(\pi 1)} - k_\mu^{(\pi 2)}\right). \tag{11}$$

We then have[3]

$$\Gamma = \left(\frac{f_{\omega\rho\pi}^2}{4\pi}\right)\left(\frac{f_{\rho\pi\pi}^2}{4\pi}\right)\left(\frac{m_\omega m_\pi^2(m_\omega - 3m_\pi)^4 W(m_\omega)}{12\times 3^{1/2}(m_\rho^2 - 4m_\pi^2)^2 m^2}\right), \tag{12}$$

where $W(m_\omega)$ is a relativistic correction factor which has been numerically estimated to be about 3.6 at $m_\omega = 787$ Mev. The constant $f_{\rho\pi\pi}^2/4\pi$ is about 2 for a ρ width of 100 Mev; the only remaining constant, $f_{\omega\rho\pi}^2/(4\pi m^2)$, can be estimated from the π^0 lifetime à la Gell-Mann and Zachariasen[4] if the ω and ρ are coupled universally to the conserved hypercharge current and the isospin current[5] with the coupling constants f_ρ and f_ω, or, equivalently, if ω and ρ dominate the dispersion integrals for the isoscalar and isovector charge form factors for every strongly interacting particle. The coupling constant for π^0 decay, defined as in Eq. (10), can then be written as

$$f_{\pi\gamma\gamma} = e^2 f_{\omega\rho\pi}/f_\rho f_\omega. \tag{13}$$

From these considerations and unitary symmetry[6] (which requires $f_\rho^2/4\pi = \frac{4}{3} f_\omega^2/4\pi$), Gell-Mann, Sharp, and Wagner[3] estimate $\Gamma \sim 400$ kev.

Along similar lines we can discuss the ω production process (2) by keeping only one-ρ-meson states that appear in the s, t, and u channels. In this ρ-dominance model, the production cross section directly measures the product $f_{\omega\rho\pi} f_{\rho\pi\pi}$, hence Γ. The differential cross section is given by

$$\frac{d\sigma}{d\Omega} = \left(\frac{f_{\rho\pi\pi}^2}{4\pi}\right)\left(\frac{f_{\omega\rho\pi}^2}{4\pi}\right)\left(\frac{p_i p_f^3 \sin^2\theta}{m^2}\right) \times \left[\frac{1}{s-m_\rho^2} + \frac{1}{t-m_\rho^2} + \frac{1}{u-m_\rho^2}\right]^2, \tag{14}$$

where[7]

$$s = 4E_{\pi i}^2,$$

$$t = 2m_\pi^2 - 2E_{\pi i}E_{\pi f} + 2p_i p_f \cos\theta,$$

$$u = 2m_\pi^2 - 2E_{\pi i}E_{\pi f} - 2p_i p_f \cos\theta,$$

$$E_{\pi i} = (p_i^2 + m_\pi^2)^{1/2}, \quad E_{\pi f} = (p_f^2 + m_\pi^2)^{1/2}.$$

The expression (14) can be integrated analytically to give the total cross section as a function of energies. The numerical results are shown in Fig. 1 where the solid curve represents the ω production cross section expected for $\Gamma = 400$ kev. Also shown is $3\pi\lambda\!\!\!^-{}^2$ (dashed curve).

We may remark that for a given value of Γ, the ρ-dominance model predicts a cross section four times smaller than the zero-range model at $s^{1/2} = 1.1$ Bev. If we are to choose between the two models, the ρ-dominance model is probably the more reasonable. In any case, it is gratifying that at least the order of magnitude of Γ can be determined from the production cross section in a model-independent way.

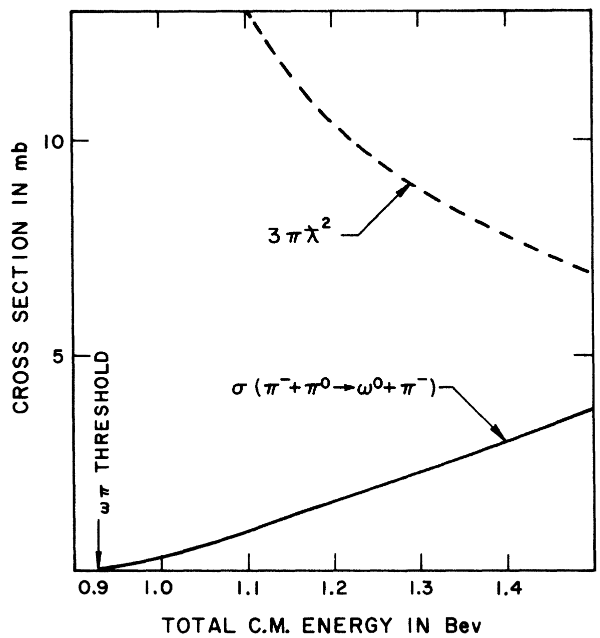

FIG. 1. The solid curve represents the cross section for $\pi^- + \pi^0 \to \omega + \pi^-$ for $\Gamma = 400$ kev as predicted by the ρ-dominance model. The dashed curve represents the p-wave unitarity limit for inelastic processes, $3\pi\lambda\!\!\!^-{}^2$.

It is hardly necessary to emphasize that the well-known Chew-Low method[1] can be used to extract the desired cross section for (2) from a study of the reaction (3). If the ω width is really of the order of 400 kev or greater, experiments along this line seem feasible in a hydrogen bubble chamber with a π^- beam of $p_{\rm lab} \sim 2$-3 Bev/c.[8]

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[1]G. F. Chew and F. E. Low, Phys. Rev. 113, 1640 (1959).

[2]Equation (5) does not agree with Eqs. (3) and (4) of G. Feinberg [Phys. Rev. Letters 8, 151 (1962)]. Our expression does agree with the $m_\rho \to \infty$ limit of Eq. (1) of reference 3.

[3]M. Gell-Mann, D. Sharp, and W. D. Wagner, Phys. Rev. Letters 8, 261 (1962). Thanks are due to the authors of this reference for many helpful discussions.

[4]M. Gell-Mann and F. Zachariasen, Phys. Rev. 124,

953 (1961).

[5]J. J. Sakurai, Ann. Phys. (New York) 11, 1 (1960).

[6]M. Gell-Mann, California Institute of Technology Synchrotron Laboratory Report CTSL-20, 1961 (unpublished); and Phys. Rev. 125, 1067 (1962). See also Y. Ne'eman, Nuclear Phys. 26, 222 (1961). If the ρ is coupled to twice the isospin current, and the ω to $\sqrt{3}$ times the hypercharge current, then the couplings of ρ and ω become "universal" in the unitary symmetry limit. Note that our f_ρ is equal to $2\gamma_\rho$ of Gell-Mann.

[7]Recall also the well-known relation $s+t+u=3m_\pi^2+m_\omega^2$.

[8]The reaction (3) is currently being studied by the Wisconsin and the Berkeley hydrogen bubble-chamber groups. This work has been stimulated in part by their experimental programs.

ERRATUM

DECAY MODES AND WIDTH OF THE η MESON. Pierre L. Bastien, J. Peter Berge, Orin I. Dahl, Massimiliano Ferro-Luzzi, Donald H. Miller, Joseph J. Murray, Arthur H. Rosenfeld, and Mason B. Watson [Phys. Rev. Letters 8, 114 (1962)].

On page 116, ninth line of text, $\Gamma(\pi^+\pi^-\pi^0)$ should read Γ(all modes).

In line 16 of the legend of Fig. 1, $\sigma(K^-p\to\Lambda\pi^0\pi^0)$ should read $\sigma(K^-n\to\Lambda\pi^0\pi^-)$.

In the ninth line of the legend of Fig. 2, $K^-p\to\Lambda\pi^+\pi^-\pi^0$ should be replaced by $K^-d\to p\Lambda\pi^+\pi^-\pi^-$.