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LOW-TEMPERATURE LIMIT OF GRÜNEISEN'S GAMMA OF GERMANIUM AND SILICON

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Gibbons[1] has measured the thermal expansion of Ge, Si, and InSb at low temperatures. Using these data and values of the heat capacity measured by various investigators, together with the Grüneisen relation,[2]

$$\gamma = \alpha V / \chi_T C_v, \tag{1}$$

he obtains a plot of γ vs T/Θ_∞ reproduced as the solid lines of Fig. 1. α is the volume coefficient of thermal expansion, V the crystal volume, χ_T the isothermal compressibility, and C_v the heat capacity at constant volume. This plot indicates an anomalous negative peak for the case of Si and InSb, but not for Ge over the range of temperatures investigated. Gibbons has indicated by the dashed lines an extrapolation of γ toward zero as T/Θ_∞ goes to zero. It is known that these materials with diamond-like structures reveal extraordinarily similar behavior in other lattice properties such as the temperature dependence of their Debye temperatures and even their lattice spectra.[3] It seemed, then, worth investigating this situation wherein a difference of behavior was observed. Sheard[4] discusses a way of obtaining high- and low-temperature limiting values of γ from a knowledge of the pressure dependence of the elastic constants of a solid, involving the averaging of a property over all the directions of a crystal. We have found a simple and quick method of obtaining the limiting value of γ as T approaches zero, from the following considerations. Derivation[2] of the relation (1) on the assumption that the Debye temperature is independent of temperature, which should be expected to be valid at very low temperatures, yields $-\gamma = d\ln\Theta / d\ln V$. It is possible to calculate the limiting value of the Debye temperature at 0°K from the values of the elastic constants, molar volume, and density of a material; de Launay[5] has prepared tables from which one can easily evaluate Θ_0 using the relation:

$$\Theta_0{}^3 = \frac{9N}{4\pi V}\left(\frac{h}{k}\right)^3 \left(\frac{C_{44}}{\rho}\right)^{3/2} \frac{9}{18+\sqrt{3}} f(s,t), \tag{2}$$

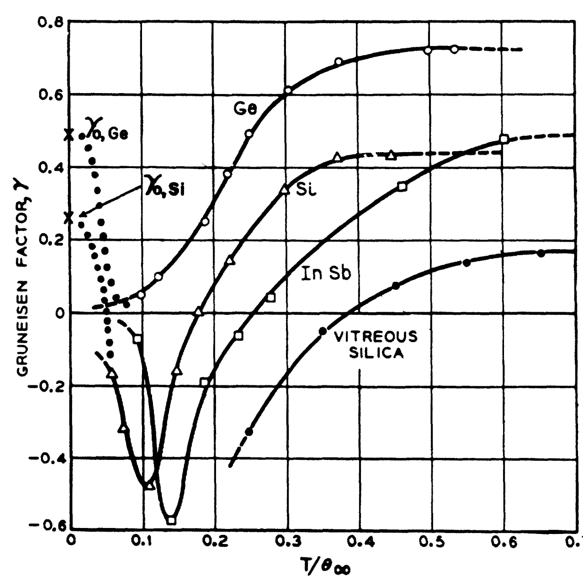

FIG. 1. Variation of Grüneisen factor with reduced temperature T/Θ_∞ for germanium ($\Theta_\infty = 400$°K), silicon ($\Theta_\infty = 674$°K), vitreous silica ($\Theta_\infty = 495$°K), and indium antimonide ($\Theta_\infty = 214$°K).[1]

where

$$s = (C_{11} - C_{44})/(C_{12} + C_{44}), \quad t = (C_{12} - C_{44})/C_{44}.$$

N is the total number of atoms, V is the volume, ρ is the density, and the remaining symbols have their usual meanings. $f(s, t)$ is presented in tabular form. From reference 2 one can obtain

$$\frac{d\ln\Theta_0}{d\ln V} = \frac{1}{2}\frac{d\ln C_{44}}{d\ln V} + \frac{1}{6} + \frac{1}{3}\frac{d\ln f(s,t)}{d\ln V},$$

which can be evaluated using the tables and the known values of C_{ij} and $d\ln C_{ij}/d\ln V$ for Ge[6] and Si.[7] Term by term the results are, for Ge and Si,

Ge: $d\ln\Theta_0/d\ln V = -\gamma_0 = -0.751 + 0.167 + 0.092 = -0.492;$

Si: $-\gamma_0 = -0.490 + 0.167 + 0.073 = -0.250.$

Note that the third term, involving the interpolation in the table and arising from the change of elastic anisotropy and Poisson ratios with volume, is a relatively small correction for these materials. The values of γ_0 so obtained are indicated by $\times$'s on the $T/\Theta_\infty = 0$ ordinate of Fig. 1. A possible interpolation of the data is indicated by a dotted line, whence the γ of Ge does exhibit the same behavior as that of Si and InSb. It seems probable that Gibbons' extrapolation given by the dashed line is incorrect in the case of Ge and that the similarity of behavior of Ga, Si, and InSb is preserved.

[1]D. F. Gibbons, Phys. Rev. 112, 136 (1958).
[2]See, for example, C. Kittel, Introduction to Solid-State Physics (John Wiley & Sons, New York, 1956), 2nd ed., pp. 153-155.
[3]J. C. Phillips, Phys. Rev. 113, 147 (1958).
[4]F. W. Sheard, Phil. Mag. 3, 1381 (1958).
[5]J. de Launay, in Solid-State Physics, edited by F. Seitz and D. Turnbull (Academic Press, Inc., New York, 1956), Vol. 2, p. 219.
[6]H. J. McSkimin, J. Acoust. Soc. Am. 30, 314 (1958).
[7]J. C. Chapman, Masters thesis, Case Institute of Technology, Cleveland, Ohio (unpublished).

NEW PHENOMENON IN MAGNETORESISTANCE OF BISMUTH AT LOW TEMPERATURE

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In the course of studying the galvanomagnetic properties at low temperatures in pure bismuth at a magnetic field above several kilo-oersteds, we have found a strong nonlinear conduction behavior; that is, a sharp change of slope in the current-voltage curve at a certain high electric field, which we call the kink field, as illustrated in Fig. 1. Each trace was taken at constant transverse magnetic field ($B \parallel$ trigonal).

Several specimens of approximate cross section 1 mm^2 and length from 0.5 to 5 mm were carefully cut from pure bismuth single crystals grown by the Czochralski technique.[1] In the present experiment, the direction of current flow was chosen parallel to the bisector direction between the binary and the bisectrix axes and the temperature was around 2°K.

The solid curve in Fig. 2 indicates the well-known Shubnikov–de Haas oscillatory behavior of our specimen in the transverse magnetoresistance with period $\Delta(1/B) = e\hbar/E_f m^* c \sim 1.5 \times 10^{-5}$ oersted^{-1}, where E_f is the Fermi energy and the magnetic field B is parallel to the trigonal axis. The lower curve in Fig. 2 shows the differential magnetoresistance derived from the straight line beyond the kink field in the current-voltage curve. The background of the latter curve is fairly independent of the magnetic field, contrary to the strong magnetic field dependency of the former curve, though the latter curve also seems to show some oscillatory effect. It should be noted

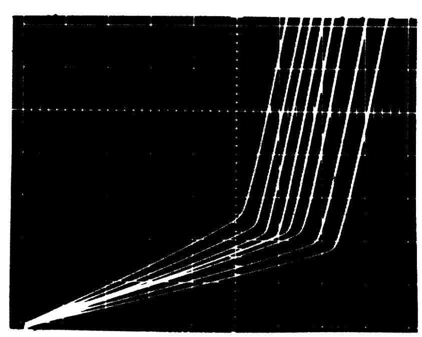

FIG. 1. Current-voltage curves showing the kink. Each curve corresponds to 14, 15, 16, 17, 18, 19, 20, and 21 kilo-oersteds from left to right. The abscissa and the ordinate are 0.2 volt/div and 100 ma/div, respectively. ($B \parallel$ the trigonal axis.)