are plotted in Fig. 2, one obtains a probability of 0.95 for $\xi = -9$ compared with a probability of 0.01 for $\xi = +2$. The apparent agreement between the detailed shape of the $\xi = -9$ curve and the experimental spectrum is consistent with the assumption that f_- and f_+ are not strongly energy dependent, but the experimental errors preclude placing useful quantitative limits on that dependence on the basis of these results.

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[1]See, for example, N. Brene, L. Egardt, and B. Qvist, Nuclear Phys. 22, 553 (1961), which also contains references to earlier theoretical work on the leptonic decay modes of K mesons.

[2]J. L. Brown, J. A. Kadyk, G. H. Trilling, R. T. Van de Walle, B. P. Roe, and D. Sinclair, Phys. Rev. Letters 7, 423 (1961).

[3]The data of reference 2 are actually consistent with constant f_+ or with rough, but quite restrictive, limits on the energy dependence of f_+.

[4]B. P. Roe, D. Sinclair, J. L. Brown, D. A. Glaser, J. A. Kadyk, and G. H. Trilling, Phys. Rev. Letters 7, 346 (1961), which also summarizes the results of earlier measurements.

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EVIDENCE AGAINST PARTIALLY CONSERVED CURRENTS IN K_{l3}^+ DECAY

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The hypothesis that the divergence of the weak current is a "gentle" operator in the sense of mild high-energy behavior of its matrix elements has been proposed by many theorists.[1,2] The motivations for such a hypothesis include the possibility of a simple derivation of the Goldberger-Treiman relation for pion decay,[3] and various symmetry considerations.[1] In the case of $|\Delta S| = 1$ decays, it was first pointed out by Bernstein and Weinberg[2] that the condition (j_ρ denotes the weak current operator),

$$(4E_K E_\pi)^{1/2}(p_K - p_\pi)_\rho \langle \pi | j_\rho | K\rangle \equiv D(s) \to 0$$

$$\text{as } s = -(p_K - p_\pi)^2 \to \infty, \tag{1}$$

together with some additional plausible assumptions, leads to complete determination of the matrix elements of $K_{\mu 3}$ and K_{e3} decays up to a constant factor. In this note we shall clarify the role of some of the assumptions made in reference 2 and show that even the weaker assumption,

$$D(s) \to \text{constant as } s \to \infty, \tag{2}$$

leads to consequences difficult to reconcile with the recent experimental data on the muon spectrum in $K_{\mu 3}^+$ decay[4] and the observed branching ratio,[5]

$$R(K_{\mu 3}^+)/R(K_{e3}^+) \equiv \rho = 1 \pm 0.2.$$

Following the notation of Bernstein and Weinberg[2] we write the matrix element for K_{l3}^+ decay in the form

$$\langle \pi^0 l^+ \nu | s | K^+\rangle = \bar{\nu}\gamma_\rho(1+\gamma_5)v(p_l)\langle \pi^0 | j_\rho | K^+\rangle (m_l/E_l)^{1/2}, \tag{3}$$

with

$$\langle \pi^0 | j_\rho | K^+\rangle = \frac{i(2\pi)^{-3}}{(4E_K E_\pi)^{1/2}}\left[f_V(s)p_{K\rho} + g_V(s)(p_K - p_\pi)_\rho\right]. \tag{4}$$

Defining $-i(2\pi)^3(4E_K E_\pi)^{1/2}\langle \pi | j_\rho | K\rangle \equiv \Lambda_\rho$ and going over to the K-π center-of-mass frame, $\vec{p}_K - \vec{p}_\pi = 0$, we find

$$\vec{\Lambda} = f_1(s)\vec{p}_K, \tag{5}$$

$$\Lambda_0 = s^{1/2} f_0(s), \quad s = -(p_K - p_\pi)^2, \tag{6}$$

where $f_1(s)$ and $f_0(s)$ denote the P- and S-wave amplitudes, respectively. The f's are related to g_V and f_V by

$$f_1(s) = f_V(s), \tag{7}$$

$$f_0(s) = g_V(s) + \tfrac{1}{2}\left(1 + \frac{m_K^2 - m_\pi^2}{s}\right) f_V(s). \tag{8}$$

Assumption (2), together with (5) and (6), implies that

$$(p_K - p_\pi)_\rho \Lambda_\rho = -s^{1/2}(p_K - p_\pi)_0 f_0(s)$$
$$= -sf_0(s) \to \text{constant as } s \to \infty. \tag{9}$$

Hence,

$$f_0(s) \to \text{constant}/s \text{ as } s \to \infty. \tag{10}$$

According to (8), $f_0(s)$ has a kinematical pole at $s=0$ with residue $(m_K^2 - m_\pi^2) f_V(0)/2$. We first show that this residue is different from zero [this is necessary for the solutions (15) and (16) below to be meaningful]. In K_{e3} decay, the term involving $g_V(s)$ in (4) becomes multiplied by the electron mass and may be neglected. The rate of K_{e3}^+ decay is therefore (note that $s = m_K^2 + m_\pi^2 - 2m_K E$, $E \equiv E_\pi$), neglecting the electron mass,

$$\int_0^{(m_K - m_\pi)^2} |f_V(s)|^2 \phi(s) ds, \tag{11}$$

where $\phi(s)$ is a phase-space factor. Recent experiments indicate that $f_V(s)$ is practically constant over the physical region in K_{e3} decay.[6] We conclude therefore that $f_V(0) \neq 0$, and may write in view of (10) an unsubtracted dispersion relation for $f_0(s)$ for $K + \pi \to l + \nu$:

$$f_0(s) = \frac{m_K^2 - m_\pi^2}{2s} f_V(0) + \frac{1}{\pi} \int_{(m_K + m_\pi)^2}^{\infty} \frac{\operatorname{Im} f_0(s')}{s' - s - i\epsilon} ds',$$
$$s = -(p_K + p_\pi)^2. \tag{12}$$

For $f_1(s)$, we make the weak assumption that

$$f_1(s) \leq \text{constant as } s \to \infty, \tag{13}$$

and write a dispersion relation with one subtraction [recall that $f_1(s) = f_V(s)$]:

$$f_1(s) = f_V(0) + \frac{s}{\pi} \int_{(m_K + m_\pi)^2}^{\infty} \frac{\operatorname{Im} f_1(s') ds'}{s'(s' - s - i\epsilon)}. \tag{14}$$

In view of the K-π resonance,[7] it is reasonable to assume that the K-π state gives the main contribution to the dispersion integrals. In this approximation $\operatorname{Im} f_l$ is simply related to δ_l, the K-π scattering phase shift for $j = l$, and (12), (13) may be solved by standard methods[8] to give

$$f_0(s) = \frac{m_K^2 - m_\pi^2}{2s} f_V(0) e^{\rho_0(s)}, \tag{15}$$

$$f_1(s) = f_V(0) e^{\rho_1(s)}, \tag{16}$$

where

$$\rho_l(s) = \frac{s}{\pi} \int_{(m_K + m_\pi)^2}^{\infty} \frac{\delta_l(s') ds'}{s'(s' - s - i\epsilon)}.$$

The form factors for K_{l3} decay are obtained from (15) and (16) by analytic continuation to $s < (m_K + m_\pi)^2$. For a sharp S-wave K-π resonance at $s = M^2$ and weak P-wave scattering[9] [i.e., with $\delta_0(s) = \pi\theta(s - M^2)$ and $\delta_1(s) \doteq 0$], (15) and (16) reduce to

$$f_V(s) \equiv f_1(s) = f_V(0)$$

and

$$f_0(s) = \frac{M^2(m_K^2 - m_\pi^2)}{2s(M^2 - s)} f_V(0),$$

or

$$g_V(s)/f_V(s) = \tfrac{1}{2}\left(\frac{m_K^2 - m_\pi^2}{M^2 - s} - 1\right),$$

which is the Bernstein-Weinberg result.[2]

We have computed the branching ratio $\rho = R(K_{\mu 3}^+)/R(K_{e3}^+)$ and the muon spectrum in $K_{\mu 3}^+$ decay on the basis of (15) and (16). For an S-wave K-π resonance at 880 Mev we find $\rho = 0.69$, and for a P-wave resonance at 880 Mev, $\rho = 0.64$.[10] Both values are below the observed value,[5] $\rho = 1 \pm 0.2$, by more than the statistical error. With either an S- or a P-wave resonance, the muon spectrum peaks at $(E_\mu - m_\mu) \doteq 65$ Mev,[11] being practically indistinguishable from the spectrum corresponding to $\xi = 2$ given in reference 4, and is in disagreement with experiment.

The conclusions of the preceding paragraph are quite insensitive to the detailed shapes of the phase shifts, and are essentially the same whether the resonance is replaced by a pole or whether one uses more accurate phase shifts, taking into account the width of the resonance (which is rather narrow).[7,10] This is mainly due to the large distance of the physical region of the decay $[m_l^2 \leq s \leq (m_K - m_\pi)^2 \doteq 7m_\pi^2]$ from the threshold of the dispersion integrals $[(m_K + m_\pi)^2 \doteq 22\, m_\pi^2]$. This large distance has the effect of rendering the form factors slowly varying functions of the momentum transfer s over the physical region of the decay, the dispersion integrals in (12) and

(14) being dominated by the inhomogeneous terms $[(m_K^2 - m_\pi^2)/2s]f_V(0)$ and $f_V(0)$. Thus, even if the weight functions of the integrals in (12) and (14) are grossly in error by, say, a factor of two owing to the neglect of higher mass states [the next state being the $K\pi\pi$ state with threshold at $(m_K + 2m_\pi)^2 \doteq 32\, m_\pi^2$; the contribution due to three-body channels, moreover, is expected to be small near threshold], the disagreement with experiment would still persist. (As an extreme and perhaps unrealistic example we put in a K-π bound state, with $l=0$ at 600 Mev, in addition to a P-wave K-π resonance at 880 Mev. The branching ratio ρ becomes 0.75 and the peak in the muon spectrum shifts from $E_\mu - m_\mu = 65$ Mev by less than 5 Mev, the shape being practically unchanged. If there are resonances in both S- and P-wave channels at 880 Mev, ρ again takes the value 0.64 with little change in the muon spectrum.)

In summary, we may state that assuming the K_{l3}^+ decay interaction is V-A, as appears to be the case at least in K_{e3}^+ decay,[6] the weakened form[2] of the partially conserved current hypothesis,[1,2] together with the additional weak assumption (13), leads to predictions about the $K_{\mu 3}/K_{e3}$ branching ratio and about the muon spectrum which seem to be in disagreement with experiment.[12] A more decisive test of these predictions might be obtained by measuring the longitudinal polarization of the muon in $K_{\mu 3}^+$ decay. Equations (15) and (16) predict small, positive polarizations at all muon energies,[11] whereas the phenomenological "negative solution"[13] described in references 4 and 13 predicts large, negative polarizations at all but the lowest muon energies.[11]

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[9]A small P-wave phase shift does not lead to a vanishing P-wave amplitude because a subtraction has been made in (14).

[10]Recent experimental work on the decay of the resonance [H. Ticho, Bull. Am. Phys. Soc. 7, 81(T) (1962)] seems to rule out the possibility of a $j=0$ resonance.

[11]More details will be given in a forthcoming paper prepared in collaboration with S. Bludman.

[12]It should be noted that $|\Delta I| = \frac{1}{2}$ has been assumed implicitly in the above discussions. If the $|\Delta I| = \frac{3}{2}$ amplitudes turn out to be large [for which there is some evidence in K_{l3}^0 decay; see R. P. Ely et al., Phys. Rev. Letters 8, 132 (1962)], $K\pi$ scattering in the $I=\frac{3}{2}$ state as well as the $I=\frac{1}{2}$ state must be considered and our conclusions may have to be modified. Conversely, if (2) and (10) are valid, the discrepancy between the predictions of (15), (16), and experiment may provide a measure of the strength of the $|\Delta I| = \frac{3}{2}$ amplitudes in K_{l3}^+ decay.

[13]If f_V and g_V are both assumed to be constants, N. Brene et al. [Nuclear Phys. 22, 553 (1961)] have shown that the branching ratio ρ is given by $0.65 + 0.124\,\xi + 0.019\,\xi^2$, where $\xi = 1 + 2g_V/f_V$. With the observed value $\rho = 1 \pm 0.2$, this gives a "positive solution" $\xi = 2$ and a "negative solution" $\xi = -9$. The "positive solution" gives muon spectrum and polarization, etc., which do not differ much from those calculated from (15) and (16).[11] The muon spectrum reported in reference 4, however, is in good agreement with the "negative solution" and in complete disagreement with the "positive solution."