RATIO OF FORM FACTORS IN THE LEPTONIC DECAYS OF THE K^+ MESON*

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The three-particle leptonic decay modes of the K^+ meson are presently assumed to be governed by a decay interaction that is the product of a $(V-A)$ leptonic current and a strangeness-nonconserving current due to strongly interacting particles.[1] In the approximation that the lepton and neutrino arise from a local interaction, the matrix element for the decay depends only on p_π and p_K, and may be written[1] as

$$M=[\tfrac{1}{2}f_+(p_K+p_\pi)+\tfrac{1}{2}f_-(p_K-p_\pi)]_\lambda \bar{u}(p_\nu)\gamma_\lambda(1+\gamma_5)v(p_L), \quad (1)$$

where the form factors f_+ and f_- may be scalar functions of $(p_K-p_\pi)^2$, and time-reversal invariance requires that f_+ and f_- be real.

Under the assumption that f_+ and f_- are constants, which is anticipated to be a reasonable approximation, the meson and electron energy spectra from the three-particle decay modes, $K_{\mu 3}^+$ and K_{e3}^+, may be calculated explicitly. According to the universal weak interaction, the matrix elements for the two decay modes should be identical except for the difference in mass of muon and electron. The latter difference leads to a considerable simplification in the form of the electron energy spectrum from K_{e3}^+, since terms containing m_e/m_K may be neglected, and the resultant expression depends only on f_+. The expression for the muon energy spectrum from $K_{\mu 3}^+$, although more complicated, is nevertheless completely specified by the parameter, $\xi=f_-/f_+$. It follows, then, that the ratio of the total transition probabilities for $K_{\mu 3}^+$ and K_{e3}^+ is also completely specified by ξ. In fact, for K^+ mesons, for the approximations stated above, this ratio is

$$R=W_{\mu 3}^{\text{tot}}/W_{e3}^{\text{tot}}=0.651+0.126\,\xi+0.0189\,\xi^2. \quad (2)$$

Recent measurements[2] of the pion momentum spectrum and of the pion-electron angular correlation from the mode K_{e3}^+ are consistent with the vector (axial vector) coupling hypothesis and with the assumption that f_+ is constant.[3] Further, the branching ratios for $K_{\mu 3}^+$ and K_{e3}^+ have been measured[4] to give $R=1.0\pm 0.2$. On this basis, then, ξ must lie in either the interval $-9.7<\xi<-7.7$ or the interval $1.0<\xi<-3.0$, where the size of the intervals is determined by the experimental error on R.

In this note we report the results of a measurement at the Brookhaven Cosmotron of the energy spectrum of muons from $K_{\mu 3}^+$ utilizing a filamentary chamber—image intensifier system to determine muon ranges. Briefly, the experimental arrangement was as follows. A counter telescope identified K^+ mesons stopping in a plastic scintillator, S, below which, in a vertical plane, stood a filamentary chamber, F, consisting of 52 alternate horizontal layers of plastic scintillator filaments and thin gold plates to provide sufficient stopping power for the charged-particle products of three-particle K^+ decays. A stopping K^+ meson generated an appropriately delayed electronic gate of duration approximately equal to $3\tau_{K^+}$. A charged particle entering F during the gate turned on the image intensifier system whose output was a photograph of the projection of the track of the particle in F. In addition to this track photograph, there were recorded for each event the time sequence of pulses in the stopping counter S and also the time sequence of pulses in F. Time information from S helped to confirm that a given photographed track originated in the decay of a K^+ meson; time information from F served to identify by their decays a number of positive pions which arise from the decay modes $K_{\pi 2}^+$ and $K_{\pi 3}^+$.

In Fig. 1(a) there is presented the observed range spectrum of charged particles that originated in the decay of a K^+ meson in S, which includes π^+ from $K_{\pi 3}^+$ (below plate 12), π^+ from $K_{\pi 2}^+$ (centered about plate 35), and μ^+ from $K_{\mu 3}^+$. In Fig. 1(b) is shown the observed range spectrum for particles identified as π^+ by their decay time. These data are the result of analysis of somewhat more than one-half of all film acquired during the experiment. Neither of the spectra in Fig. 1 is corrected for the difference in solid angle subtended by plates at different distances from the K^+ stopping region. The number of pions in (b) is not equal to the number in (a) because some muons arising from pion decays lose all or most of their energy in a gold plate and hence their parent pion is not unambiguously identified. The

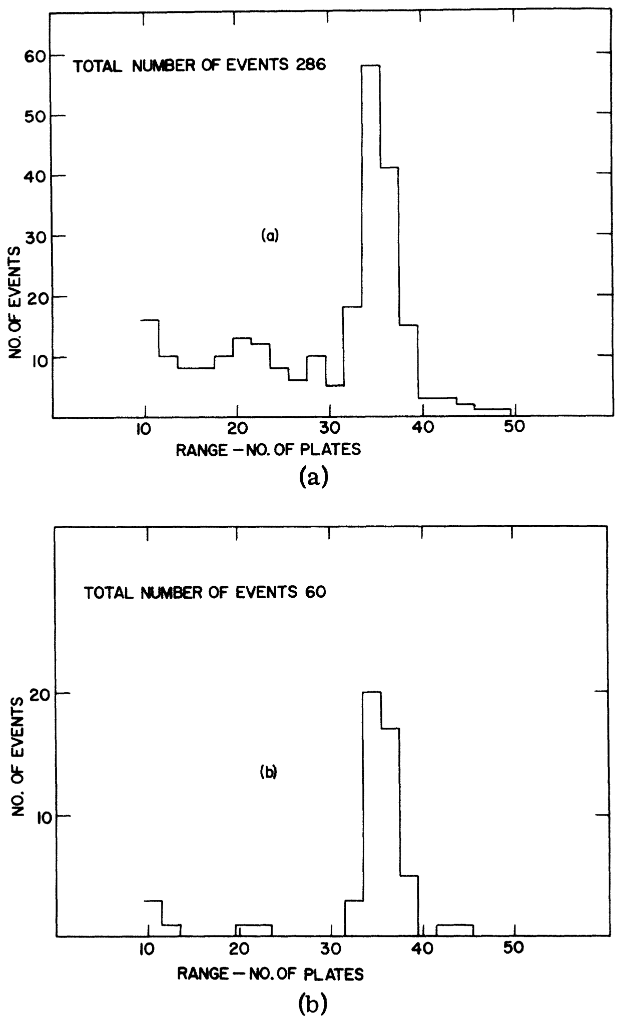

FIG. 1. (a) Total range spectrum including π^+ from $K_{\pi 3}{}^+$ (below plate 12), π^+ from $K_{\pi 2}{}^+$ (centered about plate 35), and μ^+ from $K_{\mu 3}{}^+$. (b) Range spectrum of particles with decay times less than $3\tau_\pi$. Neither spectrum is corrected for the difference in solid angle subtended by plates at different distances from the K^+ stopping region.

apparatus used here is unable to measure the ranges of mesons of kinetic energy less than about 50 Mev, where the distributions in Fig. 1 terminate.

Using a modified least-squares fit requiring smoothness of the resultant curve, we have subtracted the pions from the total spectrum of Fig. 1(a) to obtain the muon energy spectrum, corrected for solid angle, shown in Fig. 2. Since approximately 75% of the π^+ from $K_{\pi 2}{}^+$ in Fig. 1 are contained within a three-plate (~5.5 Mev) interval, the subtraction procedure leads to a large

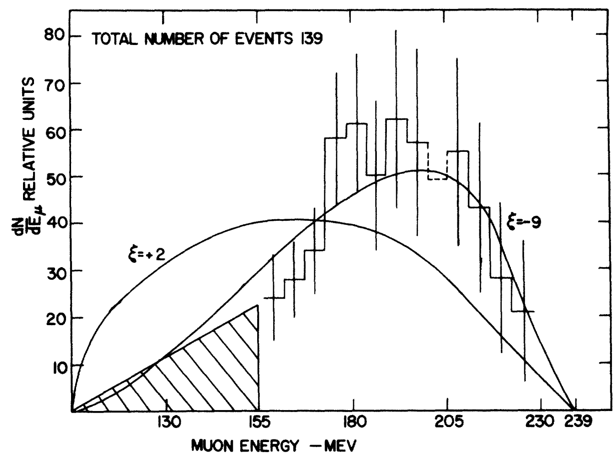

FIG. 2. Experimental muon energy spectrum (histogram) and theoretical distributions for $\xi = +2$ and $\xi = -9$. The cross-hatched area is obtained from emulsion data (reference 5). All curves are normalized to the same total transition probability.

statistical uncertainty only in that point (indicated as a dashed line) in the muon spectrum immediately below the π^+ peak. The possibility of scanning bias was investigated in an auxiliary run in which additional material was inserted between S and F such that π^+ from $K_{\pi 2}{}^+$ stopped at plate 15 and μ^+ from $K_{\mu 2}{}^+$ stopped at plate 40. The number of events observed in the two groups, when corrected for relative solid angle, gave a relative branching ratio in good agreement with the accepted value, indicating that scanning bias was small compared with the statistical uncertainty.

To aid in a detailed comparison of the experimental and theoretical spectra, we have utilized the $K_{\mu 3}{}^+$ data obtained in photographic emulsions[5] to provide an appropriately normalized value of the total number of muons between 0 and 50 Mev that should be added to the experimental spectrum of Fig. 2. This is shown as the cross-hatched area in Fig. 2 (which necessarily falls to zero at the rest energy of the muon), and permits normalization of the total transition probabilities of the theoretical spectra, calculated subject to the assumptions described above, and the experimental spectrum. A χ^2 fit yields with 95% confidence the following limits on ξ:

$$\xi \leq -2 \text{ or } \xi \geq +5,$$

which result in part from the experimental errors and in part from the fact that, for large ξ, the theoretical distribution is dominated by a term in ξ^2. For the two values of ξ specified by the measured ratio R, for which the distributions are plotted in Fig. 2, one obtains a probability of 0.95 for $\xi = -9$ compared with a probability of 0.01 for $\xi = +2$. The apparent agreement between the detailed shape of the $\xi = -9$ curve and the experimental spectrum is consistent with the assumption that f_- and f_+ are not strongly energy dependent, but the experimental errors preclude placing useful quantitative limits on that dependence on the basis of these results.

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[1]See, for example, N. Brene, L. Egardt, and B. Qvist, Nuclear Phys. 22, 553 (1961), which also contains references to earlier theoretical work on the leptonic decay modes of K mesons.

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[3]The data of reference 2 are actually consistent with constant f_+ or with rough, but quite restrictive, limits on the energy dependence of f_+.

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EVIDENCE AGAINST PARTIALLY CONSERVED CURRENTS IN K_{l3}^+ DECAY

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The hypothesis that the divergence of the weak current is a "gentle" operator in the sense of mild high-energy behavior of its matrix elements has been proposed by many theorists.[1,2] The motivations for such a hypothesis include the possibility of a simple derivation of the Goldberger-Treiman relation for pion decay,[3] and various symmetry considerations.[1] In the case of $|\Delta S| = 1$ decays, it was first pointed out by Bernstein and Weinberg[2] that the condition (j_ρ denotes the weak current operator),

$$(4E_K E_\pi)^{1/2}(p_K - p_\pi)_\rho \langle \pi | j_\rho | K \rangle \equiv D(s) \to 0$$

$$\text{as } s = -(p_K - p_\pi)^2 \to \infty, \quad (1)$$

together with some additional plausible assumptions, leads to complete determination of the matrix elements of $K_{\mu 3}$ and K_{e3} decays up to a constant factor. In this note we shall clarify the role of some of the assumptions made in reference 2 and show that even the weaker assumption,

$$D(s) \to \text{constant as } s \to \infty, \quad (2)$$

leads to consequences difficult to reconcile with the recent experimental data on the muon spectrum in $K_{\mu 3}^+$ decay[4] and the observed branching ratio,[5]

$$R(K_{\mu 3}^+)/R(K_{e3}^+) \equiv \rho = 1 \pm 0.2.$$

Following the notation of Bernstein and Weinberg[2] we write the matrix element for K_{l3}^+ decay in the form

$$\langle \pi^0 l^+ \nu | s | K^+ \rangle = \bar{\nu}\gamma_\rho (1 + \gamma_5) v(p_l) \langle \pi^0 | j_\rho | K^+ \rangle (m_l/E_l)^{1/2}, \quad (3)$$

with

$$\langle \pi^0 | j_\rho | K^+ \rangle = \frac{i(2\pi)^{-3}}{(4E_K E_\pi)^{1/2}} \left[f_V(s) p_{K\rho} + g_V(s)(p_K - p_\pi)_\rho \right]. \quad (4)$$

Defining $-i(2\pi)^3(4E_K E_\pi)^{1/2}\langle \pi | j_\rho | K \rangle \equiv \Lambda_\rho$ and going over to the K-π center-of-mass frame, $\vec{p}_K - \vec{p}_\pi = 0$, we find

$$\vec{\Lambda} = f_1(s)\vec{p}_K, \quad (5)$$

$$\Lambda_0 = s^{1/2} f_0(s), \quad s = -(p_K - p_\pi)^2, \quad (6)$$

where $f_1(s)$ and $f_0(s)$ denote the P- and S-wave amplitudes, respectively. The f's are related to g_V and f_V by

$$f_1(s) = f_V(s), \quad (7)$$

$$f_0(s) = g_V(s) + \tfrac{1}{2}\left(1 + \frac{m_K^2 - m_\pi^2}{s}\right) f_V(s). \quad (8)$$