shows a portion of the upper one expanded by ×1000 (sweep rate = 5 msec/division). The sharp spikes at the top of each excursion correspond to stimulated emission. The excursions themselves are due to the fluorescence and stray lamp light and have the same shape as the light output from the AH-6 lamp.

Figure 4(b) shows continuous maser action with dc power applied to the lamp. The upper trace shows the output of the PbSe detector, while the lower trace shows the dc light intensity as monitored by a phototube. Both traces are swept at 5 sec/division. Note that in the middle of the 50-sec sweep the lamp intensity was reduced by ≈20% for 7 sec and the maser action during this period was observed to cease. At the end of the sweep the lamp is extinguished, showing the zero level for both traces. The violent fluctuations in the maser output envelope are due to the ac ripple in the dc supply. When threshold is barely exceeded, a small ripple in the pump intensity is a large variation in that part of the light intensity which is above threshold. Power levels observed thus far are around 10 microwatts. This may well be increased by switching to continuous xenon lamps, which have more energy around 0.9 μ than do mercury lamps.

The relaxation oscillations present in the pulsed operation of CaF_2:U^{3+} maser[4] were observed to die out in a few milliseconds after the onset of maser action. Such also has been observed to be the case in the continuous Nd^{3+} maser,[9] whereas in the continuous ruby maser[10] the relaxation oscillations are observed to persist indefinitely.

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[†] The first announcement of the continuous operation of the CaF_2:U^{3+} maser was made at the January, 1962 meeting of the American Physical Society in New York.

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TEMPERATURE OF THE TRANSITION TO THE SUPERCONDUCTING STATE

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The ordinates of Fig. 1 are "observed" values of the Bardeen-Cooper-Schrieffer (BCS) parameter $[N(0)V]$ for various metals, derived by the usual formula expressing the transition temperature of a superconductor.[1] Where superconductivity has not been detected, the point of the arrow rests on the maximum value that $[N(0)V]$ can have if there is, in fact, a transition to superconductivity below the lowest temperature which has so far been investigated for that metal.[2] The point for Bi refers to the superconducting transition observed in films condensed at very low temperatures.[3]

Along the horizontal axis is plotted

$$[N(0)V]_{\text{theory}} = 3Z\frac{m}{M}\left(\frac{\mathcal{U}}{k\Theta}\right)^2 + \lambda' - \mu^*, \tag{1}$$

where Z is the valency, M the mass of an atom, and Θ the Debye temperature of the metal.[1] We take $(\lambda' - \mu^*) = 0.1$ for all metals. The value of $\mathcal{U}$ is derived from the measured electrical resistiv-

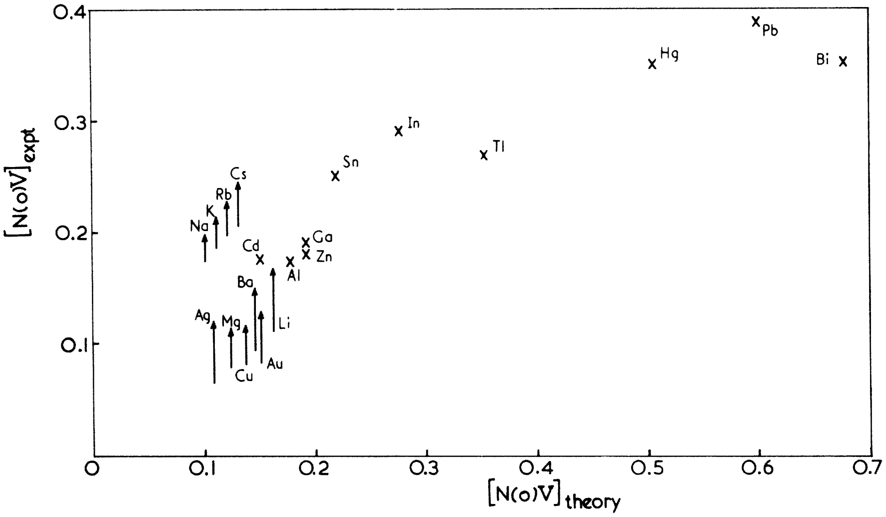

FIG. 1. Correlation of experimental and theoretical estimates of the BCS parameter for different metals. "Nonsuperconductors" have been studied down to values of $[N(0)V]$ indicated by the points of the arrows.

ity ρ_L of the liquid metal, just above its melting point:

$$\mathcal{U}^2 = \frac{\hbar e^2 Z N v_F^{\,2}}{3\pi \langle a(K)\rangle}(\rho_L - \rho_{\text{plasma}}). \tag{2}$$

v_F is the velocity of an electron on the Fermi sphere containing ZN electrons per unit volume; $\langle a(K)\rangle$ is a number—taken to be 0.4 for monovalent metals and nearly unity for polyvalent metals—derived from the observed x-ray or neutron scattering function of the liquid metal[4]; ρ_{plasma} is a theoretical[4] contribution to the electrical resistivity—a negligible correction for polyvalent metals. The electrical resistance in the liquid state is not known for Be, Ca, or Sr, nor for the transition elements Ti, Zr, V, Nb, Ta, Mo, U.[5] Magnetic metals such as Mn, Fe, Co, Ni, and the semiconductors Si and Ge are also excluded, for obvious reasons.

For superconducting elements there is obviously a close empirical correlation between experimental and theoretical estimates of $[N(0)V]$. For "nonsuperconductors" we have two alternative possibilities. It may be that the empirical correlation curve extends down to the origin, and that these elements would all eventually turn out to be superconducting if studied at sufficiently low temperatures.[1] On the other hand, it may be that $[N(0)V] > 0$ is not a sufficient criterion for superconducting, and that the parameter must be greater than about 0.15 for the transition to occur at all.

The argument for our formula is as follows. It can be shown[4] that $\mathcal{U}$ is a measure of the matrix element of the effective potential of an ion for large-angle scattering of free electrons. The same quantity will occur in the interaction between electrons and elastically displaced ions in the solid metal. That is, $\mathcal{U}$ plays the role of a rigid-ion potential in the electron-phonon interaction[6]—the quantity that appears in the old Bloch theory[7] as the constant C. The first term in Eq. (1) is then the corresponding attractive part of the BCS interaction, assuming that the density of states $N(0)$ has its free-electron value, and that the average of $(K/k_F)^2(q_D/q)^2$, over all scattering vectors K and all phonon wave vectors q, is unity.

The term $-\mu^*$ arises from the electrostatic repulsion of the electrons. According to Morel and Anderson,[1] this is about -0.1, being very insensitive to the electron density.

For the term λ' we must consider the part of the electron-phonon interaction corresponding to small-angle scattering. It can easily be shown[6,8] that the "constant" C must then tend to $-\frac{2}{3}\mathcal{E}_F$ as $K \to 0$ and must take the form

$$C(K) \approx -4\pi e^2 ZN/(k_s^{\,2} + K^2), \tag{3}$$

where k_s is a screening parameter. Indeed, Pines,[9] Morel,[1] and Morel and Anderson[1] took this to be true for *all* values of K, and made $[N(0)V]$ appear to depend only on the valency and electron density in the metal. Even after multiplication by m^*/m, these formulas were unable to distinguish adequately between Mg and Zn, between Al and Tl, or between Sn and Pb. The fact is that (3) only describes the screened Coulomb potential of an ion outside the core and is valid only for small K; when $2\pi/K$ is of the order of the diameter of the core, the repulsive pseudopotential[4,10] becomes important, and $C(K)$ be-

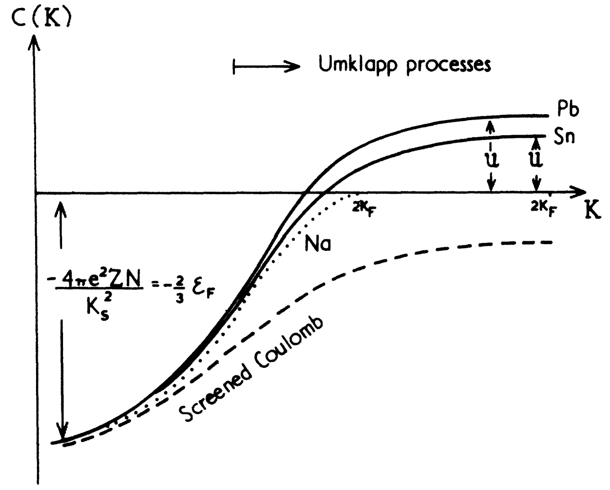

FIG. 2. The function $C(K)$ occurring in the theory of the electron-phonon interaction, showing the difference between the screened Coulomb potential assumed by Pines,[9] Morel,[1] etc., and the form suggested by the theory of pseudopotentials.

comes equal to $\mathcal{U}$, say, which has different values for different metals (Fig. 2).

But we must not neglect the contribution of this screened Coulomb term. In the liquid metal it gives rise to the plasma resistance,[4] which is not negligible in monovalent metals; in Na, where $\mathcal{U}$ is zero, the plasma resistance seems to be the whole effect. We can estimate λ', the contribution of this term to the BCS parameter, by taking over from Morel and Anderson[1] their calculated result for a standard monovalent metal. In their notation,

$$\lambda' = \tfrac{1}{2}a^2/(1+a^2) \sim 0.2. \tag{4}$$

Except for small variations in the value of a^2, this correction should be about the same for metals of all valencies, because we assume that $C(K)$ only has this value for $K < q_D$; i.e., the screened Coulomb term does not contribute to Umklapp processes.

These corrections are very approximate and should not really be quite the same for all metals. Moreover, we have no a priori reason for expecting exact numerical agreement between theoretical and experimental estimates of $[N(0)V]$. The average of $(K/k_F)^2(q_D/q)^2$ need not be exactly 1; it need not even be the same for different metals. It may be that one should draw two separate lines, for Ag, Cd, Sn, In, and for Au, Tl, Hg, Pb, as if these were a systematic "period effect." But closer numerical analysis of the problem must wait on a more exact theory of $C(K)$.

An alternative formula can be deduced by using the resistivity of the solid metal in (2) and replacing $\langle a(K)\rangle$ by an equivalent quantity—in fact[4] by $kTN\beta = kT\hbar^2 q_D^{\ 2}/Mk^2\Theta^2$. This result looks simpler, in that $[N(0)V]$ becomes independent of Θ and of M; this is actually the original formula suggested by Fröhlich[11] as a criterion for superconductivity. One would have expected good results from such a formula, since a superconductor is always a solid, but the numerical correlation with experiment is not nearly as striking as in Fig. 1. The electrical resistivity of a solid metal is very sensitive to the crystal structure, and is not determined so directly by the interaction between free electrons and individual ions as it is in the liquid state. It seems as if the superconducting transition also is not very sensitive to the details of the zone structure, lattice spectrum, etc.

I am indebted to Dr. P. W. Anderson for advice and discussion, and for a copy of his paper in advance of publication. Professor A. B. Pippard reminded me that Bi is indeed a superconductor.

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