DECAY RATES OF NEUTRAL MESONS*

M. Gell-Mann, D. Sharp,† and W. G. Wagner‡
California Institute of Technology, Pasadena, California
(Received February 15, 1962)

The partial width for the observed decay mode $\omega \to \pi^+ + \pi^- + \pi^0$ can be estimated by assuming, in the sense of dispersion theory, that the process is dominated by $\omega \to \rho + \pi$ followed by $\rho \to 2\pi$. We treat ρ as a nearly stable particle and define[1] a coupling parameter $f_{\omega\rho\pi}$. Then the width comes out to be

$$\Gamma(\omega \to 3\pi) = (m_\omega - 3m_\pi)^4 (m_\rho^{\,2} - 4m_\pi^{\,2})^{-2} m_\omega m_\pi^{\,2}\, 3^{-3/2} \times (\gamma_{\rho\pi\pi}^{\,2}/4\pi)(f_{\rho\omega\pi}^{\,2}/4\pi) W(m_\omega), \qquad (1)$$

where $\gamma_{\rho\pi\pi}^{\,2}/4\pi$ is the coupling constant[2-4] for ρ decay into 2π, in terms of which we have

$$\Gamma(\rho \to 2\pi) = \tfrac{1}{3}(\gamma_{\rho\pi\pi}^{\,2}/4\pi)(m_\rho^{\,2} - 4m_\pi^{\,2})^{3/2} m_\rho^{\,-2}. \qquad (2)$$

The factor $W(m_\omega)$ is a correction factor for relativistic kinematics and approaches unity as $m_\omega \to 3m_\pi$. For $m_\omega = 787$ Mev, we find numerically that $W = 3.56$.

A virtual photon can turn into ρ^0; at such a vertex in a diagram we insert[3,4] the constant $em_\rho^{\,2}/2\gamma_\rho$. Then the strength of the ρ resonance $m_\rho^{\,2}(m_\rho^{\,2} - t)^{-1}$ in the electric form factor of the pion is just $\gamma_{\rho\pi\pi}/\gamma_\rho$. If the ρ resonance dominates the form factor, then $\gamma_{\rho\pi\pi}/\gamma_\rho$ is of the order of unity. We may now estimate the rate of the decay $\omega \to \pi^0 + \gamma$ assuming that it is dominated, in the sense of dispersion theory, by $\omega \to \pi^0 + \rho^0$ followed by $\rho^0 \to \gamma$. As in reference 3, we obtain

$$\Gamma(\omega \to \pi^0 + \gamma) = \alpha(\gamma_\rho^{\,2}/4\pi)^{-1}(m_\omega^{\,2} - m_\pi^{\,2})^3 m_\omega^{\,-3} \times (96)^{-1}(f_{\rho\omega\pi}^{\,2}/4\pi). \qquad (3)$$

If we take a width of 100 Mev for ρ, we have $\gamma_{\rho\pi\pi}^{\,2}/4\pi = \frac{1}{2}$; assuming that $\gamma_\rho^{\,2}/4\pi$ has the same value, we find for the branching ratio

$$\Gamma(\omega \to \pi^0 + \gamma)/\Gamma(\omega \to 3\pi) = 0.17. \qquad (4)$$

Corrections for $\gamma_{\rho\pi\pi}/\gamma_\rho \neq 1$ and for different ρ widths can easily be made.

We may also try to interpret the π^0 decay as dominated by the vertex $\pi^0 \to \rho^0 + \omega^0$ followed by $\rho^0 \to \gamma$, $\omega^0 \to \gamma$. At the γ-ω vertex, we insert the constant $em_\omega^{\,2}/(2\sqrt{3}\gamma_\omega)$. If we define the vector coupling of ω to nucleons, for example, to have the strength $\sqrt{3}\gamma_{\omega NN}$, then $\gamma_{\omega NN}/\gamma_\omega$ gives the strength of the ω resonance $m_\omega^{\,2}(m_\omega^{\,2} - t)^{-1}$ in the electric isoscalar form factor of the nucleon. The factor $\sqrt{3}$ is used so that in the limit of unitary symmetry[2,4] we have $\gamma_\omega \to \gamma_\rho$. The π^0 decay rate then comes out to be

$$\Gamma(\pi^0 \to 2\gamma) = \alpha^2(\gamma_\rho^{\,2}/4\pi)^{-1}(\gamma_\omega^{\,2}/4\pi)^{-1}(192)^{-1} \times m_\pi^{\,3}(f_{\rho\omega\pi}^{\,2}/4\pi). \qquad (5)$$

[The ratio $\Gamma(\omega^0 \to \pi^0 + \gamma)/\Gamma(\pi^0 \to 2\gamma)$ in reference 3 is too small by a factor of 4.] From the measured π^0 decay width of ~3 ev, we can now estimate $f_{\rho\omega\pi}^{\,2}/4\pi$ if $\gamma_\rho^{\,2}/4\pi$ and $\gamma_\omega^{\,2}/4\pi$ are known.

Direct measurement of $\gamma_\rho^{\,2}/4\pi$ and $\gamma_\omega^{\,2}/4\pi$ is possible by means of the direct decay of the neutral vector meson into an electron or muon pair. If m_l is the lepton mass, then the decay rates are[5]:

$$\Gamma(\omega \to l^+ + l^-) = \alpha^2(\gamma_\omega^{\,2}/4\pi)^{-1}(m_\omega/36)(1 - 4m_l^{\,2}/m_\omega^{\,2})^{1/2} \times (1 + 2m_l^{\,2}/m_\omega^{\,2}), \qquad (6)$$

$$\Gamma(\rho^0 \to l^+ + l^-) = \alpha^2(\gamma_\rho^{\,2}/4\pi)^{-1}(m_\rho/12)(1 - 4m_l^{\,2}/m_\rho^{\,2})^{1/2} \times (1 + 2m_l^{\,2}/m_\rho^{\,2}). \qquad (7)$$

In the absence of such information, we crudely estimate $\gamma_{\rho\pi\pi}^{\,2} \approx \gamma_\rho^{\,2}$ as above and take $\gamma_\omega^{\,2} \approx \gamma_\rho^{\,2}$ on the basis of conjectured approximate unitary symmetry. The estimated partial widths for the various modes of ω decay are then given as in Table I, for $\gamma_\omega^{\,2}/4\pi = \gamma_\rho^{\,2}/4\pi = \gamma_{\rho\pi\pi}^{\,2}/4\pi = \frac{1}{2}$. The dependences on $\gamma_\rho^{\,2}$, $\gamma_{\rho\pi\pi}^{\,2}$, and $\gamma_\omega^{\,2}$ indicated in the table are for fixed π^0 lifetime. Other values of these parameters can be inserted at will.

Since the total width of ω is expected to be <1 Mev, exotic decay modes occur with considerable branching fractions. An important one is $\omega \to \pi^+ + \pi^-$,

Table I. Estimated partial widths of ω^0 decay.

Mode	Partial width (kev)	Dependence on γ_ρ^2, $\gamma_{\rho\pi\pi}^2$, γ_ω^2
$\omega \to \pi^+ + \pi^- + \pi^0$	395	$\gamma_\rho^2 \gamma_{\rho\pi\pi}^2 \gamma_\omega^2$
$\omega \to \pi^0 + \gamma$	69	γ_ω^2
$\omega \to \pi^+ + \pi^-$	17	Roughly $\gamma_{\rho\pi\pi}^2 \gamma_\rho^{-2} \gamma_\omega^{-2}$
$\omega \to e^+ + e^-$	2.3	γ_ω^{-2}
$\omega \to \mu^+ + \mu^-$	2.3	γ_ω^{-2}

which is caused by a small electromagnetic mixing of ω and ρ^0, as noted by Glashow.[6] If mixing occurs with amplitude y, then we have $\Gamma(\omega \to \pi^+ + \pi^-) = y^2 \times 100$ Mev, and $y^2 \sim 1/5000$ is sufficient to give a branching fraction of several percent. A very crude calculation of the mixing by means of $\omega \to \gamma \to \rho^0$ gives the entry in Table I, in agreement with reference 5.

The recently discovered neutral meson[7,8] at about 550 Mev may be pseudoscalar with $G = +1$; if so, we call it χ as in references 2 and 4. The forbidden decay rates into $3\pi^0$ and $\pi^+ + \pi^- + \pi^0$ are difficult to estimate, except that $3\pi^0/(\pi^+ + \pi^- + \pi^0) \leq \frac{3}{2}$. The remaining neutral decays are expected, however, to represent $\chi \to 2\gamma$. The decay $\chi \to 2\gamma$ may be described roughly on the assumption that the important intermediate steps are $\chi \to 2\rho^0$ (followed by $\rho^0 \to \gamma$, $\rho^0 \to \gamma$) and $\chi \to 2\omega$ (followed by $\omega \to \gamma$, $\omega \to \gamma$). We now wish to estimate the ratio of this rate to that of the hitherto unobserved charged decay mode $\chi \to \pi^+ + \pi^- + \gamma$, which should be dominated by $\chi \to 2\rho^0$, followed by $\rho^0 \to \gamma$, $\rho^0 \to \pi^+ + \pi^-$. First we ignore the dissociation of $\chi \to 2\omega$. The ratio is then easy to compute numerically and comes out

$$\Gamma(\chi \to \pi^+ + \pi^- + \gamma)/\Gamma(\chi \to 2\gamma) \approx \tfrac{1}{2}(\gamma_{\rho\pi\pi}^2/4\pi)(\gamma_\rho^2/4\pi). \tag{8}$$

If we make use of unitary symmetry, we can estimate the correction factor to be applied to (8) for the inclusion of $\chi^0 \to 2\omega \to 2\gamma$, namely $\frac{9}{4}$. We see that the mode $\pi^+ + \pi^- + \gamma$, although one order lower in α than the 2γ mode, is expected to be rarer. The actual estimate is not in violent disagreement with experiment.[8]

Some other discussions of the ω decay have recently appeared.[9] The distinctive feature of our treatment and that of reference 5 is that we express our estimates in terms of measured or measurable matrix elements involving low-mass intermediate states that we guess to be dominant.

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*Research supported by the U. S. Atomic Energy Commission and the Alfred P. Sloan Foundation.

†National Science Foundation Predoctoral Fellow.

‡Howard Hughes Doctoral Fellow.

[1] $f_{\omega\rho\pi}$ in a diagram multiples the quantity $e_{\kappa\lambda\mu\nu} k_\kappa{}^\omega e_\lambda{}^\omega k_\mu{}^\rho e_\nu{}^\rho$.

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