PHYSICAL REVIEW LETTERS

VOLUME 8 MARCH 15, 1962 NUMBER 6

OBSERVATION OF CRITICAL FLUCTUATIONS ASSOCIATED WITH PLASMA-WAVE INSTABILITIES

S. Ichimaru and D. Pines

Departments of Physics and Electrical Engineering, University of Illinois, Urbana, Illinois

and

N. Rostoker

John Jay Hopkins Laboratory for Pure and Applied Science, General Atomic Division of General Dynamics Corporation, San Diego, California

(Received February 21, 1962)

The existence of plasma-wave instabilities associated with the drift of electrons vs ions under the action of an applied electric field in a fully ionized plasma has been considered recently by a number of authors.[1-3] In this Letter we wish to point out the possibility of utilizing the scattering of electromagnetic waves to observe the onset of such instabilities. As the plasma approaches from a region of stability a critical point corresponding to the onset of an instability, a phenomenon analogous to critical opalescence is predicted, in which at certain angles enormously enhanced scattering of the electromagnetic radiation is expected to occur.

The scattering of electromagnetic waves by electron density fluctuations in a fully ionized plasma is determined by $S(\vec{k}\omega)$, the Fourier transform of the electron density-density correlation function.[4-6] The differential cross section, $d^2\sigma/d\Omega d\omega$, for the transfer of momentum $\hbar\vec{k}$ (corresponding to scattering into a solid angle $d\Omega$) and energy $\hbar\omega$ from an electromagnetic wave to the electrons in the plasma is given by

$$\frac{d^2\sigma}{d\Omega d\omega}=\left(\frac{e^2}{mc^2}\right)^2(1-\tfrac{1}{2}\sin^2\theta)S(\vec{k}\omega), \qquad (1)$$

where θ is the angle between the incident and scattered waves, and we have averaged over directions of polarization of both waves. $S(\vec{k}\omega)$ is defined by

$$S(\vec{k}\omega)=2\pi L^3\int_{L^3}d\vec{r}\int_{-\infty}^{\infty}dt\,\langle n(\vec{r}',t')n(\vec{r}'+\vec{r},t'+t)\rangle \times\exp[-i(\vec{k}\cdot\vec{r}-\omega t)], \qquad (2)$$

where N is the total number of electrons in a volume L^3, $n(\vec{r},t)$ denotes the electron number density, and the angular brackets refer to a statistical average over the electron states. In many cases of practical importance the fractional change in wavelength of the scattered radiation is small; under these circumstances a measurement of the intensity of the radiation scattered into a given angle yields directly the structure factor, $S(\vec{k})$, which is

$$S(\vec{k})=(1/N)\int_{-\infty}^{\infty}d\omega\,S(\vec{k}\omega); \qquad (3)$$

$S(\vec{k})$ is typically of order unity.

We first summarize some results of the theory of fluctuations in a fully ionized plasma. Within the random phase approximation the spectrum of electron density fluctuations $S(\vec{k},\omega)$ for a two-component plasma (electrons and positive ions with charge e) may be simply obtained by superposing the fields due to the dressed particles (electrons or ions plus their associated screening clouds).[7,8] The result is

$$S(\vec{k},\omega)=\frac{N}{k}f_-(\omega/k)\left|\frac{1+4\pi\alpha_+(\vec{k},\omega)}{\epsilon(\vec{k},\omega)}\right|^2$$

$$+\frac{N}{k}f_+(\omega/k)\left|\frac{4\pi\alpha_-(\vec{k},\omega)}{\epsilon(\vec{k},\omega)}\right|^2, \qquad (4)$$

where

$$\alpha_\pm(\vec{k},\omega)=\lim_{\delta\to 0}\ -\frac{e^2N}{L^3 m_\pm k^2}\int_{-\infty}^{\infty}\frac{k[df_\pm(v)/dv]dv}{kv-\omega-i|\delta|}, \qquad (5)$$

and $\epsilon(\vec{k}\omega)$ is the longitudinal dielectric constant,

$$\epsilon(\vec{k},\omega)=1+4\pi\alpha_-(\vec{k},\omega)+4\pi\alpha_+(\vec{k},\omega). \qquad (6)$$

$f_\pm(v)$ are the one-dimensional normalized velocity distribution functions for electrons and ions in the directions of $\vec{k}$. The expression (4) may be used for a system with arbitrary temperature ratio T_-/T_+ and electron vs ion drift velocity $\vec{V}_d$ as long as we remain in the stable region.

It is straightforward to show from (4) and (6) that for $V_d=0$ and $T_-\cong T_+$, one obtains $S(\vec{k})\cong\frac{1}{2}$ as long as $k^2\ll k_-^2$, the square of electron Debye wave vector. In this region the main contribution to $S(\vec{k})$ comes from those electrons which act to screen out the motion of the ions; the contribution from electron plasma waves is only of order k^2/k_-^2. If one considers $V_d=0$, but $T_-\gg T_+$, one finds $S(\vec{k})\cong 1$ for long wavelengths; here the main contribution to $S(\vec{k})$ comes from the ion sound waves which represent a well-defined, relatively undamped, stable excitation under these circumstances.

If one now passes to a sufficiently large value of $\vec{V}_d$, the ion sound waves become unstable; the boundary between the growing and damped waves is a sensitive function of (T_-/T_+). It may be represented by a curve $\vec{V}_d(\vec{k})$ in the $\vec{V}_d$-$\vec{k}$ plane; a critical point k_c may be defined as that wave vector for which, with increasing $\vec{V}_d$, the ion sound wave first becomes unstable; let $V_c=V_d(k_c)$ be the associated minimum drift velocity for instability. The boundary between growing and damped waves is specified by

$$\mathrm{Im}\,\epsilon[\vec{k},\omega(\vec{k})]=0, \qquad (7)$$

where $\omega(\vec{k})$, the frequency of the first stable wave, is determined by

$$\mathrm{Re}\,\epsilon[\vec{k},\omega(\vec{k})]=0. \qquad (8)$$

Since from (4), $S(\vec{k}\omega)$ is proportional to $|\epsilon(\vec{k}\omega)|^{-2}$, one finds a contribution to $S(\vec{k})$ from frequencies in the immediate vicinity of $\omega(\vec{k})$, which is

$$S_{\mathrm{res}}(\vec{k})\propto 1/\mathrm{Im}\,\epsilon[\vec{k},\omega(\vec{k})]. \qquad (9)$$

For the case of marginal stability, defined by (7), $S_{\mathrm{res}}(\vec{k})$ obviously diverges.

We have carried out an explicit evaluation of $S_{\mathrm{res}}(\vec{k})$ for the case that $T_-\gg T_+$, such that the wave of $k=0$ is the first to grow as one increases V_d. The result is

$$S_{\mathrm{res}}(\vec{k})=\frac{1}{2}\left(\frac{(m_-/m_+)^{1/2}}{(V_c-V_d\cos\chi)/V_-+(k^2/k_-^2)(V_1/V_-)}\right), \qquad (10)$$

where V_- is the average electron velocity, $(\kappa T_-/m_+)^{1/2}$, $V_c\cong(m_-/m_+)^{1/2}V_-$, χ is the angle between $\vec{k}$ and $\vec{V}_d$, and

$$(V_1/V_-)=\frac{1}{2}\left\{\left[(T_-/T_+)^{5/2}\exp\left(-\frac{[(T_-/T_+)+3]}{2}\right)\right]-(m_-/m_+)^{1/2}\right\}. \qquad (11)$$

The result, (10), is valid for $k^2\ll k_-^2$; we remark that it is identical in analytical form to the results obtained for the critical fluctuations in the vicinity of a liquid-gas phase transition.[9]

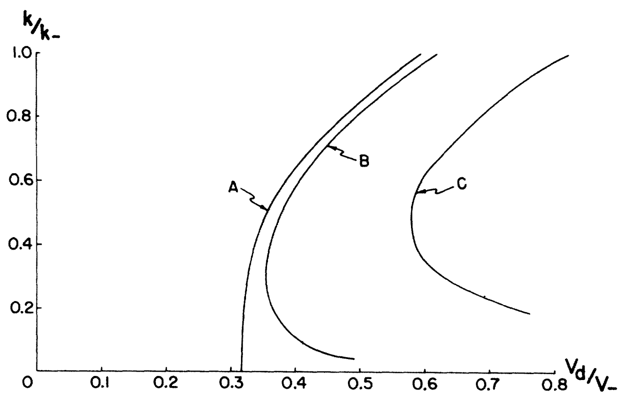

FIG. 1. Boundary between growing waves and damped waves for electron-hole plasma in InSb, with $(m_+/m_-)=14$, $(T_-/T_+)=10$. Curve A represents the boundary when hole-impurity scattering is neglected, Curve B the case for $\omega_+\tau_+=100$, Curve C the case for $\omega_+\tau_+=10$; $\omega_+=(4\pi n_+e^2/m_+\epsilon_0)^{1/2}$ and τ_+ is the lifetime of a hole against impurity scattering.

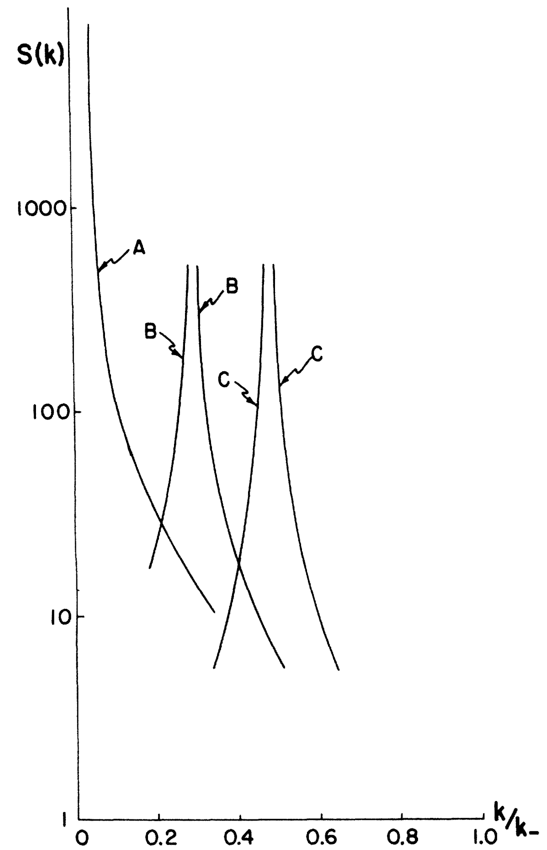

FIG. 2. The structure factor $S(k)$ at the critical point, $V_d = V_c$, for $\cos\chi = 1$ and the same physical conditions as in Fig. 1; Curves A, B, and C have the same meaning as in Fig. 1.

In the case of the instability associated with the drift of electrons vs holes in a semiconductor,[3] it is important that the influence of the scattering of the holes by impurities be taken into account. It is found that impurity scattering acts to shift the position of the onset of the instability, but does not alter appreciably the existence of critical fluctuations in the vicinity of the critical point. The results of detailed calculations for InSb are summarized in Figs. 1 and 2. We conclude that the scattering of electromagnetic waves offers a promising method of detecting the onset of such instabilities in semiconductors. For example, for case C, with 1-ev photons incident on InSb, one finds the critical scattering will display a maximum at $\theta_c \cong 90°$ for an electron density of 4×10^{15}/cc and $T_- = 200°$K. The differential scattering cross section in this vicinity is

$$d\sigma/d\Omega \cong 1.5\times10^{-10} D S(k),$$

where D is the sample thickness in cm; for the case of $\cos\chi = 1$, $S(k)$ is shown in Fig. 2.

Finally, we remark that even for $V_d < V_c$, there occurs a substantial enhancement of the scattering cross section, $S(\vec{k})$, which should be detectable experimentally. The extension of these considerations to a plasma in a magnetic field is straightforward and is presently under investigation.

---

[1]O. Buneman, Phys. Rev. Letters 1, 8 (1958); Phys. Rev. 115, 503 (1959).

[2]J. D. Jackson, J. Nuclear Energy Part C, 1, 171 (1960).

[3]D. Pines and J. R. Schrieffer, Phys. Rev. 124, 1387 (1961).

[4]J. P. Dougherty and D. T. Farley, Proc. Roy. Soc. (London) A259, 79 (1960).

[5]J. A. Fejer, Can. J. Phys. 38, 1114 (1960).

[6]E. E. Salpeter, Phys. Rev. 120, 1528 (1960).

[7]N. Rostoker, Nuclear Fusion 1, 101 (1961).

[8]W. B. Thompson and J. Hubbard, Revs. Modern Phys. 32, 714 (1960); W. B. Thompson (to be published).

[9]L. D. Landau and E. M. Lifshitz, Statistical Physics (Addison-Wesley Publication Company, Inc., Reading, Massachusetts, 1958), p. 366.