which tunneling could be used to detect radiation.[4] A theoretical calculation[9] yields a value for τ of 0.43×10^{-7} sec, which is consistent with our experimental results.

The sensitivity of this experiment is being increased by operating at lower temperatures and by producing junctions with a lower resistance, so that electrons may be injected into the superconductor more rapidly.

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†A. P. Sloan Fellow.

[1]J. Bardeen, L. N. Cooper, and J. R. Schrieffer, Phys. Rev. 108, 1175 (1957).

[2]I. Giaever and K. Megerle, Phys. Rev. 122, 1101 (1961).

[3]I. Giaever, H. R. Hart, and K. Megerle, Phys. Rev. (to be published).

[4]E. Burstein, D. N. Langenberg, and B. N. Taylor, Phys. Rev. Letters 6, 92 (1961).

[5]R. H. Parmenter, Phys. Rev. Letters 7, 274 (1961).

[6]J. Bardeen, G. Rickayzen, and L. Tewordt, Phys. Rev. 113, 982 (1959).

[7]J. Bardeen, Phys. Rev. Letters 6, 57 (1961).

[8]R. G. Chambers, Proc. Roy. Soc. (London) A215, 481 (1952).

[9]J. R. Schrieffer and D. M. Ginsberg (to be published).

CALCULATION OF THE QUASIPARTICLE RECOMBINATION TIME IN A SUPERCONDUCTOR*

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The previous Letter[1] gives an experimental upper limit for the average time required for quasiparticle recombination in a superconductor. Burstein, Langenberg, and Taylor[2] have calculated the contribution to the recombination rate from photon emission, and obtain a result which is much too small to account for the experimental results.[1] It is therefore of interest to calculate the recombination rate due to phonon emission.

Using the BCS theory[3] in the limit in which k_BT is small compared with the gap parameter, Δ, the recombination rate $\Gamma_{\vec{k}}$ for a quasi-particle in the state $\vec{k}$ is given by

$$\Gamma_{\vec{k}}=\frac{2\pi}{\hbar}\sum_{\vec{k}',\lambda}|v(\vec{k},\vec{k}',\lambda)|^2 \times\frac{\hbar}{2\omega_{\vec{q},\lambda}}|\mu(\vec{k},\vec{k}')|^2 f_{\vec{k}'}\delta(E_{\vec{k}}+E_{\vec{k}'}-\hbar\omega_{\vec{q},\lambda}), \quad (1)$$

where $\vec{q}=\vec{k}-\vec{k}'+\vec{K}$ ($\vec{K}$ = reciprocal lattice vector),

$$|\mu(\vec{k},\vec{k}')|^2=\tfrac{1}{2}\left(1+\frac{\Delta_{\vec{k}}\Delta_{\vec{k}'}-\epsilon_{\vec{k}}\epsilon_{\vec{k}'}}{E_{\vec{k}}E_{\vec{k}'}}\right), \quad (2)$$

$$E_{\vec{k}}=(\epsilon_{\vec{k}}^2+\Delta_{\vec{k}}^2)^{1/2}, \quad (3)$$

$$f_{\vec{k}}=[\exp(E_{\vec{k}}/k_BT)+1]^{-1}, \quad (4)$$

and $v(\vec{k},\vec{k}',\lambda)$ = phonon-electron matrix element. Here $\epsilon_{\vec{k}}$ is the Bloch single-particle energy in the normal state measured relative to the Fermi energy E_F, and $\omega_{\vec{q},\lambda}$ is the frequency of phonons of wave vector $\vec{q}$ and polarization λ.

Due to the complexities of the band structure in lead and uncertainties of the phonon-electron matrix elements we choose a spherical band model with deformation potential phonon-electron matrix elements,

$$|v(\vec{k},\vec{k}',\lambda)|^2=C_\lambda^{\,2}q^2/\rho_m, \quad (\vec{K}=0) \quad (5)$$

where ρ_m is the mass density and C_λ is the Bloch interaction constant. Also, we set $\omega_{\vec{q},\lambda}=S_\lambda q$ and we work in a box of unit volume. Replacing the summation in Eq. (1) by an integration with respect to q, k' and the azimuthal angle, we find

$$\Gamma_k=\frac{m^*\Delta^2}{\pi\rho_m\hbar^5k_F}\sum_\lambda\frac{C_\lambda^{\,2}}{S_\lambda^{\,4}}\int_{-\infty}^{\infty}d\epsilon_{k'}f_{k'}, \quad (6)$$

where k_F is the Fermi momentum. For states of interest $E_k\simeq\Delta_k$ so that the coherence factor μ is approximately unity.

Due to the complicated band structure of lead it is reasonable to assume that the Bloch constants for longitudinal and transverse phonons are comparable. Thus we set $C_\lambda = C$, where C is chosen to fit the high-temperature resistivity ρ:

$$\frac{\rho}{k_B T} = \frac{9\pi^3}{16}\left(\frac{\hbar k_F}{n_a e^2 k_D{}^2 M}\right)\left(\frac{C}{E_F}\right)^2\left(\frac{1}{S_L{}^2}+\frac{2}{S_T{}^2}\right). \quad (7)$$

Here, n_a is the number of free electrons per atom, k_D is the Debye wave number, M is the atomic mass, and S_L and S_T are the longitudinal and transverse sound velocities. We use the values for lead: $\Delta = 1.34\times10^{-3}$ ev,[1] $M = 3.46\times10^{-22}$ g, $S_L = 2.39\times10^5$ cm/sec,[4] $S_T = 1.27\times10^5$ cm/sec,[4] $n_a = 1.24$,[5] $k_D = 0.810\times10^8$ cm^{-1}, $k_F = 1.06\times10^8$ cm^{-1}, $m^* = m$, $\rho/k_B T = 5.78\times10^{-4}$ esu. One obtains the reasonable value $C/E_F = 1.16$. With the value 3.44×10^{-20} erg for the integral in Eq. (6) appropriate to the temperature 1.44°K used in the experiment,[1] we find for the recombination time $1/\Gamma = 0.43\times10^{-7}$ sec. This result is an order of magnitude smaller than the experimental upper limit at this temperature.

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†A. P. Sloan Fellow.

[1]D. M. Ginsberg (to be published).

[2]E. Burstein, D. N. Langenberg, and B. N. Taylor, Phys. Rev. Letters 6, 92 (1961).

[3]J. Bardeen, L. N. Cooper, and J. R. Schrieffer, Phys. Rev. 108, 1175 (1957).

[4]W. P. Mason and H. E. Bömmel, J. Acoust. Soc. Am. 28, 930 (1956).

[5]R. G. Chambers, Proc. Roy. Soc. (London) A215, 481 (1952).

RESONANT ABSORPTION OF ULTRASOUND BY OPEN-ORBIT ELECTRONS IN CADMIUM*

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A true spatial resonance has been observed in the absorption of high-frequency sound waves in cadmium for a particular direction of applied magnetic field. The normal magnetoacoustic effect is not a resonance phenomenon, since increasing the ratio of electron mean free path to sound wavelength does not produce a corresponding narrowing of the absorption maxima. Furthermore, the effect reported here differs from the ultrasonic cyclotron resonance reported by Roberts[1] in that the former is a spatial resonance, whereas the latter is a temporal resonance.

Figure 1 shows the variation in attenuation of ultrasound propagated in the [10$\bar{1}$0] direction as a function of frequency divided by field for a field directed along [$\bar{1}$2$\bar{1}$0]. The relative width of the absorption line at $\nu/H = 0.053$ Mc/oersted decreases with increasing frequency (and decreasing temperature) as one would expect for a resonance phenomenon. The first of a series of about 25 magnetoacoustic maxima is included in the figure to emphasize the narrowness of the resonance line.

The resonance appears to be caused by conduc-

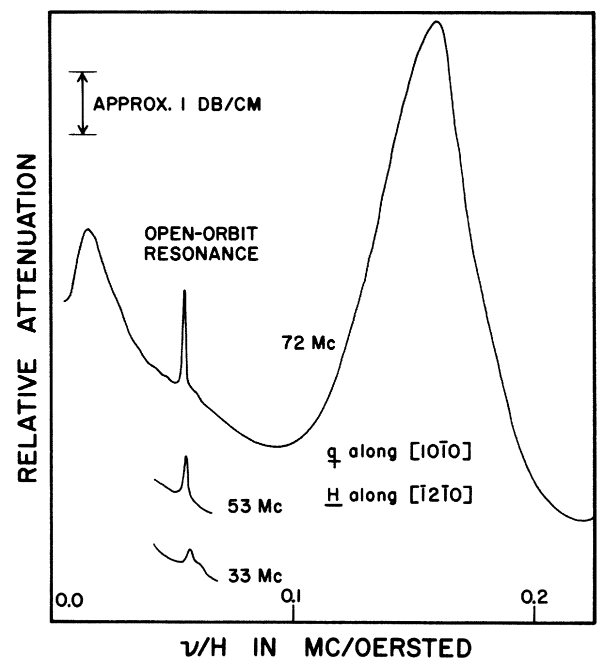

FIG. 1. Shape of the open-orbit resonance line contrasted with shape of the normal magnetoacoustic maximum on the right.