in quantitative agreement with theory, while samples for which large-k spin waves may be expected give no indication of the spin-wave enhancement.

Recent observations[8] of the dependence of ΔH_k on k could cast doubt as to whether spin waves excited by nonlinear mechanisms alone are z directed (i.e., give the lowest threshold) as predicted by Suhl. However, since a significant spin-wave contribution would be predicted if the lowest threshold were for large θ_k, regardless of the nature of the inhomogeneities, our experimental results give evidence that the Suhl spin waves are z directed.

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UPPER LIMIT FOR QUASI-PARTICLE RECOMBINATION TIME IN A SUPERCONDUCTOR*

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In a superconductor at finite temperature a dynamic equilibrium exists, with clothed electrons (quasi-particles) being continuously excited above the gap and recombining in pairs to fall below the gap again.[1] The purpose of this experiment is to gain information concerning the rate of these processes. We shall discuss this rate in terms of the mean time τ for recombination with any particular excited electron.

At temperatures low compared with the critical temperature T_c, one expects τ to be roughly proportional to $e^{\Delta/kT}$, where Δ is one half of the energy gap width, k is Boltzmann's constant, and T is the absolute temperature. This arises from the fact that the number of excited electrons available for pairing with a given electron is roughly proportional to $e^{-\Delta/kT}$.

In the BCS theory,[1] it is assumed that τ is very large compared with Planck's constant divided by Δ, so that the recombination process does not significantly broaden the energy levels of the electron system. That this assumption is valid for $T \ll T_c$ is borne out perhaps most convincingly by tunneling measurements,[2,3] which indicates a gap edge so sharp that τ must be on the order of 10^{-11} sec or longer in tin at 0.30°K. This result shows that recombination is probably too slow to have a significant effect on the usual electronic transport properties at extremely low temperatures, although it may be important nearer to T_c. The finite size of τ is a crucial factor in the use of superconductors for some devices in which a nonequilibrium state is desired.[4,5] The only upper limit for τ available until now has been provided by a calculation by Burstein, Langenberg, and Taylor[4] of τ_{opt}, the mean time for recombination accompanied by the emission of a photon. It was shown that in lead at 2°K, $\tau_{opt} = 0.4$ sec. However, there exists an alternate process, in which the excess momentum and energy are carried away by a phonon, so that τ may be much shorter than τ_{opt}. Indeed, the present experiment indicates that $\tau = 2.2\times10^{-7}$ sec.

The principle of this experiment can be described as follows. Electrons are injected by tunneling from a normal metal through an insulating layer into a superconducting metal.[2] After being injected, an electron may (a) tunnel back through the same junction, (b) recombine and fall below the gap, or (c) diffuse and tunnel through another junction into a second piece of normal metal. The measured characteristics and currents of the junctions provide a measure of how many electrons

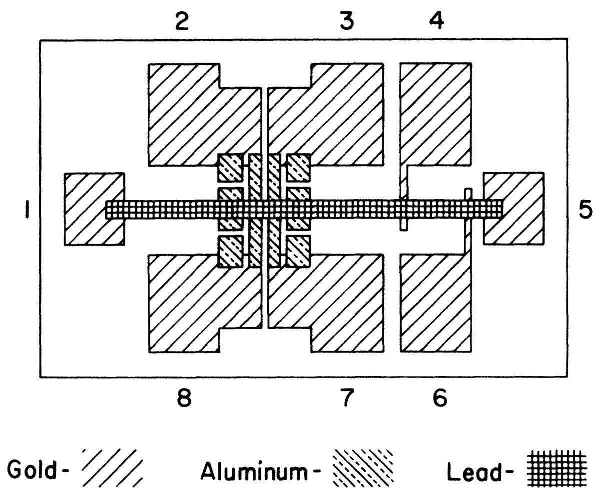

FIG. 1. Diagram of slide with evaporated films. Not drawn to scale.

have fallen below the gap before reaching the second junction, from which one can calculate τ, as described in detail below.

A schematic picture of the sample appears in Fig. 1. The sample preparation proceeded as follows. Gold electrodes were evaporated onto a clean slide made of crystal quartz. Then aluminum was evaporated onto the slide through a mask which was constructed of sheet metal and three wires in such a way that the evaporated aluminum formed four areas. The middle two were each 0.0089 cm wide, and were separated by a distance of 0.00254 cm. After the aluminum was deposited on the quartz, it was oxidized for a short time, and a strip of lead 0.158 cm wide was then evaporated onto the slide at room temperature in a vacuum of 2×10^{-5} mm of mercury. The slide was removed from the evaporator, and metal was removed with a scriber under a microscope to render the outer two areas of aluminum ineffective and to divide the gold areas which contacted the inner two aluminum electrodes as shown in Fig. 1. The sample was then mounted on an apparatus which permitted electrical contact to the gold electrodes, and was placed in a Dewar which was then evacuated. The scribing, mounting, and evacuation were completed in about 10 min. The Dewar was then cooled down. The resistance of part of the lead film was measured at room temperature and at 77°K by using terminals 1 and 5 for current contacts and potentiometrically measuring the voltage between terminals 4 and 6. (Any thermal emf was negligible.) The average thickness of the film, which is needed for analysis of the data, was found from the resistance values at these two temperatures by assuming that the film's resistivity was the sum of two parts, a temperature-independent part due to boundary scattering and a temperature-dependent part which was the same as for pure bulk lead (Matthiessen's rule).

The main measurements were made with the sample in direct contact with superfluid helium at 1.44°K. First the differential conductance, dI/dV, of each junction was determined. This was done using an ac method, with an ac voltage of 30 μv at 39 cps. The resulting plot of dI/dV as a function of the dc bias voltage V corresponds fairly well to the theoretically derived[1] density of electronic states in the superconductor when the theoretical curve is scaled to a gap width of 1.34 ev. This has been found previously by other methods.[2,3] This shows that the sample was a good one. The resistances of the two junctions were 10.9 kohm and 9.2 kohm at a bias of 4 v. During these readings, electrodes 1, 2, and 3 were used to supply the current, and electrodes 5, 7, and 8 were used for the voltage measurements.

To obtain a value for τ, we reversed a dc current going through each junction in turn, and looked for a change in current coming through the other junction, using a high-sensitivity galvanometer with a resistance of 570 ohm. With up to 44.3 μa of current coming through each junction, the current induced at the other junction was less than 2.45×10^{-5} μa. Data obtained by us and by others[2] using junctions with lower resistance per unit area indicate that the heating of the sample was not significant at the level of power dissipation used in these measurements.

In order to obtain an upper limit for the mean recombination time τ from these data, we make the following assumptions: (1) The electrons which are injected into the superconductor assume a Fermi-Dirac distribution in the states above the gap in a time short compared to τ, since recombination must wait until an electron finds a partner to pair off with. (2) The electrons above the gap diffuse in a manner effectively described by a one-dimensional random walk with a free path approximately equal to the average film thickness λ, and with a velocity given by the group velocity[6] $v=(v_0/\sqrt{3})\epsilon/E$. Here v_0 is the Fermi velocity in the normal state, ϵ is the normal-state energy of the occupied state relative to the Fermi energy, and E is the excitation energy,[1] $(\epsilon^2+\Delta^2)^{1/2}$. (3) The recombination time τ is approximately the same for all the electrons of interest. This assumption is justified by the fact that at temperatures small compared with Δ/k, the excited electrons are almost all in states fairly near the top of the gap so that the coherence factor[1] $h=\frac{1}{2}(1-\epsilon/E)$ for all of them is approximately the same. (4) An electron in the superconducting metal lying over a junction will tunnel through that junction with a probability γ per unit time which is independent of ϵ. The validity of this assumption can be inferred directly from the fact that the experimentally determined dI/dV curve yields the expected density-of-states curve in the superconductor, a fact which has received a theoretical explanation.[7] (5) So few electrons actually tunnel through the receiving junction that the number left behind is essentially the same as if there were no such tunneling. This assumption can be shown from our value for γ to be on extremely secure grounds.

Using these assumptions, the analysis proceeds as follows. If an electron is injected into the superconductor at the position $x=0$ at time $t=0$, the probability of its being at the position x at time t and still being excited above the gap is given by $(2\pi tv\lambda)^{-1/2}\exp(-x^2/2tv\lambda)e^{-t/\tau}$. Averaging over positions of injection and allowing the electron an infinite amount of time to diffuse and tunnel into the entire area of the other junction, we obtain an expression for the probability P that an electron injected at one junction will eventually tunnel through the other:

$$P=\frac{\gamma}{\delta(2\pi v\lambda)^{1/2}}\times\int_0^\infty dt\int_0^\delta dx'\int_{\beta+x'}^{\beta+\delta+x'}dx\,t^{-1/2}\exp(-x^2/2tv\lambda)e^{-t/\tau},$$

where δ is the width of the junctions and β is the distance between them. When the integrations are performed, one obtains

$$P=(\gamma\tau/2\delta\theta)e^{-\beta\theta}(1-e^{-\delta\theta})^2, \qquad (1)$$

where $\theta\equiv(2/tv\lambda)^{1/2}$.

The slowest electrons, for which E is extremely close to Δ, do not contribute much to P, according to Eq. (1). The reason is that they decay before they are able to reach the other junction. All of the electrons which contribute significantly to P at the temperature of our experiment have velocities not extremely different from v_0. For these, one can show from Eq. (1) that the exponentials, of the form e^x, can be well approximated by $1+x$. We do this to simplify the calculation, and obtain the relation

$$\tau=2P^2v\lambda/\gamma^2\delta^2.$$

In the spirit of our calculation, we use for v the thermal average, which is

$$v_0e^{-\Delta/kT}\left(\sqrt{3}\int_0^\infty dy\,e^{-\sigma}\right)^{-1},$$

where

$$\sigma=[y^2+(\Delta/kT)^2]^{1/2}.$$

We have assumed that $e^{\Delta/kT}\gg1$, as in this experiment. The parameter γ can be found from the resistance R of the junction at any voltage V which is $\gg\Delta/e$, where e is the electronic charge. For such a voltage, we should have a current equal to $e^2VN_0\lambda A\gamma$, where A is the area of the junction, 1.0×10^{-3} cm^2, and N_0 is the normal-state density of states per unit volume.

Summing up, we have

$$\tau=2P^2\lambda^3R^2e^4N_0^2A^2v_0e^{-\Delta/kT}\left(\delta^2\sqrt{3}\int_0^\infty dy\,e^{-\sigma}\right)^{-1}. \qquad (2)$$

We estimate N_0 and v_0 from a free electron model with 1.24 conduction electrons per atom, as indicated by anomalous skin effect measurements.[8] The value for R in this experiment was 9170 ohm, and the film was 1250 A thick. The maximum possible value for P consistent with our results was 5.54×10^{-7}. The integral in Eq. (2) is 8.7×10^{-5}. Our value for τ is finally

$$\tau\leq2.2\times10^{-7}\text{ sec}.$$

In view of the various approximations in the analysis, this upper limit for τ should be considered as only approximate. Our analysis has implicitly assumed that in the experiment the injection of electrons produced only a small fractional change in the volume density of excited electrons. Our upper limit for τ permits one to show that this is true.

This upper limit for τ is very much less than the mean time associated with radiative recombination.[4] Therefore the recombination process is much more likely to be accompanied by the emission of a phonon than a photon. This places severe limitations on the use of the tunneling phenomenon as a possible source of radiation. The shortness of τ also limits the ease with which tunneling could be used to detect radiation.[4] A theoretical calculation[9] yields a value for τ of 0.43×10^{-7} sec, which is consistent with our experimental results.

The sensitivity of this experiment is being increased by operating at lower temperatures and by producing junctions with a lower resistance, so that electrons may be injected into the superconductor more rapidly.

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CALCULATION OF THE QUASIPARTICLE RECOMBINATION TIME IN A SUPERCONDUCTOR*

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The previous Letter[1] gives an experimental upper limit for the average time required for quasiparticle recombination in a superconductor. Burstein, Langenberg, and Taylor[2] have calculated the contribution to the recombination rate from photon emission, and obtain a result which is much too small to account for the experimental results.[1] It is therefore of interest to calculate the recombination rate due to phonon emission.

Using the BCS theory[3] in the limit in which k_BT is small compared with the gap parameter, Δ, the recombination rate $\Gamma_{\vec{k}}$ for a quasi-particle in the state $\vec{k}$ is given by

$$\Gamma_{\vec{k}}=\frac{2\pi}{\hbar}\sum_{\vec{k}',\lambda}|v(\vec{k},\vec{k}',\lambda)|^2 \times\frac{\hbar}{2\omega_{\vec{q},\lambda}}|\mu(\vec{k},\vec{k}')|^2 f_{\vec{k}'}\delta(E_{\vec{k}}+E_{\vec{k}'}-\hbar\omega_{\vec{q},\lambda}),\quad(1)$$

where $\vec{q}=\vec{k}-\vec{k}'+\vec{K}$ ($\vec{K}$ = reciprocal lattice vector),

$$|\mu(\vec{k},\vec{k}')|^2=\tfrac{1}{2}\left(1+\frac{\Delta_{\vec{k}}\Delta_{\vec{k}'}-\epsilon_{\vec{k}}\epsilon_{\vec{k}'}}{E_{\vec{k}}E_{\vec{k}'}}\right),\quad(2)$$

$$E_{\vec{k}}=(\epsilon_{\vec{k}}^2+\Delta_{\vec{k}}^2)^{1/2},\quad(3)$$

$$f_{\vec{k}}=[\exp(E_{\vec{k}}/k_BT)+1]^{-1},\quad(4)$$

and $v(\vec{k},\vec{k}',\lambda)$ = phonon-electron matrix element. Here $\epsilon_{\vec{k}}$ is the Bloch single-particle energy in the normal state measured relative to the Fermi energy E_F, and $\omega_{\vec{q},\lambda}$ is the frequency of phonons of wave vector $\vec{q}$ and polarization λ.

Due to the complexities of the band structure in lead and uncertainties of the phonon-electron matrix elements we choose a spherical band model with deformation potential phonon-electron matrix elements,

$$|v(\vec{k},\vec{k}',\lambda)|^2=C_\lambda{}^2q^2/\rho_m,\quad(\vec{K}=0)\quad(5)$$

where ρ_m is the mass density and C_λ is the Bloch interaction constant. Also, we set $\omega_{\vec{q},\lambda}=S_\lambda q$ and we work in a box of unit volume. Replacing the summation in Eq. (1) by an integration with respect to q, k' and the azimuthal angle, we find

$$\Gamma_k=\frac{m^*\Delta^2}{\pi\rho_m\hbar^5k_F}\sum_\lambda\frac{C_\lambda{}^2}{S_\lambda{}^4}\int_{-\infty}^{\infty}d\epsilon_{k'}f_{k'},\quad(6)$$

where k_F is the Fermi momentum. For states of interest $E_k\simeq\Delta_k$ so that the coherence factor μ is approximately unity.

Due to the complicated band structure of lead