tion of real, soft photons for small momentum transfer in the center-of-mass system.

[1]K. E. Eriksson and A. Peterman, Phys. Rev. Letters 5, 444 (1960). K. E. Eriksson, Nuovo cimento 19, 1044 (1961).

[2]Yung Su Tsai, Phys. Rev. 120, 269 (1960).

[3]These processes offer also the advantage that since the skeleton graphs contain only internal electron lines, the contribution of other charged fields, e.g., to vacuum polarization can enter only in graphs of at least the sixth order in "e" (the standard radiative corrections we are calculating take into account graphs up to the fourth order). This means that what one could call the "natural breakdown" of quantum electrodynamics (i.e., the deviation due to other existing fields) is particularly negligible for these processes. Really neutral fields decaying into photons can give some contributions at higher order which could become appreciable for high c.m. energies [e.g., through the process $e^+ - e^- \to \pi^0 + \gamma$; see G. Furlan, Nuovo cimento 19, 840 (1961)].

[4]I. Harris and L. M. Brown, Phys. Rev. 105, 1656 (1957).

TOTAL CROSS SECTIONS AT HIGH ENERGIES*

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Total cross sections which are well known experimentally[1-5] are nearly constant and seem to satisfy the following inequalities from several Bev to the highest energy available at the present moment ($\gtrsim 20$ Bev):

$$\sigma(\pi^-, p) > \sigma(\pi^+, p), \tag{1}$$

$$\sigma(K^-, p) > \sigma(K^+, p), \tag{2}$$

$$\sigma(\bar{p}, p) > \sigma(p, p). \tag{3}$$

Pomeranchuk[6] and other authors[7] have proved that the total cross section for a particle on a target must be identical with that for its antiparticle on the same target at sufficiently high energy under some conditions.

It is our purpose to explain inequalities (1) to (3) and to predict other relations among total cross sections at high energies under a hypothesis on generalized isospin independence. The three inequalities will be explained on the basis of the following facts: (α) the isospin of the Λ particle is zero; (β) all hyperons have strangeness of the same sign; (γ) the strangeness of all hyperons is negative; and (δ) the pion and K meson have baryon number zero. Inequality (1) is explained by (α), inequality (2) by (α) and (γ), and inequality (3) by (α), (β), and (δ). If hyperons had positive strangeness, then the direction of inequality (2) would have been reversed. If the hyperons did not all have the same sign, then there would be no basis for inequality (3).

Let us start with the total cross sections for the (π^-, p) and (π^+, p) and take only charge-independent strong interactions into account. The (π^-, p) has two isospin states ($T = \frac{3}{2}$ and $T = \frac{1}{2}$) whereas (π^+, p) has only one isospin state ($T = \frac{3}{2}$). Final states with the same kinds and same number of particles, irrespective of their z components of isospin, T_3, will be designated as being in the same channel. The (π^-, p) includes all the channels that belong to the (π^+, p), but there are channels with pure $T = \frac{1}{2}$ that belong to the (π^-, p) alone.

Therefore from charge independence one obtains

$$\sigma(\pi^-, p) - \sigma(\pi^+, p) = \tfrac{2}{3}\sum_i {}_c[\sigma^{(i)}(T = \tfrac{1}{2}) - \sigma^{(i)}(T = \tfrac{3}{2})] + \tfrac{2}{3}\sum_i {}_n\sigma^{(i)}(T = \tfrac{1}{2}), \tag{4}$$

where $\sum_c$ stands for the summation over all the channels common to both (π^-, p) and (π^+, p) and $\sum_n$ for the summation over all the possible channels that are included only in the (π^-, p). The channels that belong to $\sum_n$ are exhausted by

$$\begin{aligned} &\Lambda + K^0 + m(\Lambda + \bar{\Lambda}), \\ &n + \Lambda + \bar{\Lambda} + m(\Lambda + \bar{\Lambda}), \qquad (m = 0, 1, 2, \cdots) \\ &\bar{\Xi}^0 + \Lambda + \Lambda + m(\Lambda + \bar{\Lambda}). \end{aligned} \tag{5}$$

We now make the dynamical assumption that

$$\sum_i {}_c\sigma^{(i)}(T = \tfrac{1}{2}) = \sum_i {}_c\sigma^{(i)}(T = \tfrac{3}{2}) \tag{6}$$

at high energies. The difference in the total cross sections comes from noncommon channels. Then the relations

$$\sigma(\pi^-, p) = \sigma(\pi^0, p) = \sigma(\pi^+, n) = \sigma(\pi^0, n) > \sigma(\pi^+, p) = \sigma(\pi^-, n) \tag{7}$$

are obtained at high energies from the same considerations.

The contribution of a particular channel to the total cross section would decrease with energy. As was shown by Eq. (5), on the other hand, the number of noncommon channels increases with energy. We may argue that this increase may explain why the experimental difference between $\sigma(\pi^-, p)$ and $\sigma(\pi^+, p)$ still remains even at 20 Bev.[1,2] It would be worth while to note that $\sigma(\pi^-,p) = \sigma(\pi^+,p)$ provided that the isospin of the Λ particle is not zero.

We would like to compare $\sigma(K^+, p)$ $[\sigma(p, p)]$ with $\sigma(K^-, p)$ $[\sigma(\bar{p}, p)]$, etc. In the pion-nucleon states, only T_3 is different. In general, the final states that appear may have different baryon number B, strangeness S, and isospin T_3. The definition of channels is therefore extended so that final states with the same kinds and same number of particles are in the same channel irrespective of B and S as well as T_3. We shall assume in general that common channels, as a whole, have the same contribution to total cross section in each initial state. This hypothesis is an extension of Eq. (6).

Turning now to K-meson-nucleon states, let us first compare the total cross sections for (K^-, p) $[(K^0, p)]$ and $(\bar{K}^0, p)$ $[(K^+, p)]$. Then, any final state of the (K^-, p) $[(K^0, p)]$ has the same B and S as the $(\bar{K}^0, p)$ $[(K^+, p)]$, but a different T_3. This situation is the same as that in the pion-nucleon state. The noncommon channels belonging to (K^-, p) [or (K^0, p)] but not to $(\bar{K}^0, p)$ [or (K^+, p)] consist of $T = 0$ states alone with strangeness $S = -1$ $[S = 1]$. All such noncommon channels are given by

$$K^- + p \to \Lambda + n(\Lambda + \bar{\Lambda}), \quad (n = 1, 2, 3, \cdots); \tag{8}$$

there is no such channel for final states of $K^0 + p$. This means that the relations $\sigma(K^-, p) > \sigma(\bar{K}^0, p)$ and $\sigma(K^0, p) = \sigma(K^+, p)$ hold.

Let us compare $(\bar{K}^0, p)$ and (K^+, p). These have the same B and T_3 but $(\bar{K}^0, p)$ has $S = -1$ and (K^+, p) has $S = 1$. One may show that all noncommon channels belonging to $(\bar{K}^0, p)$ but not to (K^+, p) are given by

$$\begin{aligned} &(\Lambda \text{ or } \Sigma) + n_1(\pi) + n_2(\Lambda + \bar{\Lambda}) + n_3(\Sigma + \bar{\Sigma}) \\ &\qquad + n_4(\Lambda + \bar{\Sigma}) + n_5(\bar{\Lambda} + \Sigma), \\ &\qquad (n_1 + n_2 + n_3 + n_4 + n_5 = 1, 2, 3, \cdots) \\ &(\Lambda \text{ or } \Sigma) + n_6(\pi) + n_7(\Xi + \bar{\Xi}), \\ &\qquad (n_6 + n_7 = 1, 2, 3, \cdots) \\ &\Xi + K + n_8(\pi) + n_9(\Xi + \bar{\Xi}), \\ &\qquad (n_8 + n_9 = 0, 1, 2, \cdots) \end{aligned} \tag{9}$$

where charge superscripts are dropped. From Eqs. (8) and (9) one obtains

$$\begin{aligned} \sigma(K^-, p) - \sigma(K^+, p) = {} & \tfrac{1}{2}\sum_i{}_c[\sigma_{S=-1}^{(i)}(0) - \sigma_{S=-1}^{(i)}(1)] \\ & + \sum_i{}_{c'}[\sigma_{S=-1}^{(i)}(1) - \sigma_{S=1}^{(i)}(1)] \\ & + \tfrac{1}{2}\sum_i{}_n \sigma_{S=-1}^{(i)}(0) + \sum_i{}_{n'}\sigma_{S=-1}^{(i)}(1), \end{aligned} \tag{10}$$

where $\sum_c$ and $\sum_n$ [or $\sum_{c'}$ and $\sum_{n'}$] are the common and noncommon channels for (K^-, p) and $(\bar{K}^0, p)$ [or $(\bar{K}^0, p)$ and (K^+, p)], respectively. Our hypothesis means that

$$\begin{aligned} & \sum_i{}_c[\sigma_{S=-1}^{(i)}(0) - \sigma_{S=-1}^{(i)}(1)] \\ & \qquad = \sum_i{}_{c'}[\sigma_{S=-1}^{(i)}(1) - \sigma_{S=1}^{(i)}(1)] = 0; \end{aligned} \tag{11}$$

then $\sigma(K^-, p) > \sigma(K^+, p)$ is obtained. Continuing the same considerations leads to

$$\sigma(K^-, p) = \sigma(\bar{K}^0, n) > \sigma(\bar{K}^0, p) = \sigma(K^-, n) > \sigma(K^0, p) = \sigma(K^+, n) = \sigma(K^+, p) = \sigma(K^0, n). \tag{12}$$

Return now to inequality (3). The $(\bar{p}, p)$ and $(\bar{n}, n)$ have the same B and S as $(p, \bar{n})$, but different values of T_3. Similar considerations apply to the comparison of (p, p) and (n, n) with (p, n). If the method mentioned above for the case of $\Delta B = \Delta S = 0$ and $\Delta T_3 \neq 0$ is applied to these pairs, one can show that $\sigma(\bar{p}, p) = \sigma(\bar{n}, n) > \sigma(p, \bar{n})$ and $\sigma(p, p) = \sigma(n, n) = \sigma(p, n)$. It remains to compare $(p, \bar{n})$ with (p, p). These two states have the same S and T_3 but the former has $B = 0$ and the latter $B = 2$. The noncommon channels that belong to

$(p, \bar{n})$ alone are given by

$$n_1(\pi) + n_2(K + \bar{K}) + n_3(\Lambda + \bar{\Lambda}) + n_4(\Sigma + \bar{\Sigma})$$
$$+ n_5(\Lambda + \bar{\Sigma}) + n_6(\bar{\Lambda} + \Sigma) + n_7(\Xi + \bar{\Xi}),$$

with

$$\sum_{i=2}^{7} n_i = 0 \text{ and } n_1 = 2, 3, 4, \cdots,$$

$$\sum_{i=2}^{6} n_i = 0, \quad n_7 = 1, 2, 3, \cdots, \text{ and } n_1 = 0, 1, 2, \cdots,$$

$$n_2 = 0, \quad \sum_{i=3}^{6} n_i = 1, 2, 3, \cdots, \quad n_7 = 0, \quad \text{and } n_1 = 0, 1, 2, \cdots,$$

$$n_2 = 0, \quad \sum_{i=3}^{6} n_i = 1, \quad n_7 = 1, 2, 3, \cdots, \quad \text{and } n_1 = 0, 1, 2, \cdots,$$

$$n_2 = 1, \quad \sum_{i=3}^{6} n_i = 0, \quad n_7 = 0, 1, 2, \cdots, \quad \text{and } n_1 = 0, 1, 2, \cdots,$$

$$n_2 = 2, 3, 4, \cdots, \quad \sum_{i=3}^{7} n_i = 0, \quad \text{and } n_1 = 0, 1, 2, \cdots, \tag{13}$$

and

$$[(\Xi + \bar{\Lambda} + K) \text{ or } (\bar{\Xi} + \Lambda + \bar{K}) \text{ or } (\Xi + \bar{\Sigma} + K)$$
$$\text{or } (\bar{\Xi} + \Sigma + \bar{K})] + n_8(\pi) + n_9(\Xi + \bar{\Xi}),$$
$$(n_8 + n_9 = 0, 1, 2, \cdots).$$

Therefore,

$$\sigma(\bar{p}, p) = \sigma(\bar{n}, n) > \sigma(\bar{n}, p) > \sigma(p, n) = \sigma(n, n) = \sigma(p, p). \tag{14}$$

Because of invariance under charge conjugation, $\sigma(p, \bar{n}) = \sigma(\bar{p}, n)$, etc. Our speculation is supported by the fact that Ashmore et al.[5] have shown that $\sigma(p, p) = \sigma(p, n)$ within experimental error.

As shown above, any channel of the 21 possible combinations of 2-particle states can be analyzed in three steps: $\Delta B \neq 0$, $\Delta S = 0$, and $\Delta|T_3| = 0$; $\Delta B = 0$, $\Delta S \neq 0$, and $\Delta|T_3| = 0$; and $\Delta B = 0$, $\Delta S = 0$, and $\Delta|T_3| \neq 0$. Any case in which there is a change in two of the quantities B, S, and T_3 must be examined individually. The following additional relations may be of experimental interest.

$$\begin{aligned}
&\sigma(\pi^+, \pi^-) > \sigma(\pi^+, \pi^+) = \sigma(\pi^-, \pi^-),\\
&\sigma(\Sigma^-, p) = \sigma(\Sigma^+, n) > \sigma(\Sigma^+, p) = \sigma(\Sigma^-, n),\\
&\sigma(\bar{\Sigma}^+, p) = \sigma(\bar{\Sigma}^-, n) > \sigma(\bar{\Sigma}^-, p) = \sigma(\bar{\Sigma}^+, n),\\
&\sigma(\Xi^-, p) > \sigma(\Xi^-, n), \quad \sigma(\bar{\Xi}^-, p) = \sigma(\bar{\Xi}^-, n),\\
&\sigma(\pi^-, d) = \sigma(\pi^+, d),^8\\
&\sigma(K^-, d) > \sigma(K^+, d),\\
&\sigma(\bar{p}, d) > \sigma(p, d).
\end{aligned} \tag{15}$$

The relations between $\sigma(\Sigma^+, p)$ and $\sigma(\bar{\Sigma}^+, p)$, and between $\sigma(\Xi^-, n)$ and $\sigma(\bar{\Xi}^-, n)$, have not been written. The reason is that, for example, (Ξ^-, n) and $(\bar{\Xi}^-, n)$ states have channels not common with each other; that is, $n + \Xi^- \rightarrow \Sigma^0 + \Sigma^-$ and $n + \bar{\Xi}^- \rightarrow K^0 + K^+$.

The previous analysis of cross sections was based on the elementary particles π, K, N, Λ, Σ, and Ξ. It is to be noted that Eqs. (7), (12), (14), and (15) still hold even if there exists a particle with $B = S = T = 0$ as well as all resonances due to strong interactions.[9] It is also worth while to note that if the total cross sections of two states [for example $\sigma(\pi^+, p)$ and $\sigma(\pi^-, p)$] should cross each other at high energies where all resonances disappear, then this would rule out our hypothesis. If the contribution from noncommon channels does not vanish at sufficiently high energy then our results are in conflict with the Pomeranchuk theorem,[10] but if it does then our results agree with the theorem.

It is very interesting that the asymmetry with respect to the intrinsic quantum numbers of elementary particles may be reflected in the inequalities among total cross sections within the framework of our hypothesis.

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[1]G. von Dardel, R. Mermod, P. A. Piroué, M. V. Vivargent, G. Weber, and K. Winter, Phys. Rev. Letters 7, 127 (1961); S. J. Lindenbaum, W. A. Love, J. A. Niederer, S. Ozaki, J. J. Russell, and L. C. L. Yuan, Phys. Rev. Letters 7, 352 (1961).

[2]G. von Dardel, D. H. Frisch, R. Mermod, R. H. Milburn, P. A. Piroué, M. V. Vivargent, G. Weber, and K. Winter, Phys. Rev. Letters 5, 333 (1960).

[3]E. W. Jenkins, W. F. Baker, R. L. Cool, T. F. Kycia, R. H. Phillips, and A. L. Read, Bull. Am. Phys. Soc. 24, 433 (1961).

[4]M. J. Longo, J. A. Helland, W. N. Hess, B. J. Moyer, and V. Perez-Mendez, Phys. Rev. Letters 3, 568 (1959); R. Armenteros, C. A. Coombes, B. Cook, G. R. Lambertson, and W. A. Wenzel, Phys. Rev. 119, 2068 (1960); S. J. Lindenbaum, W. A. Love, J. A. Niederer, S. Ozaki, J. J. Russell, and L. C. L. Yuan, Phys, Rev. Letters 7, 185 (1961).

[5]A. Ashmore, G. Cocconi, A. N. Diddens, and A. M. Wetherall, Phys. Rev. Letters 5, 576 (1960).

[6]I. Ia. Pomeranchuk, J. Exptl. Theoret, Phys. (U.S.S.R.) 34, 725 (1958) [translation: Soviet Phys.–JETP 34, 499 (1958)].

[7]D. Amati, M. Fierz, and V. Glaser, Phys. Rev. Letters 4, 89 (1960); M. Sugawara and A. Kanazawa, Phys. Rev. 123, 1895 (1961); S. Weinberg (to be published).

[8]The relation between cross sections for the target deuteron can be obtained by similar considerations. These relations are of immediate experimental interest.

[9]The final state with bound states (for example d) is included in the same channel where the bound states are replaced by their corresponding free particles (for example, free pn state with $T=0$).

[10]The dispersion relations for forward scattering possibly might not hold in the present form because the properties of dynamics at short distances are reflected in the total cross sections at high energies that partially include the phenomena with large momentum transfer.