[10]An example of the effect under consideration is the case of the $K^{\pm}\rightarrow \pi^{\pm}+\pi^0$ decay, which is prohibited by the $\Delta T=\pm\frac{1}{2}$ selection rule but which, nevertheless, competes with the allowed $K^{\pm}\rightarrow 3\pi$ decays as a consequence of the greater phase space available to the former. However, in this case the phase-space factor has only to overcome an inhibition of ~20-25 in the 2π-decay matrix element. Furthermore, the larger mass of the ω (780 Mev compared to 495 Mev for the $K^{\pm}$) favors the allowed 3π-decay mode in the competition.

[11]See, for example, E. Fermi, Elementary Particles (Yale University Press, New Haven, Connecticut, 1951). However, it is necessary to use the exact, relativistic expression for $d\rho_3$, and to evaluate the necessary integrals by numerical methods. I am indebted to Robert Zier for the numerical work involved in this computation.

[12]A similar argument predicts the decay $\omega\rightarrow\pi^0+\gamma$ with about the same branching as the 2π decay. This decay is being looked for by the Johns Hopkins–Northwestern group.[3] However, its detection is obviously much more difficult than the decay $\omega\rightarrow\pi^+ + \pi^-$ here under consideration.

[13]The main difficulty in establishing the experimental value of this ratio arises from problems of normalization between samples containing numbers of pions differing by one in the reactions $p+\bar{p}\rightarrow n\pi+\omega$.

[14]The $\pi^0\rightarrow 2\gamma$ lifetime is $\sim 10^{-16}$ sec [see R. G. Glasser, N. Seeman, and B. Stiller, Proceedings of the 1960 Annual International Conference on High-Energy Physics at Rochester (Interscience Publishers, Inc., New York, 1960), p. 30]; an η with $J=0^-$ would be expected to exhibit a comparable lifetime for 2γ decay.

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RADIATIVE CORRECTIONS TO PAIR ANNIHILATION TOTAL CROSS SECTION

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The recent discussions on the possibility of testing the validity of quantum electrodynamics at short distances have generated new interest concerning the behavior of radiative corrections to electrodynamical cross sections at high energies.

In particular it is very well known that the contribution of real photons to radiative corrections is such as to diminish their absolute value. More precisely, Eriksson and Peterman[1] have shown that for high-energy processes with large momentum transfer ($q^2\gg m^2$), taking into account the emission of real, soft (in the center-of-mass system) photons lowers the expected $\alpha\ln^2(q^2/m^2)$ dependence of the radiative corrections to an $\alpha\ln(q^2/m^2)$ dependence.

In computing the contribution of real photons to the radiative corrections of a differential cross section, one is faced with the problem of introducing in the formula the resolving power of the experimental device, which amounts to taking into account the particular geometry and efficiency of the detecting apparatus,[2] which may in turn hinder the clarity of the theoretical discussion. This difficulty can be avoided for electrodynamical processes such as Compton scattering and pair annihilation for which a total cross section can be defined.[3]

We have computed radiative corrections to the total cross section for the annihilation of the e^+e^- pair, taking into account terms up to e^6.

As is known, the differential cross section for this process corrected for virtual photon contributions can be put in the form[4]:

$$d\sigma_v = d\sigma_0\left\{1+\frac{\alpha}{\pi}\left[F_1(\gamma)\ln\frac{\Lambda}{m}+f_2(\kappa,\tau)\right]\right\}, \qquad (1)$$

where $d\sigma_0$ is the Born cross section, $m^2\kappa=2p_1\cdot k_1=2p_2\cdot k_2$, $m^2\tau=2p_1\cdot k_2=2p_2\cdot k_1$, Λ is the usual fictitious mass of the photon introduced to regularize the infrared divergence [f_2 is independent of Λ for Λ small], and $\gamma=E_+/m$. The right side of (1) can be integrated to give the correction due to virtual photons to the total cross section:

$$\sigma_v=\sigma_0\left\{1+\frac{\alpha}{\pi}\left[F_1(\gamma)\ln\frac{\Lambda}{m}+F_2(\gamma)\right]\right\}. \qquad (2)$$

[(2) is invariant because of the invariant infrared regularization.]

In order to get the contribution of real photons to the total cross section, we have inserted the fictitious mass Λ in the differential cross section for annihilation into three photons and then integrated to obtain the total cross section. The details of this calculation will be published sep-

Table I. Radiative correction up to e^6 to the total cross section for e^+e^- annihilation into photons. $\gamma_L = E_+/m$ in the laboratory system; $\gamma_{c.m.}$ is the same in the center-of-mass system. δ_T represents the total correction; δ_v and δ_3 are connected with the virtual and real photon contribution, respectively (for the precise meaning see the text).

γ_L	δ_T from Eq. (5)	δ_T exact	δ_v	δ_3	$\gamma_{c.m.}$
2×10^2	0.0164	0.0136	-0.0399	0.0564	10
2×10^3	0.0291	0.0286	-0.0933	0.122	31.6
2×10^4	0.0458	0.0457	-0.167	0.213	10^2
10^6	0.0833	0.0832	-0.340	0.423	7.07×10^2
10^8	0.1424	0.1424	-0.619	0.761	7.07×10^3

arately. The result can be put in the form:

$$\sigma_3 = \sigma_0 \frac{\alpha}{\pi}\left[F_1(\gamma)\ln\frac{m}{\Lambda} + F_3(\gamma)\right]. \tag{3}$$

Adding this to (2), one obtains the total cross section with corrections both from virtual and real photons:

$$\sigma_T = \sigma_0\left\{1 + \frac{\alpha}{\pi}\left[F_2(\gamma) + F_3(\gamma)\right]\right\} = \sigma_0(1 + \delta_T). \tag{4}$$

The terms containing Λ cancel as they should. F_2 comes from the virtual-photon corrections and F_3 from the three-photon annihilations, although separately they have no simple physical meaning (since divergent terms have been subtracted, the separation into F_2 and F_3 may depend on the particular regularization method adopted).

The exact expression for δ_T is rather complicated and will be given in another paper, but from it the following expression can be obtained that is valid for $\gamma \gg 1$, $\beta \simeq 1$ (in the lab system):

$$\delta_T = \frac{\alpha}{6\pi}\left\{\ln^2(2\gamma) - \tfrac{1}{2}\ln(2\gamma) + 2\pi^2 - \tfrac{13}{2} + \frac{11 - 5\pi^2}{2[\ln(2\gamma) - 1]}\right\}. \tag{5}$$

In Table I we give δ_T for some values of γ and also, for comparison, the values obtained by numerical computation from the exact formula. [The terms left out are of the type $\alpha n \ln^2(2\gamma)/\gamma$ with n of the order of unity; in fact, with $n \simeq 1.4$, Eq. (5) is valid within 2 % down to $\gamma \simeq 200$.]

The interesting feature of (5) is that, contrary to what one could have expected from previous work,[2] the leading term is still of the $\ln^2(2\gamma)$ type.

In order to understand this result we have separated the contribution of soft and hard real photons in the center-of-mass system. The total cross section for annihilation into three photons, each with energy $\geq\Delta$ (valid for $\Delta \ll m$ in the c.m. system), is

$$\sigma_3(>\Delta) = \sigma_0^c \frac{\alpha}{\pi}\left[-F_1^c(\gamma)\ln\frac{2\Delta}{m} - \varphi_3(\gamma) + F_3^c(\gamma)\right]. \tag{6}$$

The functions with index c are the transformed expressions of formula (4) in the center-of-mass system, while γ also refers to this system, and

$$\varphi_3(\gamma) = \frac{2\gamma^2 - 1}{\gamma^2\beta}\left\{\mathcal{L}\left(\frac{2\gamma}{\gamma(1+\beta)}\right) + \tfrac{1}{2}\mathcal{L}[\gamma^4(1-\beta)^4] - \frac{\pi^2}{3} + 2\ln\gamma(1+\beta)\ln\frac{\gamma(1+\beta)}{2\beta\gamma} + \ln\gamma(1+\beta)\ln\frac{\gamma(1+\beta)}{4\gamma^2}\right\} + \frac{2}{\beta}\ln\gamma(1+\beta), \tag{6'}$$

where $\mathcal{L}(x)$ is the Spence function defined as

$$\mathcal{L}(x) = -\int_0^x \frac{\ln|1-t|}{t}dt.$$

For high γ,

$$\varphi_3(\gamma) \simeq -2\ln^2(2\gamma) + 2\ln(2\gamma) - \pi^2/3, \tag{6''}$$

and Eq. (6) becomes

$$\sigma_3(>\Delta) = \frac{\alpha\sigma_0}{12\pi}\left[24(2\ln 2\gamma - 1)\ln\frac{m\gamma}{\Delta} - 24\ln(2\gamma) + 12 + \frac{36 - 8\pi^2}{2\ln(2\gamma) - 1}\right] = \sigma_0\delta_3 - \frac{2\alpha}{\pi}\sigma_0(2\ln 2\gamma - 1)\ln\frac{2\Delta}{m}. \tag{7}$$

Similarly, one can give the correction due to virtual photons plus that due to real photons with energy (in the c.m. system) $\leq\Delta\ll m$:

$$\sigma_v(<\Delta)=\sigma_0\left\{1+\frac{\alpha}{\pi}\left[-F_1^{\ c}(\gamma)\ln\frac{m}{2\Delta}+\varphi_3(\gamma)+F_2^{\ c}(\gamma)\right]\right\}, \tag{8}$$

which for high γ becomes

$$\sigma_v(<\Delta)=\sigma_0\left\{1+\frac{\alpha}{12\pi}\left[-24(2\ln 2\gamma-1)\ln\frac{m\gamma}{\Delta}+8\ln^2(2\gamma)+22\ln(2\gamma)+4\pi^2-25+\frac{3\pi^2-25}{2\ln(2\gamma)-1}\right]\right\}$$
$$=\sigma_0\delta_v+\frac{2\alpha}{\pi}\sigma_0(2\ln 2\gamma-1)\ln\frac{2\Delta}{m}. \tag{9}$$

It is interesting to observe that formulas (6) and (8) or (7) and (9), although not Lorentz invariant, now have a precise physical meaning. The introduction of a minimum energy of the photons (in the c.m. system) which can be regarded as a sort of regularization of the infrared divergence, allows a separation of the contributions of virtual + real, soft photons (energy $<\Delta$) from real, hard ones (energy $>\Delta$). It is also seen that these contributions contain large terms which compensate each other exactly. This is exemplified in Table I, where in columns IV and V the values of δ_3 and δ_v have been tabulated from (7) and (9).

If we consider $\Delta/m\gamma$ to be the relative resolving power and to be independent of γ, we have from (7) that the contribution of hard photons (in the c.m. system) does not give $\ln^2(2\gamma)$ terms, which instead are contained in (9), namely, in the sum of virtual photons and real, soft photons. This point can be analyzed further if we proceed to the differential cross section corrected for virtual and real, soft photons in the c.m. system. To this end let us consider the (regularized) cross section for annihilation into two hard photons plus one soft with energy $\leq\Delta\ll m$ in the c.m. system:

$$d\sigma_{3\text{soft}}=d\sigma_0\frac{\alpha}{\pi}\left[F_1^{\ c}(\gamma)\ln\frac{2\Delta}{\Lambda}+\varphi_3(\gamma)\right]. \tag{10}$$

It is to be pointed out that the term in square brackets is independent of the angle; this means that the emission of soft photons is isotropic as it should be. If we add to (10) Eq. (1) expressed in the c.m. system, we obtain the desired differential cross section. Its relativistic limit can be easily calculated for the following limiting cases:

$\kappa=\tau=2\gamma^2\gg 1$,

$$d\sigma_c=d\sigma_0\left\{1+\frac{\alpha}{\pi}\left[2(2\ln 2\gamma-1)\ln\frac{\Delta}{m\gamma}+3\ln 2\gamma(1+\ln 4)-\frac{\pi^2}{6}-6.97-\tfrac{3}{2}\ln^2 2\right]\right\}; \tag{11}$$

$\kappa=4\gamma^2\gg\tau\simeq 1$,

$$d\sigma_c=d\sigma_0\left\{1+\frac{\alpha}{\pi}\left[2(2\ln 2\gamma-1)\ln\frac{\Delta}{m\gamma}+2\ln^2(2\gamma)-4-\frac{\pi^2}{2}\right]\right\}, \tag{12}$$

which correspond to 90° and 0° annihilation in the c.m. system, respectively. It is seen that while for the first case the $\ln^2(2\gamma)$ terms are absent, in the second case they are present. This can be understood if it is recalled that while the corrections coming from virtual photons are negative and large ($\ln^2$ terms) at 90° and small (no $\ln^2$ terms) in the forward direction, the soft, real photon contribution is positive and large ($\ln^2$ terms) and isotropic [see (10) with (6′′)]. In particular, the latter compensates the $\ln^2$ terms of the virtual photon contribution at large angles (high momentum transfer),[2] but these terms are still present in the forward direction, since in this situation the virtual photon contribution is small (no $\ln^2$ terms).

It is clear that, upon integrating the differential cross section to obtain again the total one, we will still get the $\ln^2(2\gamma)$ terms.

In conclusion we can say that the corrected total cross section for positron-electron annihilation contains $\ln^2(2\gamma)$ terms. The above discussions enable us to ascribe their origin to the contribution of real, soft photons for small momentum transfer in the center-of-mass system.

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[3]These processes offer also the advantage that since the skeleton graphs contain only internal electron lines, the contribution of other charged fields, e.g., to vacuum polarization can enter only in graphs of at least the sixth order in "e" (the standard radiative corrections we are calculating take into account graphs up to the fourth order). This means that what one could call the "natural breakdown" of quantum electrodynamics (i.e., the deviation due to other existing fields) is particularly negligible for these processes. Really neutral fields decaying into photons can give some contributions at higher order which could become appreciable for high c.m. energies [e.g., through the process $e^+ - e^- \rightarrow \pi^0 + \gamma$; see G. Furlan, Nuovo cimento 19, 840 (1961)].

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TOTAL CROSS SECTIONS AT HIGH ENERGIES*

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Total cross sections which are well known experimentally[1-5] are nearly constant and seem to satisfy the following inequalities from several Bev to the highest energy available at the present moment (≳20 Bev):

$$\sigma(\pi^-, p) > \sigma(\pi^+, p), \tag{1}$$

$$\sigma(K^-, p) > \sigma(K^+, p), \tag{2}$$

$$\sigma(\bar{p}, p) > \sigma(p, p). \tag{3}$$

Pomeranchuk[6] and other authors[7] have proved that the total cross section for a particle on a target must be identical with that for its antiparticle on the same target at sufficiently high energy under some conditions.

It is our purpose to explain inequalities (1) to (3) and to predict other relations among total cross sections at high energies under a hypothesis on generalized isospin independence. The three inequalities will be explained on the basis of the following facts: (α) the isospin of the Λ particle is zero; (β) all hyperons have strangeness of the same sign; (γ) the strangeness of all hyperons is negative; and (δ) the pion and K meson have baryon number zero. Inequality (1) is explained by (α), inequality (2) by (α) and (γ), and inequality (3) by (α), (β), and (δ). If hyperons had positive strangeness, then the direction of inequality (2) would have been reversed. If the hyperons did not all have the same sign, then there would be no basis for inequality (3).

Let us start with the total cross sections for the (π^-, p) and (π^+, p) and take only charge-independent strong interactions into account. The (π^-, p) has two isospin states ($T = \frac{3}{2}$ and $T = \frac{1}{2}$) whereas (π^+, p) has only one isospin state ($T = \frac{3}{2}$). Final states with the same kinds and same number of particles, irrespective of their z components of isospin, T_3, will be designated as being in the same channel. The (π^-, p) includes all the channels that belong to the (π^+, p), but there are channels with pure $T = \frac{1}{2}$ that belong to the (π^-, p) alone.

Therefore from charge independence one obtains

$$\sigma(\pi^-, p) - \sigma(\pi^+, p) = \frac{2}{3}\sum_i{}_c[\sigma^{(i)}(T = \tfrac{1}{2}) - \sigma^{(i)}(T = \tfrac{3}{2})] + \frac{2}{3}\sum_i{}_n\sigma^{(i)}(T = \tfrac{1}{2}), \tag{4}$$

where $\sum_c$ stands for the summation over all the channels common to both (π^-, p) and (π^+, p) and $\sum_n$ for the summation over all the possible channels that are included only in the (π^-, p). The channels that belong to $\sum_n$ are exhausted by

$$\Lambda + K^0 + m(\Lambda + \bar{\Lambda}),$$
$$n + \Lambda + \bar{\Lambda} + m(\Lambda + \bar{\Lambda}), \quad (m = 0, 1, 2, \cdots) \tag{5}$$
$$\bar{\Xi}^0 + \Lambda + \Lambda + m(\Lambda + \bar{\Lambda}).$$

We now make the dynamical assumption that

$$\sum_i{}_c\sigma^{(i)}(T = \tfrac{1}{2}) = \sum_i{}_c\sigma^{(i)}(T = \tfrac{3}{2}) \tag{6}$$

at high energies. The difference in the total cross sections comes from noncommon channels. Then the relations

$$\sigma(\pi^-, p) = \sigma(\pi^0, p) = \sigma(\pi^+, n) = \sigma(\pi^0, n) > \sigma(\pi^+, p) = \sigma(\pi^-, n) \tag{7}$$