VIOLATION OF ISOTOPIC SPIN CONSERVATION IN THE DECAY OF EXCITED MESONIC STATES*

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Experimental evidence now indicates a number of unstable mesons of "strangeness" 0. Two of these have been shown to have spin and parity $J=1^-$: the $\rho\to 2\pi$ (isotopic spin $T=1$, mass $M \simeq 750$ Mev, width $\Gamma \simeq 100$ Mev),[1] and the $\omega\to 3\pi$ ($T=0$, $M\simeq 780$ Mev, $\Gamma < 25$ Mev).[2] A third, the $\eta\to 3\pi$ ($T=0$, $M\simeq 550$ Mev, $\Gamma<50$ Mev)[3] has been conjectured to have $J=1^-$ on the basis of arguments linking it to the mesonic state responsible for the isoscalar nucleon form factor; however, there appear to be experimental indications[4] that it has $J=0^-$. A fourth meson of possible $J=1^-$, the $\zeta\to 2\pi$ ($T=1$, $M\simeq 575$ Mev, $\Gamma<70$ Mev) has been reported by the Saclay group.[5]

Although all of these states have energy sufficient to decay into at least four pions, considerations of available phase space strongly favor their decay into two pions when permitted. However, 2π decay is forbidden to both of the isoscalar ($T=0$) mesons mentioned above, for either of the spin assignments suggested. In the case of $J=0^-$, the 2π decay would violate parity conservation; since such a violation can only be condoned for the weak interactions, a situation which would lead to <u>much</u> too long a lifetime for this state, we may, for the purposes of the following considerations, regard this as an absolute selection rule. On the other hand, in the case of a particle of $J=1^-$, $T=0$, 2π decay is forbidden only by virtue of the conservation of G parity (as is also the case for the 3π decay for $J=0^-$, $T=0$). This conservation law[6] is an expression of the expectation that the strong interactions are invariant with respect to "reflection in charge space." Such charge symmetry is, of course, a special aspect of the requirement of charge independence (isotopic spin conservation) for the strong interactions. The selection rules for the decay of excited mesonic states are summarized in Table I.

However, recent observations[4,7] indicate an alternative 2π-decay mode of the ω meson, with a decay probability between 1 and 10% of the dominant 3π decay. In explanation of this observation, there have recently appeared two communications in this journal. In one, Fubini[8] ascribes the phenomenon to a fundamental breakdown of the principle of charge symmetry in the strong interactions responsible for the pion binding in the ω meson. In the second, Glashow[9] has noted that the small mass difference and overlapping widths of the ρ and ω mesons can result in a "mixing" of the two states by virture of the electromagnetic interactions.

It is the purpose of this note to point out that the observation of a weak 2π decay of the ω meson is, indeed, to be expected on the basis of simple and straightforward considerations of the effects of the electromagnetic interactions on the ω decay, and to present a quantitative estimate of the expected $2\pi/3\pi$ branching ratio. If these considerations are valid, the explanation of Fubini is unnecessarily drastic, while that of Glashow, although indicating an elegant and sufficient cause, is not necessary to account for the observations. Furthermore, these considerations can provide a means for choosing between the two spin assignments suggested for the η meson.

The point is that the violation of charge symmetry by the electromagnetic interactions can be expected to give rise to an appreciable decay $\omega\to\pi^+ + \pi^-$ despite the fact that the matrix element for this decay mode contains the factor $\alpha = 1/137$,

Table I. Selection rules for pionic decay of excited mesonic states.[a]

Spin and parity	Isotopic spin	2π	3π	4π
0^+	0	yes	no (P, G)	yes
	1	no (G)	no (P)	no (G)
0^-	0	no (P)	no (G)	no (P)
	1	no (P, G)	yes	no (G)
1^+	0	no (P, G)	yes	no (G)
	1	no (P)	no (G)	yes
1^-	0	no (G)	yes	no (G)
	1	yes	no (G)	yes

[a]These selection rules are most easily derived following E. Fermi and C. N. Yang, Phys. Rev. 76, 1739 (1949), by considering the mesonic states as appropriate nucleon-antinucleon combinations. Since, on this model, the mesonic states of lowest mass are most likely to arise from N-$\bar{N}$ combinations in an orbital S state, the spin assignments $J=0^-$ and 1^- would be favored.

and even in the absence of a nearby ρ-meson state. There are two reasons for the relative enhancement of the 2π as compared to the 3π decay mode. The first is simply the greater phase space available to the 2π decay. However, it is easily shown that this factor alone cannot overcome the factor $\sim\alpha^2$ favoring the allowed 3π decay.[10]

The second reason, however, weighs much more heavily in favor of the 2π decay. This arises from the angular momentum barriers which inhibit the 3π decay, owing to the spin 1^- of the ω. Thus, the simplest (and presumed most probable) decay $\omega \to 3\pi$ is one in which two of the pions possess internal orbital angular momentum $L=1$, while the third is in a state of orbital angular momentum $l=1$ with respect to the c.m. of the system. Even though the 2π decay also requires an orbital angular momentum of $1\hbar$, the fact that the 3π decay involves two barriers, and appreciably less momentum per pion, may be shown to provide the required inhibition as compared to the "forbidden" 2π decay.

The competition between the two decay rates may be estimated as follows: The ratio of decay rates is given by

$$\frac{\lambda_3}{\lambda_2}=\frac{\int |H_3(p_3,q)|^2 d\rho_3(p_3)}{|H_2|^2\rho_2}, \tag{1}$$

where the H's represent the decay matrix elements and the ρ's the phase-space factors. q is the internal momentum of the di-pion, while p_3 is the momentum of the third pion in the 3π decay; the momenta p of the pions in the 2π decay are, of course, uniquely determined by the mass M of the ω. Now, we assume that the interaction leading to the decays is characterized by the range $R=m^{-1}$ ($\hbar=c=1$) and that the matrix element for the 3π decay depends on the distribution of momenta among the products only through the angular momentum barrier penetration factors,

$$|H_2|^2=\frac{\Omega}{V}A_2\frac{p^2}{p^2+m^2}, \tag{2}$$

$$|H_3|^2=\frac{\Omega^2}{V^2}A_3\frac{q^2}{q^2+m^2}\frac{p_3^{\,2}}{p_3^{\,2}+m^2}, \tag{3}$$

in which $\Omega=4\pi/3m^3$ is the interaction volume and V is a normalization volume (which is cancelled by the dependence of the phase-space factors on V).

The phase-space factors for the 2π and 3π decays being well known,[11] the integration in Eq. (1) may be performed, and the result obtained in the form

$$\lambda_3/\lambda_2=C(M,m/\mu)A_3/A_2 \tag{4}$$

(μ is the pion mass). The numerical constant $C(M,m/\mu)$ has been obtained for the two values of M corresponding to the η and ω mesons, and for a number of values of the range parameter m/μ. These results are given in Table II.

The values of C shown in Table II testify to the sensitivity of the decay competition both on M and on m/μ. Assuming that the electromagnetic interaction leads to a ratio of the "allowed" to the "forbidden" decay constants of

$$A_3/A_2\simeq\alpha^{-2}\cong 2\times 10^4, \tag{5}$$

we note that the (reasonable) choice $m/\mu=2$ ($R\cong 0.7$ fermi) gives rise to the predictions

$$\begin{aligned}\lambda_3/\lambda_2 &\cong 1.6 \text{ for } M=550 \text{ Mev } (\eta \text{ meson})\\ &\cong 70 \quad\text{for } M=780 \text{ Mev } (\omega \text{ meson}).\end{aligned} \tag{6}$$

An increase in the range parameter m (decrease in the range) would reduce these numbers.[12]

The available evidence in the case of the ω meson[13] indicates $\lambda_3/\lambda_2(\omega)\simeq 10$-$100$. This is not inconsistent with the predictions for $m/\mu\simeq 2$; the same interaction range would predict roughly equal 2π- and 3π-decay rates for the η ($M\simeq 550$ Mev) if it has $J=1^-$, but there is no compelling reason for assuming either precisely the same range or the same value of A_3/A_2 for the two mesonic states.

From the preceding we may conclude that the observation of an appreciable 2π-decay mode of the ω meson (and of η, if it has $J=1^-$) does not imply any unexpected violation of the accepted conservation laws for the strong interactions. On the other hand, that these G-violating decays are plausible does not necessarily imply that they must appear with the predicted strength. Rather, the determination of an accurate experimental

Table II. Inhibition factor $C(M,m/\mu)$, arising from phase-space and angular momentum barrier penetration, for the 3π- vs the 2π-decay mode of a meson with $J=1^-$, $T=0$.

Mass M (Mev)	$C(M,m/\mu)$ $m/\mu=1$	2	4
550	2.7×10^{-3}	8.3×10^{-5}	1.8×10^{-6}
780	8.4×10^{-2}	3.8×10^{-3}	1.2×10^{-4}

value for the ratio λ_3/λ_2 should shed considerable light on the nature of the strong interactions responsible for these excited mesonic states, and on the nature of their coupling to the electromagnetic field.

Table II also provides a means for estimating the ratio of the widths of the ω and ρ resonances. Thus, neglecting the small ω-ρ mass difference, and assuming $A_3/A_2 \sim 1$ (i.e., both decays strongly allowed),

$$\Gamma_\omega/\Gamma_\rho \simeq C(780, 2) \simeq 4\times10^{-3} \text{ (for } m/\mu \simeq 2). \tag{7}$$

Taking $\Gamma_\rho \simeq 100$ Mev, Eq. (7) yields $\Gamma_\omega \simeq \frac{1}{2}$ Mev, not inconsistent with the observations, albeit small.

Returning to the η meson, if it has $J=0^-$, $T=0$, Table I shows that all decays into pions only are forbidden, although the 3π decay could proceed by virtue of the electromagnetic interactions. Since, for a particle of spin 0, the decay $\eta \to \pi+\gamma$ is forbidden by angular momentum conservation (the photon carries spin 1 along its direction of emission), the only decay which would be expected to compete favorably with $\eta \to 3\pi$ would be $\eta \to 2\pi+\gamma$. Observation of an alternate $\eta \to 2\pi$ decay mode would permit the unambiguous choice of $J=1^-$ over $J=0^-$.

Finally, we may attempt to estimate the widths of the η and ζ resonances. Thus, if the ζ meson should be just a lighter version of the ρ, considerations of phase space and angular momentum barriers predict

$$\Gamma_\zeta = 0.41(A_\zeta/A_\rho)\Gamma_\rho \approx 40 \text{ Mev}, \tag{8}$$

assuming $m/\mu \simeq 2$ and $A_\zeta \approx A_\rho$. If the η has $J=1^-$, we may combine this result with the preceding computation (taking $M_\eta \approx M_\zeta$, $A_\eta \approx A_\zeta$):

$$\Gamma_\eta \approx 2C(550, 2)\Gamma_\zeta, \tag{9a}$$

$$\Gamma_\eta \approx \tfrac{1}{2}\times10^{-4}\Gamma_\rho \sim \tfrac{1}{2}\times10^{-2} \text{ Mev}. \tag{9b}$$

On the other hand, if η has $J=0^-$ its 3π decay is inhibited both by the α^2 factor and by phase space, but there are no longer any angular momentum barriers to surmount. In this case, a computation along lines similar to those outlined above gives, for $m/\mu = 2$,

$$\Gamma_\eta/\Gamma_\rho = \lambda_\eta/\lambda_\rho \simeq 2\times10^{-3}A_\eta/A_\rho \simeq 10^{-7}, \tag{10}$$

assuming $A_\eta/A_\rho \approx \alpha^2$.

On both spin assumptions, the width of the η resonance is expected to be immeasurably small. The corresponding decay lifetimes are $\sim10^{-19}$ and 10^{-16} sec for $J=1^-$ and 0^-, respectively. In the latter case ($J=0^-$), however, the lifetime for the 3π decay may be sufficiently long to allow appreciable competition by an $\eta \to 2\gamma$ decay mode.[14]

To summarize: Heavy mesons with spin $J=1^-$ and $T=0$ are expected to exhibit observable 2π-decay modes as a result of the breakdown of G parity through the electromagnetic interactions. This decay mode appears to have been observed for the ω meson,[7] with a branching ratio consistent with the predictions of a crude computation which takes into account phase space and angular momentum barrier penetration. If the η meson also has $J=1^-$ this decay mode should be competitive with the allowed 3π decay. However, for a $J=0^-$ η meson, the 2π decay is strictly forbidden, while the decay $\eta \to 2\gamma$ should be comparable to the observed $\eta \to 3\pi$ decay mode. Although numerical estimates of all of these effects have been given, these should be regarded as having only order-of-magnitude reliability.

A number of other questions still demand an answer: Are there two states with $J=1^-$, $T=0$, as predicted by Sakurai?[15] Is it sheer coincidence that the masses of the ω and ρ (and of the η and ζ) differ by only a few percent? How does the so-called ABC meson[16] fit into the picture? Answers to these questions are necessary before we can resolve the profound perplexity which now surrounds the observations on the strongly interacting particles.

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[2]B. C. Maglić et al., Phys. Rev. Letters 7, 178 (1961). I am indebted to Dr. Maglić for illuminating discussions on most of the problems considered in this paper, and for communication of some of his own observations before publication. See, also, M. L. Stevenson et al., Phys. Rev. 125, 687 (1962).

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[8]S. Fubini, Phys. Rev. Letters 7, 466 (1961).

[9]S. Glashow, Phys. Rev. Letters 7, 469 (1961).

[10]An example of the effect under consideration is the case of the $K^{\pm} \rightarrow \pi^{\pm} + \pi^0$ decay, which is prohibited by the $\Delta T = \pm\frac{1}{2}$ selection rule but which, nevertheless, competes with the allowed $K^{\pm} \rightarrow 3\pi$ decays as a consequence of the greater phase space available to the former. However, in this case the phase-space factor has only to overcome an inhibition of ~20-25 in the 2π-decay matrix element. Furthermore, the larger mass of the ω (780 Mev compared to 495 Mev for the $K^{\pm}$) favors the allowed 3π-decay mode in the competition.

[11]See, for example, E. Fermi, Elementary Particles (Yale University Press, New Haven, Connecticut, 1951). However, it is necessary to use the exact, relativistic expression for $d\rho_3$, and to evaluate the necessary integrals by numerical methods. I am indebted to Robert Zier for the numerical work involved in this computation.

[12]A similar argument predicts the decay $\omega \rightarrow \pi^0 + \gamma$ with about the same branching as the 2π decay. This decay is being looked for by the Johns Hopkins–Northwestern group.[3] However, its detection is obviously much more difficult than the decay $\omega \rightarrow \pi^+ + \pi^-$ here under consideration.

[13]The main difficulty in establishing the experimental value of this ratio arises from problems of normalization between samples containing numbers of pions differing by one in the reactions $p + \bar{p} \rightarrow n\pi + \omega$.

[14]The $\pi^0 \rightarrow 2\gamma$ lifetime is $\sim 10^{-16}$ sec [see R. G. Glasser, N. Seeman, and B. Stiller, Proceedings of the 1960 Annual International Conference on High-Energy Physics at Rochester (Interscience Publishers, Inc., New York, 1960), p. 30]; an η with $J = 0^-$ would be expected to exhibit a comparable lifetime for 2γ decay.

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RADIATIVE CORRECTIONS TO PAIR ANNIHILATION TOTAL CROSS SECTION

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The recent discussions on the possibility of testing the validity of quantum electrodynamics at short distances have generated new interest concerning the behavior of radiative corrections to electrodynamical cross sections at high energies.

In particular it is very well known that the contribution of real photons to radiative corrections is such as to diminish their absolute value. More precisely, Eriksson and Peterman[1] have shown that for high-energy processes with large momentum transfer ($q^2 \gg m^2$), taking into account the emission of real, soft (in the center-of-mass system) photons lowers the expected $\alpha \ln^2(q^2/m^2)$ dependence of the radiative corrections to an $\alpha \ln(q^2/m^2)$ dependence.

In computing the contribution of real photons to the radiative corrections of a differential cross section, one is faced with the problem of introducing in the formula the resolving power of the experimental device, which amounts to taking into account the particular geometry and efficiency of the detecting apparatus,[2] which may in turn hinder the clarity of the theoretical discussion. This difficulty can be avoided for electrodynamical processes such as Compton scattering and pair annihilation for which a total cross section can be defined.[3]

We have computed radiative corrections to the total cross section for the annihilation of the e^+e^- pair, taking into account terms up to e^6.

As is known, the differential cross section for this process corrected for virtual photon contributions can be put in the form[4]:

$$d\sigma_v = d\sigma_0 \left\{ 1 + \frac{\alpha}{\pi} \left[F_1(\gamma) \ln\frac{\Lambda}{m} + f_2(\kappa, \tau) \right] \right\}, \qquad (1)$$

where $d\sigma_0$ is the Born cross section, $m^2\kappa = 2p_1 \cdot k_1 = 2p_2 \cdot k_2$, $m^2\tau = 2p_1 \cdot k_2 = 2p_2 \cdot k_1$, Λ is the usual fictitious mass of the photon introduced to regularize the infrared divergence [f_2 is independent of Λ for Λ small], and $\gamma = E_+/m$. The right side of (1) can be integrated to give the correction due to virtual photons to the total cross section:

$$\sigma_v = \sigma_0 \left\{ 1 + \frac{\alpha}{\pi} \left[F_1(\gamma) \ln\frac{\Lambda}{m} + F_2(\gamma) \right] \right\}. \qquad (2)$$

[(2) is invariant because of the invariant infrared regularization.]

In order to get the contribution of real photons to the total cross section, we have inserted the fictitious mass Λ in the differential cross section for annihilation into three photons and then integrated to obtain the total cross section. The details of this calculation will be published sep-