TOTAL CROSS SECTIONS FOR PIONS ON PROTONS IN THE MOMENTUM RANGE 10 TO 20 Gev/c

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(Received January 29, 1962)

We have previously measured the total cross sections of π^+ and π^- on protons up to 10 Gev/c.[1] Lindenbaum et al.[2] have explored these cross sections further up to 20 Gev/c. The smallness of the variations with energy found in this region makes more precise measurements desirable, and we have therefore extended our previous measurements to 20 Gev/c. The method used was essentially identical to that previously described. The rejection against particles heavier than pions was improved by the use, in addition to the selective Čerenkov counter, of a 10-m threshold gas Čerenkov counter in coincidence. The muon component was rejected by an anticoincidence counter behind 1.6 m of iron. Measurements showed that the electron component was negligible. Impurities in the beam were thus less than 1 % and do not influence the accuracy of the measurements. For each energy the transmission at zero solid angle was extrapolated from measurements in at least 3 geometries, with solid angles between 0.2 and 1 msr at 10 Gev/c and between 0.1 and 0.5 msr at 20 Gev/c.

The results are given in Table I.

The errors given include, in addition to the statistical uncertainties, the errors due to rate effects, and to the extrapolation procedure. The absolute values of all the cross sections may have an additional common error of 0.5 mb, due to uncertainties in the dummy calibration, the effective target length, and other systematic sources, which, however, do not influence the energy dependence.

Our data between 4.5 and 20 Gev/c can be fitted very well with a functional behavior

$$\sigma(\pi^\pm p) = \sigma_\infty + b^\pm p^{-\beta}. \qquad (1)$$

The cross section at infinite energy σ_∞ and the coefficient $b^\pm$ for positive and negative pions are given in Table II for different values of the exponent β. From the value χ^2 of column 5, whose expected value is 12 when the 4-parameter expression (1) is fitted to our 16 measurements, it is seen that a wide range of exponents between 0.3 and 1 give excellent fits to our measurements. The fact that the minimum value χ^2 is considerably below the expected value, and that the fit is thus rather better than one would expect, is an indication that the errors are not entirely statistical but also include some systematic errors. The best fit is obtained for an exponent of about -0.7.

The goodness of the fit between the data and the expression (1) is also demonstrated by Fig. 1, where the momentum scale is chosen such that a linear relation is obtained for $\beta = 0.7$. This figure also includes our previous measurements from 4.5 to 10 Gev/c.[1] The data of Lindenbaum et al.[2] are shown for comparison. For negative pions they agree within the errors with our points and with the functional behavior. For positive pions their measurement in the range between 11 and 14 Gev/c is lower than ours, but outside this range there is good agreement. The data of Longo[3] for π^+p below 4 Gev/c fit very well to the energy dependence (1) extrapolated with the same constants as ours down to 2.7 Gev/c. The same is true for the (π^-p) cross sections of Vovenko et al.[4] in the range from 3.4 to 9.2 Gev/c.

The good fit to a momentum dependence of the type (1) over this wide momentum range may be taken as a strong support of Pomeranchuk's theorem[5] that positive and negative total cross sections approach the same limit at high energies. As

Table I. Total cross sections for ($\pi^\pm p$).

Momentum (Gev/c)	$\sigma(\pi^-p)$ (mb)	$\sigma(\pi^+p)$ (mb)	$\Delta\sigma$ (mb)
10	26.5 ±0.35	25.0 ±0.5	1.5 ±0.45
12	26.0 ±0.25	24.8 ±0.3	1.2 ±0.4
14	26.0 ±0.20	24.7 ±0.3	1.3 ±0.4
17	25.7 ±0.20	24.6 ±0.2	1.1 ±0.3
20	25.6 ±0.50	24.1 ±0.5	1.5 ±0.7

Table II. Best fits to the data for $\sigma_T(\pi^\pm p)$, 4.5-20 Gev/c. $\sigma_T = \sigma_\infty + b^\pm p^{-\beta}$.

β	α_∞	b^+ mb (Gev/c)$^\beta$	b^- mb (Gev/c)$^\beta$	χ^2	$-\delta f^2$
0.2	12.92	19.87	22.37	18	0.0079
0.3	17.39	16.03	19.18	15	0.0062
0.5	20.94	13.97	19.04	9	0.0049
0.7	22.48	14.37	22.10	5	0.0042
1.0	23.63	17.19	31.28	7	0.0036

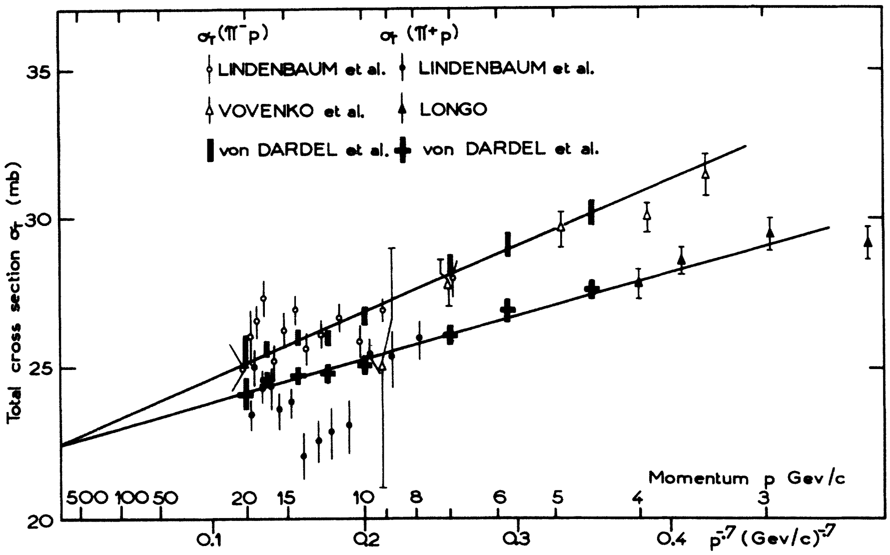

FIG. 1. Total (π^+p) and (π^-p) cross sections versus $p^{-0.7}$. The heavy symbols combine our present data and those previously published[1]; the lines represent the power law dependence, Eq. (1), for best fit. Other measurements are from references 2, 3, and 4.

seen from Fig. 1, a more conclusive proof will require very accurate measurements at much higher energies than are presently available.

An energy dependence of the cross sections of the type (1) guarantees the convergence of the integral in the sum rule,[6]

$$\frac{1}{6\mu}(1+\mu/M)(a_1-a_3)=f^2/\mu^2+\frac{1}{8\pi^2}\int_\mu^\infty[\sigma_-(\omega)-\sigma_+(\omega)]\frac{d\omega}{k}, \tag{2}$$

for pion-nucleon scattering. From this formula Spearman[7] evaluated a coupling constant $f^2=0.082 \pm 0.008$ on the assumption that the integrand disappears above 2 Gev/c. This value was in good agreement with other determinations. The last column in Table II gives the correction to f^2 for a high-energy dependence of the cross sections above 4.5 Gev/c according to (1), assuming that in the range 2-4.5 Gev/c the difference $[\sigma(\pi^-p)-\sigma(\pi^+p)]$ has the value at 4.5 Gev/c. The correction is small compared to the uncertainty due to errors in the low-energy parameters.

Recently, Chew and Frautschi[8] have predicted that at high laboratory energies, E, the asymptotic behavior of the total cross sections is dominated by terms of the type $E^{-[1-\alpha_i(0)]}$, where the Regge exponent for zero momentum transfer, $\alpha_i(0)$, is characteristic for the particle or resonance state exchanged in the process.[9]

Applying these ideas to the $(\pi^\pm p)$ total cross sections, Udgaonkar[10] concludes that the difference between $\sigma(\pi^+p)$ and $\sigma(\pi^-p)$ will approach zero, and the sum a constant value, as inverse power laws with exponents characteristic of, respectively, the $T=1$, $J=1$ two-pion state (ρ), and the $T=0$ two-pion resonance (ABC) found by Booth, Abashian, and Crowe.[11]

In terms of this interpretation our data indicate that $\alpha_\rho(0)\simeq\alpha_{\rm ABC}(0)\simeq 0.3$. Due to the experimental uncertainties this is, however, only a very rough estimate, especially as far as $\alpha_{\rm ABC}$ is concerned. Choosing $\alpha_{\rm ABC}\approx 0$, as one would expect from the properties of the ABC resonance and $\alpha_\rho \simeq 0.3$, hardly makes the fit to the data worse.

We are grateful for the whole-hearted cooperation of the proton synchrotron machine division in the operation of the synchrotron and our hydrogen target. We also want to thank G. Gendre and L. Velati for their assistance in preparing and running the experiment.

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DETERMINATION OF THE Σ PARITY*

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(Received January 29, 1962)

In a previous Letter we reported a hydrogen-bubble-chamber experiment on the K^-p interaction in the vicinity of 400-Mev/c incident K^- momentum.[1] The existence of an excited hyperon of 1520-Mev mass and 16-Mev full width was established; the state was found to have isotopic spin 0, spin 3/2, even parity with respect to K^-p, and a $\bar{K}N$:$\Sigma\pi$:$\Lambda 2\pi$ branching ratio of 3:5:1. In this Letter we report the study of the angular distributions and polarizations of the different $\Sigma\pi$ charge states, from which we conclude that the $Kp\Sigma$ parity is odd.

The determination of the $Kp\Sigma$ parity in the reaction

$$K^- + p \to \Sigma + \pi \tag{1}$$

rests on establishing the parity of the transition operator M defined by $\psi_f = M\chi_i$, where $\psi_f(\theta,\phi)$ is the final-state wave function and χ_i is the initial spin function. For a reaction in which a new pair of particles is created, M may be either scalar or pseudoscalar. The problem can be conveniently discussed in terms of a generalization of the Minami transformation.[2] Defining $\vec{k}_i$ and $\vec{k}_f$ as unit vectors in the incident K^- and outgoing π directions, respectively, and $\vec{n} = \vec{k}_i \times \vec{k}_f / |\vec{k}_i \times \vec{k}_f|$ as the unit normal to the scattering plane, one can write four expressions for M—two scalar and two pseudoscalar. These are listed in column 1 of Table I. Here A and B are functions of the center-of-mass (c.m.) scattering angle θ between the K^- and the π. For final $S_{1/2}$, $P_{1/2}$, and $P_{3/2}$ waves (abbreviated S, P_1, P_3), they are[3]

$$A = \tfrac{1}{2}\lambda[S + (2P_3 + P_1)\cos\theta]; \quad B = \tfrac{1}{2}i\lambda[(P_3 - P_1)\sin\theta]. \tag{2}$$

Since the operators $\vec{\sigma}\cdot\vec{k}_i$ and $\vec{\sigma}\cdot\vec{k}_f$ change the parities of all initial- and final-state partial waves, respectively, the transformation from the first to the second row represents the usual Minami transformation, whereas rows 3 and 4 represent transformations of either the initial- or final-state partial waves of row 1. Thus for rows 2 and 4 we have $S \to P_1$, $P_1 \to S$, and $P_3 \to D$ in Eq. (2). The cross section I can be obtained immediately from $I = \langle \psi_f^\dagger, \psi_f \rangle$, and the polarization $\vec{P}$ from $I\vec{P} = \langle \psi_f^\dagger, \vec{\sigma}\psi_f \rangle$. Substituting $\psi_f = M\chi_i$ and taking spin

Table I. Generalized Minami ambiguities.

Transition matrix M	Parity of M	Cross section I'	$I\vec{P}$	Example initial → final
$(A + B\vec{\sigma}\cdot\vec{n})$	+	$I + I\vec{P}\cdot\vec{P}_i$	$2\mathrm{Re}(A^*B)\vec{n}$	$SP_1P_3 \to SP_1P_3$
$\vec{\sigma}\cdot\vec{k}_f(A + B\vec{\sigma}\cdot\vec{n})\vec{\sigma}\cdot\vec{k}_i$	+	$I + I\vec{P}\cdot\vec{P}_i$	$-2\mathrm{Re}(A^*B)\vec{n}$	$P_1SD \to P_1SD$
$(A + B\vec{\sigma}\cdot\vec{n})\vec{\sigma}\cdot\vec{k}_i$	-	$I - I\vec{P}\cdot\vec{P}_i$	$2\mathrm{Re}(A^*B)\vec{n}$	$P_1SD \to SP_1P_3$
$\vec{\sigma}\cdot\vec{k}_f(A + B\vec{\sigma}\cdot\vec{n})$	-	$I - I\vec{P}\cdot\vec{P}_i$	$-2\mathrm{Re}(A^*B)\vec{n}$	$SP_1P_3 \to P_1SD$