DECAY PRODUCTS OF VECTOR MESONS*

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In the past year, several experimental groups have reported[1] evidence for the existence of peaks in the Q-value plots of multipion systems produced in different ways. One plausible interpretation of such peaks is that they are due to the production of new particles, elementary or otherwise, which decay rapidly into the pions actually seen. Indeed, in some models of the strong interactions,[2] such particles are specifically predicted.

While the existence of a specific case of such a particle has been recognized by its decay into a 2-pion or 3-pion channel, it is well to remember that the decay mode by which a particle is recognized may not be its dominant decay mode at all. It is the purpose of this note to point out that this may well be true for the two reported mesons with $T=0$ and probably $J=1$, which have been recognized by their 3π "decay modes." The fact that the 3π mode is probably not dominant for these particles may lead to a clarification of some recently rumored experimental data.

We have computed the partial widths for decay into various channels of the two $T=0$, $J=1$ mesons, called η^0 and ω^0, with masses $m_\eta \sim 550$ Mev and $m_\omega \sim 790$ Mev. The calculations done are all elementary. We assume that the matrix element has the simplest form consistent with the reported spin and parity assignments, neglect final-state interactions, and evaluate all strong dimensionless coupling constants by $g^2/4\pi = 1$. It is to be expected that such a calculation will give the correct order of magnitude, and that the qualitative features that emerge would persist in a more accurate calculation.

Let us first consider the 3π modes. The matrix element for the decay is

$$M = g\epsilon_\mu R^3 \frac{\epsilon_{\mu\alpha\beta\gamma} p_\alpha^{\ 1} p_\beta^{\ 2} p_\gamma^{\ 3}}{(16\omega_1\omega_2\omega_3 m)^{1/2}}. \tag{1}$$

Here g is a dimensionless coupling constant, R is an unknown radius characterizing the decay, p^i are the 4-momenta of the pions, which must be π^+, π^-, π^0, and m is the mass of the decaying meson. The decay rates (or widths) obtained from this are

$$\Gamma_{\omega\to 3\pi} \simeq \frac{g_\omega^{\ 2}}{4\pi}(m_\omega R_\omega)^6 \frac{m_\omega}{9\times 10^5} \simeq (m_\omega R_\omega)^6 \text{ kev}, \tag{2}$$

$$\Gamma_{\eta\to 3\pi} \simeq \frac{g_\eta^{\ 2}}{4\pi}(m_\pi R_\eta)^6 \frac{m_\eta}{10^3} \simeq (m_\pi R_\eta)^6 \times 500 \text{ kev}. \tag{3}$$

To obtain these, we have treated the pions as relativistic in the ω decay, and as nonrelativistic in the η decay. For the radius R, it seems reasonable to take a value of about $1/3m_\pi$, which is the lowest mass that can occur in intermediate states.[3] With this R, we get

$$\Gamma_{\omega\to 3\pi} \sim 30 \text{ kev}, \tag{4}$$

$$\Gamma_{\eta\to 3\pi} \sim 1 \text{ kev}. \tag{5}$$

These anomalously small partial widths are the result of the strong momentum dependence of the matrix elements (1), which is required by the spin and isospin assignments of the ω and η. No "higher symmetry" of strong interactions has been assumed to get them. Indeed any such symmetry is likely to be strongly violated by mass differences.

In view of the small values of $\Gamma_{3\pi}$ it is very unlikely that these are the dominant decay modes. It is therefore necessary to investigate other modes.

For the ω meson, one such alternate mode is provided by a mechanism suggested by Glashow.[4] He has pointed out that the existence of a ρ^0 meson, with a mass near that of the ω^0 and the same spin and parity, will lead to an enhancement of the virtual electromagnetic effects which would allow the ω^0 to decay into 2π. Specifically, the states which will have well-defined lifetimes will be linear combinations of ω^0 and ρ^0, which may be written as

$$|\bar\rho\rangle = (|\rho\rangle - \lambda|\omega\rangle)/(1+\lambda^2)^{1/2}, \tag{6}$$

$$|\bar\omega\rangle = (|\omega\rangle + \lambda|\rho\rangle)/(1+\lambda^2)^{1/2}. \tag{7}$$

Here λ is a mixing parameter which depends on the rate of electromagnetic transitions between ρ^0 and ω^0, as well as on the mass difference of ρ and ω. Glashow has estimated that λ may be about $\frac{1}{10}$. The states $\bar\rho$ and $\bar\omega$ will have mass distributions separated by some amount Δm, and widths related to their decay constants $\Gamma_{\bar\rho}$, $\Gamma_{\bar\omega}$ in the usual way. They will not have well-defined isotopic spin and so the strong production reactions must be analyzed in terms of ρ and ω. The same is true for the decay into a particular isospin state, such as 2π or 3π. In order to determine the time development of the decays of a meson produced as an ω, it is necessary to first express ω in terms of $\bar{\omega}, \bar{\rho}$ to find the wave function at any time, and then re-express $\bar{\omega}, \bar{\rho}$ in terms of ω, ρ in order to compute the decays into a particular channel. A similar situation exists in the K^0-$\bar{K}^0$ complex when there is a selection rule such as $\Delta S = \Delta Q$ operating. However, in that case the "particles" K_1, K_2 are eigenstates of CP, while in this case the $\bar{\rho}^0, \bar{\omega}^0$ do not differ in any obvious internal quantum numbers, and their physical significance is only that they are mostly ρ^0, or mostly ω^0.

Such an analysis has been done for states which on production have a well-defined isotopic spin. These states may be produced by specific reactions, such as

$$p + n \to d + \rho^0, \tag{8}$$

$$d + d \to \mathrm{He}^4 + \omega^0. \tag{9}$$

We find for the time dependence of a state which is pure $T = 0$ at production, which we call $|\omega(t)\rangle$,

$$|\omega(t)\rangle = \frac{|\omega\rangle}{1+\lambda^2}[\exp(im_{\bar{\omega}}t - \Gamma_{\bar{\omega}}t/2) + \lambda^2 \exp(im_{\bar{\rho}}t - \Gamma_{\bar{\rho}}t/2)]$$

$$+ \frac{\lambda|\rho\rangle}{1+\lambda^2}[\exp(im_{\bar{\omega}}t - \Gamma_{\bar{\omega}}t/2) - \exp(im_{\bar{\rho}}t - \Gamma_{\bar{\rho}}t/2)]. \tag{10}$$

A state with $T = 1$ at production will at a later time be

$$|\rho(t)\rangle = \frac{|\rho\rangle}{1+\lambda^2}[\exp(im_{\bar{\rho}}t - \Gamma_{\bar{\rho}}t/2) + \lambda^2 \exp(im_{\bar{\omega}}t - \Gamma_{\bar{\omega}}t/2)]$$

$$+ \frac{\lambda|\omega\rangle}{1+\lambda^2}[\exp(im_{\omega}t - \Gamma_{\omega}t/2) - \exp(im_{\rho}t - \Gamma_{\rho}t/2)]. \tag{11}$$

It may be seen that there will be time-dependent oscillations for the decay of such a state even into modes of a given isospin, which can be reached only from one of $|\rho\rangle$ or $|\omega\rangle$. However, because of the short lifetime involved, it seems unlikely that these oscillations will be measurable. What is therefore of interest is the time integral of the decay rate into a particular channel. We first consider the decay probabilities into definite isospin states, such as 2π or 3π. These are labelled $P_{\omega^0 \to 2\pi}$, etc., where ω^0 means that state is produced as an ω^0.

$$P_{\omega^0 \to 2\pi} = \frac{\lambda^2}{(1+\lambda^2)^2}\left[\frac{1}{\Gamma_{\bar{\omega}}} + \frac{1}{\Gamma_{\bar{\rho}}} - \frac{4(\Gamma_{\bar{\omega}} + \Gamma_{\bar{\rho}})}{\Delta m^2 + (\Gamma_{\bar{\rho}} + \Gamma_{\bar{\omega}})^2}\right]\Gamma_{\rho \to 2\pi}, \tag{12}$$

$$P_{\omega^0 \to 3\pi} = \frac{\Gamma_{\omega \to 3\pi}}{(1+\lambda^2)^2}\left[\frac{1}{\Gamma_{\bar{\omega}}} + \frac{\lambda^4}{\Gamma_{\bar{\rho}}} + \frac{4\lambda^2(\Gamma_{\bar{\omega}} + \Gamma_{\bar{\rho}})}{\Delta m^2 + (\Gamma_{\bar{\rho}} + \Gamma_{\bar{\omega}})^2}\right], \tag{13}$$

$$P_{\rho^0 \to 2\pi} = \frac{\Gamma_{\rho \to 2\pi}}{(1+\lambda^2)^2}\left[\frac{1}{\Gamma_{\bar{\rho}}} + \frac{\lambda^4}{\Gamma_{\bar{\omega}}} + \frac{4\lambda^2(\Gamma_{\bar{\omega}} + \Gamma_{\bar{\rho}})}{\Delta m^2 + (\Gamma_{\bar{\omega}} + \Gamma_{\bar{\rho}})^2}\right], \tag{14}$$

$$P_{\rho^0 \to 3\pi} = \frac{\lambda^2\Gamma_{\omega \to 3\pi}}{(1+\lambda^2)^2}\left[\frac{1}{\Gamma_{\bar{\rho}}} + \frac{1}{\Gamma_{\bar{\omega}}} - \frac{4(\Gamma_{\bar{\omega}} + \Gamma_{\bar{\rho}})}{\Delta m^2 + (\Gamma_{\bar{\omega}} + \Gamma_{\bar{\rho}})^2}\right]. \tag{15}$$

We see that even states which are produced with a given isospin can decay into modes with a different isospin, because of the admixture of the other meson at finite times. Furthermore, if such a state is produced, its decay into a given mode will show a double peak in the Q value of the decay products, the peaks coming in the neighborhood of the average masses of the $\bar{\rho}$ and $\bar{\omega}$. This may provide an explanation of the two nearby peaks in the 2π Q-value plot reported by the Berkeley antiproton group.[5] In order for this to be so, it is first necessary that the mass difference Δm between $\bar{\rho}$ and $\bar{\omega}$ not be smaller than the width of either $\bar{\rho}$ or $\bar{\omega}$. The relative height of the two peaks will depend both on the relative probability for production of ρ and ω and on the branching ratios of ρ and ω into the 2π mode. It is not unlikely that the probabilities of producing ρ and ω, in a strong interaction where there is no selection rule forbidding either, are roughly equal.

The branching ratios ω and ρ into 2π can be obtained from Eqs. (12)-(15), provided that we know the total decay rates $\Gamma_{\bar{\omega}}, \Gamma_{\bar{\rho}}$ for the "true" particles. For the $\bar{\rho}$ meson, a calculation of the 2π-decay rate similar to the one we have done for the $\omega \to 3\pi$ gives a width of about 20 Mev. The main other competing decay mode is likely to be $\bar{\rho}^0 \to \eta^0 + \pi^0$, whose signature will depend on the main η-decay mode. A calculation of the $\eta\pi$ mode, using the matrix element

$$M_{\rho \to \eta\pi} = \frac{g}{\mathfrak{M}^2}\frac{\epsilon_{\mu\nu\alpha\beta}k_\alpha^{(\rho)}k_\beta^{(\eta)}\epsilon_\mu^{(\rho)}\epsilon_\nu^{(\eta)}}{(8m_\rho E_\eta E_\pi)^{1/2}}, \tag{16}$$

$$\Gamma_{\rho\to\eta\pi} = \frac{g^2}{4\pi} \frac{[(m_\rho^2 - m_\eta^2)^2 - 2m_\pi^2(m_\rho^2 + m_\eta^2)]^{3/2}}{24\mathfrak{M}^2 m_\rho^3}. \quad (17)$$

For $\mathfrak{M} \simeq 2m_\pi$, this width is about 15 Mev. We therefore expect that the modes $\bar{\rho} \to \eta + \pi$ and $\bar{\rho} \to 2\pi$ will have roughly equal probabilities, and the total $\bar{\rho}$ width will be $\Gamma_{\bar{\rho}} \sim 40$ Mev.

The mode $\rho \to \eta + \pi$ also exists for the charged $\rho^\pm$. Since the branching ratio for the ρ into these modes is near $\frac{1}{2}$, it would be useful to try to detect them. This could be done by using the fact that a π with a rather well-defined kinetic energy of 50 Mev in the ρ rest system comes out. So if ρ events are chosen by looking at the nucleon recoil in $\pi^+ + p \to \rho^+ + p$, for example, the decay pion of the ρ could be recognized by its sharp energy in the ρ rest system. These considerations are independent of the decay mode of the η^0.

For the $\bar{\omega}$ meson, the partial width for the 2π mode will be

$$\Gamma_{\bar{\omega}\to 2\pi} \simeq \lambda^2, \quad \Gamma_{\rho\to 2\pi} \simeq 0.2 \text{ Mev if } \lambda = 0.1.$$

Thus $\Gamma_{\bar{\omega}\to 2\pi}$ is likely to be much larger than $\Gamma_{\bar{\omega}\to 3\pi}$. This will be true provided that there is no enhancement of the 3π state. Such an enhancement would occur if a $T=1$ state with a mass comparable to the η should exist. Some evidence for such a state, called the ζ meson, has also been reported.[6] If the ζ does exist, then the decay $\omega \to \zeta + \pi$ can occur. The rate for this would be comparable to the rate for $\rho \to \eta + \pi$, or about 15 Mev.[7] In this case it would almost certainly be the dominant decay of the ω and hence the $\bar{\omega}$. Since the ζ will decay into 2π, most of the time, it is to be expected that if the ζ exists, most ω decays will appear as 3π decays. However, the Dalitz plot will show a peculiar band structure, as emphasized by Nauenberg and Pais,[8] due to the Q-value peak in the 2π subsystems. As no evidence of such bands has been reported, it may be that there is no ζ meson.

If the ζ does not exist, another likely decay mode for $\bar{\omega}$ is probably the mode $\bar{\omega} \to \pi^0 + \gamma$. An estimate for the rate of this mode[9] gives a width $\Gamma_{\bar{\omega}\to\pi^0\gamma}$ of about 0.2 Mev also, which means that it is comparable to our estimate of $\Gamma_{\bar{\omega}\to 2\pi}$. Finally, there is the mode $\omega^0 \to \eta^0 + \pi^0$, which also occurs via the electromagnetic mixing with the ρ^0. The rate for this decay should be $\Gamma_{\bar{\omega}\to\eta\pi} = \lambda^2\Gamma_{\rho\to\eta\pi}$ or about 0.15 Mev, again comparable to the 2π-decay rate. If the η decays into $\pi^0\gamma$ as we expect, this will not be easy to detect.

To summarize the situation for ω decays, we find that unless the ζ meson exists, the partial widths for $\omega \to 2\pi$, $\omega \to \pi^0\gamma$, and $\omega \to \eta^0\pi^0$ should be of the same order of magnitude, and perhaps an order of magnitude larger than the direct decay $\omega \to 3\pi$. In this case, we would expect that the height of the 2π peak at the $\bar{\omega}$ mass should be of the same order as the height at the $\bar{\rho}$ mass, and not two orders of magnitude smaller. This is because if the $\bar{\omega}$ decays a sizable fraction of the time into 2π, as we expect, the height of the peak will be determined by the intensity of ρ and ω at production, and not on the widths $\Gamma_{\bar{\rho}\to 2\pi}$, $\Gamma_{\bar{\omega}\to 2\pi}$.

We turn now to the η meson. If the ζ meson does not exist, it is likely that the main decay modes of the η will be $\eta \to \pi^0\gamma$, with a width of 100 kev, compared to 1 kev for $\eta \to 3\pi$. On the other hand, if ζ does exist, then the mixing of the neutral modes will be repeated at this mass, and the phenomena of the double peak in the 2π modes will occur here also. An experimental search for both the $\pi^0\gamma$ and the double peak should surely be carried out.

In conclusion, the main results of our analysis are the following, and are given in tabular form in Table I.

(1) The partial widths for direct $\omega \to 3\pi$ and $\eta \to 3\pi$ are likely to be rather small, and so these will not be the main decay modes of ω and η.

(2) If a $T=1, J=1$ ζ meson exists with a mass near 550 Mev, the decay $\omega \to \zeta + \pi \to 3\pi$ with a width of about 10 Mev will be the dominant ω decay. In this case the 2π decay of ω will be a λ^2 effect of about 1%, and the second peak in the $\pi^+\pi^-$ system will be very hard to find. However, an additional 3π peak of the ρ mass should exist from $\rho \to \zeta + \pi$. This will also be of order λ^2 in intensity.

(3) If the ζ does not exist, the electromagnetic mixing of ω with ρ^0 will provide a mechanism for the decay $\omega \to 2\pi$ with a width of about 200 kev. This will produce a second peak in the $\pi^+\pi^-$ system, whose Q value will be shifted from the ρ-meson peak, and whose height may be comparable to the ρ peak. No second peak will occur in the $\pi^+\pi^0$ system.

(4) The decay into π^0 and γ will go for both ω and η with a width of several hundred kev. For the η this is likely to be the dominant mode, while for the ω it may be comparable to $\omega \to 2\pi$.

(5) The ρ meson should decay about 50% of the time into η and π. In view of (4), this will show up as a large component of $2\pi + \gamma$ in the ρ decay.

It is therefore strongly urged that experimental

Table I. Summary of vector meson decay modes.

Particle	Decay modes	Partial widths (approximate)	Comment
$\bar{\omega}^0$	$\pi^+\pi^-\pi^0$	30 kev	
	$\pi^+\pi^-$	200 kev	Via ρ^0 admixture.
	$\eta^0\pi^0$	150 kev	Via ρ^0 admixture.
	$\pi^0\gamma$	200 kev	
	$\zeta^+\pi^-$, etc.	15 Mev	If ζ exists, this will look like 3π decay.
$\bar{\rho}^0$	$\pi^+\pi^-$	20 Mev	
	$\eta^+\pi^-$, etc.	15 Mev	Gives pion of 50-Mev kinetic energy.
	$\zeta^+\pi^-$, etc.	200 kev	If ζ exists, via ω^0 admixture.
	$\pi^0\gamma$	200 kev	
η^0 (or $\bar{\eta}^0$ if ζ^0 exists)	$\pi^+\pi^-\pi^0$	1 kev	
	$\pi^0\gamma$	100 kev	
	$\pi^+\pi^-$	100 kev	If ζ exists, via admixture of ζ^0 into $\bar{\eta}^0$.
ζ^0	$\pi^+\pi^-$	20 Mev	

physicists try to find the alternate decay modes of the meson resonances and their branching ratios.

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[2]J. J. Sakurai, Ann. Phys. (New York) 11, 1 (1960). M. Gell-Mann, California Institute of Technology Report CTSL-20 (unpublished); Phys. Rev. (to be published).

[3]The radius R may be estimated by having the $\omega^0 \to 3\pi$ decay go via a $\rho^0 + \pi^0$ intermediate state. In this case one gets an R which is slightly energy dependent, but averaging about $1/3\, m_\pi$. If the width of $\omega^0 \to 3\pi$ or $\eta^0 \to 3\pi$ is much larger than the value quoted here, it will be necessary to find some explanation of the large radius required to give this.

[4]S. Glashow, Phys. Rev. Letters 7, 469 (1961). Related considerations are given by H. P. Dürr and W. Heisenberg (to be published).

[5]B. Maglić, colloquium delivered at Brookhaven National Laboratory.

[6]R. Barloutaud, J. Heughebaert, A. Leveque, J. Meyer, and R. Omnes, Phys. Rev. Letters 8, 32 (1962).

[7]The decay $\bar{\rho}^0 \to \zeta + \pi$ will also occur if ζ exists, with a width $\lambda^2 \Gamma_{\omega \to \zeta + \pi}$. In this case a double peak will exist in the 3π mode as well, but the $\bar{\rho}$ peak will be very small.

[8]M. Nauenberg and A. Pais, Phys. Rev. Letters 8, 82 (1962).

[9]The matrix element for the $\pi^0\gamma$ decay mode is $M = g_{\pi\gamma} R \epsilon_{\mu\nu\rho\tau} \epsilon_\mu^{(\omega)} p_\nu^{(\omega)} q_\rho^{(\gamma)} \epsilon_\tau^{(\gamma)} / (8 m_\omega E_\pi E_\gamma)^{1/2}$, giving a rate $R_{\pi\gamma} = (g_{\pi\gamma}^2/4\pi)(m_\omega^3/24)R^2$. An estimate of $g_{\pi\gamma}R$ using an intermediate state of one ρ meson gives $g_{\pi\gamma}R \sim e/m_\rho$ so that $R_{\pi\gamma} \simeq 0.2$ Mev.