HIGH-ENERGY CROSS SECTIONS, POMERANCHUK THEOREMS, AND REGGE POLES*

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The Pomeranchuk theorems[1] have played an important role in recent discussions of the high-energy asymptotic behavior of scattering amplitudes. The assumed constancy of high-energy cross sections has been used by Chew and Frautschi[2,3] in their statement of the principle of maximum strength for strong interactions. Recent experiments have shown, however, that the $\pi^{\pm}$-p, K^--p, and $\bar{p}$-p total cross sections are still falling at 20 Bev, and that they are doing so at different rates. On the other hand, the K^+-p and p-p total cross sections appear to have attained constant values. This has often led to the statement that the conditions for the validity of the Pomeranchuk theorems have not yet been attained in any of the elementary particle scattering processes, at energies accessible to the present-day accelerators. Discussions on this question have been clouded by the complete absence of a theoretical framework prescribing how high the energy has to be for the validity of these theorems, what is the rate at which the different cross sections are to approach the constant limit postulated by Pomeranchuk, or how the differences of particle-antiparticle cross sections are to approach zero. Therefore, it has not been possible so far, even empirically, to get a clear and reliable idea regarding the trends of these cross sections at high energies.[4,5] It is the purpose of this Letter to point out that one does have such a framework now, following from the proposal of Chew and Frautschi that the asymptotic behavior of the elastic scattering amplitude for any process is dominated by the Regge poles[6] in the crossed channels. We can already recognize why certain cross sections are falling more rapidly than others at high energies while certain others are more or less constant. With more extensive and accurate experimental data on the cross sections at <u>high</u> energies on the one hand, and with a better theoretical understanding of the trajectories and residues of the Regge poles corresponding to the pionic resonances in the <u>low</u>-energy region on the other, it should be possible in the near future to understand how the different cross sections are approaching the Pomeranchuk limit.

The relation between the asymptotic behaviors of the cross sections and the Regge poles in the crossed channels will be seen quite clearly if we consider the example of $\pi^{\pm}$-p cross sections in some detail. These cross sections may be expressed as

$$\tfrac{1}{2}[\sigma(\pi^+p)+\sigma(\pi^-p)]=\frac{1}{q_L}\operatorname{Im}T^+(\theta=0),$$

$$\tfrac{1}{2}[\sigma(\pi^-p)-\sigma(\pi^+p)]=\frac{1}{q_L}\operatorname{Im}T^-(\theta=0),$$

where q_L is the laboratory-system momentum of the pion and $T^{\pm}$ are combinations of $I=\frac{1}{2}$ and $I=\frac{3}{2}$ amplitudes given by

$$T^+=\tfrac{1}{3}(T^{1/2}+2T^{3/2}),\quad T^-=\tfrac{1}{3}(T^{1/2}-T^{3/2}).$$

The sum and difference of the π^+-p and π^--p cross sections are therefore governed by the Regge poles that are present in the $\pi+\pi \leftrightarrow N+\bar{N}$ channel of the T^+ and T^- amplitudes, respectively. Now T^+ and T^- for the $\pi+\pi \longrightarrow N+\bar{N}$ channel have $I=0$, $G=+1$ and $I=1$, $G=+1$, respectively, and so the only known Regge poles that could be present in them are the vacuum pole and ABC pole[7] in the case of T^+, and the ρ-meson pole in the case of T^-. The vacuum pole trajectory, of course, passes through 1 at $s=0$ in order to ensure constancy of high-energy cross sections; if the trajectories of the ABC pole and ρ-meson pole pass through $\alpha_{\mathrm{ABC}}(0)$ and $\alpha_\rho(0)$, respectively, at $s=0$, then we immediately see[8] that

$$\sigma(\pi^-p)-\sigma(\pi^+p)\propto E^{-[1-\alpha_\rho(0)]},$$

and

$$\sigma(\pi^-p)+\sigma(\pi^+p)\propto a+bE^{-[1-\alpha_{\mathrm{ABC}}(0)]},$$

where E is the energy of the pion in the laboratory system. The ABC pole shows itself as a virtual state near the threshold of π-π scattering, i.e., at $s\approx 4$, and so has a value close to zero at $s\approx 4$. Therefore, it is expected to have a value ≈ 0 at $s=0$. If we put $\alpha_{\mathrm{ABC}}(0)=0$, we get

$$\sigma(\pi^+p)+\sigma(\pi^-p)=a+b/E.$$

It is interesting to note that such a fit was actually used by Lindenbaum <u>et al.</u>[5] for their data purely on empirical grounds. One now sees a possible reason why it works so well. It is worth while to emphasize here the close relationship between the zero-energy $I=0$ scattering of the π-π system (ABC pole) and the sum of high-energy $\pi^{\pm}$-p cross sections. The coefficient b of the $E^{-[1-\alpha_{\mathrm{ABC}}(0)]}$ term should eventually be related to the parameters of the $I=0$ π-π scattering at low energies.

The ρ-meson pole passes through 1 at $s=29$, and has a value <1 at $s=0$, and is thus responsible for the decrease with energy of the difference of $\pi^{\pm}$-p cross sections. The CERN data[9] from 4.5 Bev/c to 10 Bev/c can be fitted by

$$\sigma(\pi^- p) - \sigma(\pi^+ p) \propto E^{-1/2},$$

indicating $\alpha_\rho(0) \approx 0.5$. The Brookhaven data at higher energies[5] also seem to be consistent with such a fit. But in view of the small value of the difference of these cross sections, and the consequent large error in the experimental values, one has to wait for more data before making a more definitive fit.

One important point which already emerges from the above discussion is that in attempting empirical fits to the high-energy cross sections, one must use certain definite linear combinations of the cross sections. Thus it is meaningless to say that $\sigma(pp)$ or $\sigma(K^+p)$ alone satisfies the conditions for the validity of Pomeranchuk theorem.

In Table I we have shown the Regge poles governing various sums and differences of cross sections, and the expected asymptotic behaviors arising therefrom.[10,11] The Regge poles in question are those corresponding to vacuum, η, ρ, ω, π, ABC,..., i.e., six in all. It is therefore not surprising that the high-energy cross sections for different channels approach the Pomeranchuk limit quite differently.

The coefficients $a, b, \ldots$ occurring in Table I are related to the residues of the various Regge poles in the corresponding channels, and it should be possible to obtain information on them from an analysis of data on the electromagnetic structure of the nucleon, and low-energy π-π, π-N, N-N, K-N, and π-K scattering. Some of these relationships are being considered in detail. In the meantime, it appears on empirical grounds that the signs of most of the coefficients as written in the table are positive. For any given two-particle channel, the vacuum pole alone contributes to the forward coherent amplitude at the highest energies, giving rise to a constant total cross section and the Pomeranchuk theorems. At lower energies, more and more Regge poles show up and produce "deviations" from the Pomeranchuk theorems.[12] We have already seen in the case of $[\sigma(\pi^-p)+\sigma(\pi^+p)]$ that even though $\alpha_{\mathrm{vac}}(0) - \alpha_{\mathrm{ABC}}(0) \approx 1$, the ABC pole makes a substantial contribu-

Table I. Regge poles contributing to various cross sections.

Cross sections	Contributing Regge poles	Expected high-energy behavior
$\sigma(\pi^-p)+\sigma(\pi^+p)$	vacuum, ABC	$a+bE^{-[1-\alpha_{\mathrm{ABC}}(0)]}$
$\sigma(\pi^-p)-\sigma(\pi^+p)$	ρ	$cE^{-[1-\alpha_\rho(0)]}$
$\sigma(K^-p)-\sigma(K^+p)$	η,ω,ρ	$dE^{-[1-\alpha_\eta(0)]}+eE^{-[1-\alpha_\omega(0)]}-gE^{-[1-\alpha_\rho(0)]}$
$\sigma(K^-p)+\sigma(K^+p)$	vacuum, ABC	$f+hE^{-[1-\alpha_{\mathrm{ABC}}(0)]}$
$\sigma(\bar{p}p)-\sigma(pp)$	η,ω,ρ	$lE^{-[1-\alpha_\eta(0)]}+mE^{-[1-\alpha_\omega(0)]}-qE^{-[1-\alpha_\rho(0)]}$
$\sigma(\bar{p}p)+\sigma(pp)$	vacuum, π, ABC	$p+nE^{-[1-\alpha_\pi(0)]}+rE^{-[1-\alpha_{\mathrm{ABC}}(0)]}$
$\sigma(pp)-\sigma(np)$	ρ,π	$qE^{-[1-\alpha_\rho(0)]}+nE^{-[1-\alpha_\pi(0)]}$
$\sigma(pp)+\sigma(np)$	vacuum, ABC, η,ω	$p+rE^{-[1-\alpha_{\mathrm{ABC}}(0)]}-lE^{-[1-\alpha_\eta(0)]}-mE^{-[1-\alpha_\omega(0)]}$
$\sigma(K^+p)+\sigma(K^+n)$	vacuum, ABC, η,ω	$f+hE^{-[1-\alpha_{\mathrm{ABC}}(0)]}-dE^{-[1-\alpha_\eta(0)]}-eE^{-[1-\alpha_\omega(0)]}$
$\sigma(K^+p)-\sigma(K^+n)$	ρ	$gE^{-[1-\alpha_\rho(0)]}$

tion in the energy region of approximately 20 Bev/c, making this cross section fall by about 15% between 5 and 20 Bev/c. Since all the Regge poles we are considering have trajectories lying between the vacuum pole and the ABC pole, we may expect that whenever more than one of them are capable of contributing, their contributions will be comparable in this energy range. Now in the case of $\sigma(pp)$ and $\sigma(K^+p)$, all or almost all of them are contributing, some with positive sign and some with negative. The observed near constancy of these cross sections therefore seems to be a rather complicated effect arising from the more or less complete cancellation of the contributions of all except the vacuum pole.

It should be noticed that some of the coefficients occur more than once in the table. For example, the coefficients of the ρ-pole term are equal but opposite in sign in $[\sigma(\bar{p}p)-\sigma(pp)]$ and $[\sigma(pp)-\sigma(np)]$; the coefficients of η and ω poles are equal but opposite in sign in $[\sigma(\bar{p}p)-\sigma(pp)]$ and $[\sigma(pp)+\sigma(np)]$; and so on. This means, for example, that whereas $\sigma(\bar{p}p)$ approaches the Pomeranchuk limit from above, $\sigma(np)$ should do so from below. The scanty information on $\sigma(np)$ is consistent with this expectation. At 5 Bev,[13] $\sigma(np)=33.6\pm1.6$ mb, and thus is about 9 mb less than $\sigma(pp)$, and this difference is comparable with $\sigma(\bar{p}p)-\sigma(pp)$ at the same energy,[14] as expected. It is to be hoped that enough data will become available in the near future to test these predictions in detail.

Recent data[14,15] on $K^{\pm}$-p, p-p, and $\bar{p}$-p scattering seem to bear out the above expectations, though since differences of cross sections are to be fitted, and these have large errors, a detailed quantitative comparison will have to be postponed. $[\sigma(K^+p)+\sigma(K^-p)]$ seems consistent with $\alpha_{\text{ABC}}(0)\approx0$, and $[\sigma(K^-p)-\sigma(K^+p)]$ can be fitted with a single Regge pole with an effective $\alpha(0)\approx0.4$, which we may for the time being attribute to the η.[16] Actually ρ, η, and ω are all involved in the latter, and with more accurate data it should be possible to separate their effects. For the present, it is very encouraging that the value of $\alpha_\rho(0)$ from $[\sigma(K^+p)+\sigma(K^-p)]$ is consistent with that from $[\sigma(\pi^-p)-\sigma(\pi^+p)]$ and that $\alpha_\eta(0)$ has a value consistent with our ideas regarding the slopes of the Regge trajectories.[3] The data on $\sigma(\bar{p}p)$ and $\sigma(pp)$ also fit into the above pattern. An analysis of the combined data to get $\alpha(0)$ for all the poles is in progress.

To conclude, we would like to emphasize that even though there may be no structure in the cross sections, there is considerable information hidden in the gentle slopes with which the total cross sections are approaching the Pomeranchuk limit.

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[1]I. Ya. Pomeranchuk, J. Exptl. Theoret. Phys. (U.S.S.R.) 34, 725 (1958) [translation: Soviet Phys.–JETP 7, 499 (1958)]; L. B. Okun' and I. Ya. Pomeranchuk, J. Exptl. Theoret. Phys. (U.S.S.R.) 30, 424 (1956) [translation: Soviet Phys. – JETP 3, 307 (1956)].

[2]G. F. Chew and S. C. Frautschi, Phys. Rev. Letters 7, 394 (1961).

[3]G. F. Chew and S. C. Frautschi, Phys. Rev. Letters 8, 41 (1962).

[4]For example, in a recent paper Lindenbaum et al. (reference 5) have given an empirical fit for their data on $\pi^{\pm}$-p cross sections which allows the difference of these cross sections to approach a finite limit (≈2 mb) as energy becomes infinite. Such a behavior would, of course, be inconsistent with the Pomeranchuk theorem.

[5]S. J. Lindenbaum, W. A. Love, J. A. Niederer, S. Ozaki, J. J. Russell, and L. C. L. Yuan, Phys. Rev. Letters 7, 352 (1961).

[6]T. Regge, Nuovo cimento 14, 951 (1959); 18, 947 (1960).

[7]The ABC pole refers to the $I=0$ two-pion anomaly observed by Abashian, Booth, and Crowe near threshold of π-π scattering [N. E. Booth, A. Abashian, and K. M. Crowe, Phys. Rev. Letters 7, 35 (1961)].

[8]Note, for example, that we are taking the leading term in T^- to be proportional to $P_{\alpha_\rho(s)}(-\cos\theta_s)$ which $\sim(\cos\theta_s)^{\alpha_\rho(s)}$ as $\cos\theta_s\to\infty$; and that at $s=0$, $\cos\theta_s\propto E$, the laboratory energy in the πN channel. In conformity with the notation of reference 3, we are using s to denote the square of the center-of-mass energy in the crossed channel in which the Regge pole is introduced.

[9]G. von Dardel, D. H. Frisch, R. Mermod, R. H. Milburn, P. A. Piroué, M. Vivargent, G. Weber, and K. Winter, Phys. Rev. Letters 7, 127 (1961).

[10]Note that only $B=0$, $S=0$ trajectories contribute. In preparing this table we have assumed that η and ω have $I=0$, $G=-1$; and that ρ has $I=1$, $G=+1$. The Regge poles which contribute can be determined from considerations of G parity and isospin. In the case of $\sigma(\bar{p}p)\pm\sigma(pp)$ they follow directly from the concept of J parity introduced by Chew and Frautschi (reference 3). I am thankful to Dr. Evan J. Squires for several interesting discussions on this point.

[11]We have retained only the leading term in the expan-

sion of $P_\alpha(z)$ for large z. The more exact expressions will be considered in another communication in connection with the angular distributions in the diffraction peak.

[12]In our table we have included only the known Regge poles whose trajectories lie high enough for them to show up as particles. These appear to be sufficient for a description of the cross sections in the region of several Bev. At low energies more Regge poles may be expected to contribute as well.

[13]J. H. Atkinson, W. N. Hess, V. Perez-Mendez, and R. W. Wallace, Phys. Rev. 123, 1850 (1961).

[14]S. J. Lindenbaum, W. A. Love, J. A. Niederer, S. Ozaki, J. J. Russell, and L. C. L. Yuan, Phys. Rev. Letters 7, 185 (1961).

[15]E. W. Jenkins, W. F. Baker, R. L. Cool, T. F. Kycia, R. H. Phillips, and A. L. Read, Bull. Am. Phys. Soc. 6, 433 (1961); and private communication from Dr. Rodney L. Cool to Dr. Gerson Goldhaber. I am thankful to Dr. Goldhaber for informing me about these results and to Dr. Kycia for permission to use them prior to publication.

[16]We have assumed here that η has spin 1. There seems to be some uncertainty now about this assignment. If the spin turns out to be 0, the effect of η on high-energy cross sections may not be very large.

P-WAVE PION-PION RESONANCE

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In a recent paper[1] solutions of the coupled S- and P-wave amplitudes in pion-pion scattering were obtained for the range in the coupling constant, $0.20 \leq |\lambda| \leq 0.45$, in which the P wave exhibits a resonance, and the S-wave scattering length for the $I=0$ isotopic spin state is greater than unity.[2] We present here the results for the solution corresponding to $\lambda = -0.1$, which satisfies the requirements of analyticity, unitarity, and crossing symmetry at low energies.[3] This solution possesses the features of the recent experimental results on the P-wave resonance in the two-pion system.[4-7]

In obtaining the solutions for smaller $|\lambda|$, the same methods were used as in reference 1. In particular the values of the two P-wave constants ξ_1 and a_1 (quoted in reference 1) were determined by the first and second derivative crossing conditions at $\nu = -\frac{2}{3}$.[8] The system of equations for low-energy pion-pion scattering solved in reference 1 was reprogrammed for numerical solution on the 7090 computer, and the results presented in reference 1 were reproduced. The greater speed of the 7090 computing machine provided the possibility of increasing the number of cycles necessary to reach convergence for smaller $|\lambda|$, and thirty cycles determined the phase shifts to a sufficient accuracy. As explained in reference 1, for the value $\lambda = -0.1$, the calculations are carried out for a number of ξ_1, a_1, the final values being those which satisfy the crossing conditions. As was found previously, the approximate determination of ξ_1, a_1 by the crossing conditions is nontrivial and the width Γ of the P-wave resonance is sensitive to the value of ξ_1 obtained from these conditions.

For the value $\lambda = -0.1$ the calculated values of a_1, ξ_1, the S-wave scattering lengths, and the resonance parameters are shown in Table I.[9] The width Γ is the total width at half maximum, and is related to the reduced width γ by $\Gamma = [\nu_R^3/(\nu_R+1)]^{1/2}\gamma$. The resonance position quoted in Table I corresponds in units of the pion mass to 700 Mev, while the total resonance width at half maximum is of the order 63 Mev.[10] The scattering length α_0 is less than unity, and therefore will not contradict the experiments on pion production in pion-nucleon collisions in processes of the type $\pi + N \rightarrow \pi + \pi + N$.[11-14]

Table I. Calculated values of a_1^1, ξ_1, resonance parameters ν_R, Γ, and S-wave scattering lengths α_0, α_2 for $\lambda = -0.1$.

a_1^1	ξ_1	ν_R	Γ	α_0	α_2
-0.0050	-199.5	5.25	0.2	0.67	0.2