[10]M. Gell-Mann and A. Pais, Phys. Rev. 97, 1387 (1955). See also footnote 8.

[11]All existing evidence on $\Delta S=0$ leptonic transitions is consistent with CP invariance. See M. T. Burgy, V. E. Krohn, T. B. Novey, G. R. Ringo, and V. L. Telegdi, Phys. Rev. Letters 1, 324 (1958).

[12]B. Cork, L. Kerth, W. A. Wenzel, J. W. Cronin, and R. L. Cool, Phys. Rev. 120, 1000 (1960), give the most recent results and references to earlier work. See M. Gell-Mann and A. Rosenfeld, Ann. Rev. Nuclear Sci. 7, 407 (1957), for interpretation.

[13]It is sometimes said that even if $K^0\rightarrow 2\pi$ transitions are not CP invariant, the $\Delta I=\frac{1}{2}$ rule insures that $K_2{}^0\rightarrow 2\pi$ decay would be forbidden, up to small corrections of the sort considered here. This is, however, not necessarily so. If the $K\rightarrow 2\pi$ amplitude has an energy-dependent, CP-nonconserving phase, then virtual, off-the-energy-shell transitions $K^0\rightarrow 2\pi\rightarrow \bar{K}^0$ would induce $K_2{}^0\rightarrow 2\pi$ decay. This was not treated in the analysis given by S. Weinberg, Phys. Rev. 110, 782 (1958). Even aside from this, we do not expect the $\Delta I=\frac{1}{2}$ rule to be exact, as witness the occurrence of $K^+\rightarrow \pi^+ + \pi^0$ decay. In neutral K-meson decay, if the amplitude for a final two-pion state of isotopic spin $I=2$ is only 1/20 that for $I=0$ (a value suggested by $K^+\rightarrow\pi^+ +\pi^0$ decay), and if the two amplitudes are not in phase, then the $K_2{}^0\rightarrow 2\pi$ branching ratio would be of order unity. This latter point was called to our attention by Dr. B. Sakita.

TESTS OF THE SINGLE-PION EXCHANGE MODEL

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The single-pion exchange model (SPEM) of high-energy particle reactions provides an attractively simple picture of seemingly complex processes and has accordingly been much discussed in recent times.[1] The purpose of this note is to call attention to the possibility of subjecting the model to certain tests precisely in the domain where the model stands the best chance of making sense.

Consider a collision between particles p and k (labelled here by their four-momenta) which results in two groups of outgoing particles, $(p_1{}', \ldots, p_m{}')$ and $(k_1{}', \ldots, k_n{}')$. We restrict ourselves to configurations where the outgoing particles, as viewed in the barycentric system, form two well-defined narrow cones and we partition the particles accordingly into the two groups $\{p_i{}'\}$ and $\{k_i{}'\}$. We suppose, in addition, that the selection rules permit the exchange of a single pion: $p+k\rightarrow\{p_i{}'\}+\pi+k\rightarrow\{p_i{}'\}+\{k_i{}'\}$. Define the invariant momentum transfer $\Delta=p-\sum p_i{}'=\sum k_i{}'-k$. Regarded as a function of Δ^2, the transition amplitude has a pole at $\Delta^2=-\mu^2$ (μ=pion mass), corresponding to the diagram of Fig. 1. The residue involves a product of the amplitudes, $M(p+\pi\rightarrow\{p_i{}'\})$ and $M(k+\pi\rightarrow\{k_i{}'\})$, which describe, respectively, the indicated physical processes. The point $\Delta^2=-\mu^2$ of course lies outside of the physical domain for the reaction $p+k\rightarrow\{p_i{}'\}+\{k_i{}'\}$. Nevertheless, in the SPEM picture one accepts the diagram of Fig. 1 as representing the dominant contribution at small enough physical Δ^2. Indeed one hopes that the only configurations which are ever very probable are those in which there is some partition of final particles corresponding to not too large Δ^2.

It is clear that, given information on the physical reactions $p+\pi\rightarrow\{p_i{}'\}$ and $k+\pi\rightarrow\{k_i{}'\}$, one is led on the basis of the diagram of Fig. 1 to quite definite, and testable, predictions. In less optimistic applications, however, one envisages allowing for at least some additional, unspecified dependence on the variable Δ^2, to correct for off-the-mass-shell effects at the vertices and in the pion propagator.[2]

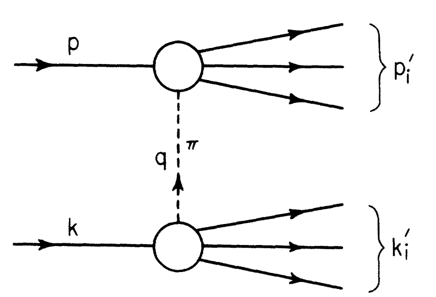

FIG. 1. Diagram for single-pion exchange.

Nevertheless, even if the Δ^2 dependence is left unspecified, and even if, moreover, the vertex functions of Fig. 1 are regarded as completely unknown, the general structure of the diagram leads to certain concrete and testable restrictions on the properties of the over-all reaction spectrum. This comes about because the structure of Fig. 1 implies that there is no correlation between the two groups of final particles, $\{p_i'\}$ and $\{k_i'\}$, beyond what follows from kinematics. This result, reflected in Eq. (2) below, depends in an essential way on the fact that the exchanged pion is a spinless particle.

The differential reaction cross section $d\sigma$ is given by

$$Jd\sigma = f\prod_i dp_i'\delta(p_i'^2+m_i^2) \times \prod_j dk_j'\delta(k_j'^2+\mu_j^2)\delta(p+k-\sum p_i'-\sum k_j'). \quad (1)$$

where J is the relative current of the incident particles, f is the square of the invariant transition amplitude, and all energies are positive-definite. The crucial remark is that, on the peripheral collision picture, f has the structure

$$f = G(p, p_i')H(k, k_i'). \quad (2)$$

The implications of this restriction on the structure of f are best brought out in the reference frames in which one or another of the initial particles is at rest. Thus:

1. In the system where p is at rest (the laboratory system, if p is in fact the target particle), the differential cross section should be invariant under the simultaneous rotation of all three-vectors $\vec{p}_i'$ about the momentum vector $\vec{q}$ of the virtual meson: $\vec{q} = \vec{k} - \sum_i \vec{k}_i' = \sum_i \vec{p}_i'$. This result follows from inspection of Eqs. (1) and (2).

2. Similarly, in the system where k is at rest the differential cross section should be invariant under simultaneous rotation of all three-vectors $\vec{k}_i'$ about $\vec{q} = -\sum_i \vec{k}_i' = \sum_i \vec{p}_i' - \vec{p}$.

It is easy to prove that the above two tests are exhaustive for fixed incoming energy. There are further implications of the model having to do with relations between differential cross sections at different incoming energies, but these implications are rather complicated and lend themselves less easily to experimental testing.

As an example, consider the reaction $\pi(k)+N(p) \to \pi(k_1')+\pi(k_2')+N(p')$.[3] The peripheral collision model for this process involves exchange of a single pion according to $k+p \to k+p'+\pi \to p'+k_1'+k_2'$. In the rest frame of the initial pion the implication of the model is that, for given $\vec{p}$ and $\vec{p}'$, the differential cross section should be independent of the orientation of the plane defined by $\vec{k}_1'$ and $\vec{k}_2'$ about the line $\vec{q} = -\vec{k}_1' - \vec{k}_2' = \vec{p}' - \vec{p}$. Of course for those limiting configurations in which $\vec{p}$ and $\vec{p}'$ are collinear this assertion is an empty one.[4] But where $\vec{p}$ and $\vec{p}'$ are not collinear one could, outside of the peripheral collision model, envisage a correlation between the directions defined by $\vec{k}_1' \times \vec{k}_2'$ and $\vec{p} \times \vec{p}'$. The detection of an appreciable correlation effect of this sort, especially for the subclass of events with small momentum transfer Δ^2, would weigh heavily against the single-pion exchange model.[5]

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[1]For excellent reviews of the model, and references to the literature, see: S. D. Drell, Revs. Modern Phys. 33, 458 (1961); F. Salzman and G. Salzman, Phys. Rev. (to be published).

[2]See, for example, S. D. Drell and K. Hiida, Phys. Rev. Letters 7, 199 (1961); E. Ferrari and F. Selleri, Phys. Rev. Letters 7, 387 (1961).

[3]The single-pion exchange model was first applied here by C. Goebel, Phys. Rev. Letters 1, 337 (1958); G. F. Chew and F. E. Low, Phys. Rev. 113, 1640 (1959). Experimental results have been discussed on this model by A. R. Erwin, R. March, W. D. Walker, and E. West, Phys. Rev. Letters 6, 628 (1961); E. Pickup, D. K. Robinson, and E. O. Salant, Phys. Rev. Letters 7, 192 (1961).

[4]This is peculiar to the case where the set $\{p_i'\}$ contains only one member. For reactions in which the set $\{p_i'\}$ contains two or more particles, rotational invariance of the $\vec{k}_i'$ about $\Sigma_i \vec{k}_i'$ would be a nontrivial result even when $\vec{p}'$ and $\vec{p}$ are collinear.

[5]A second restrictive consequence of the model, as applied to the present example, is perhaps worth mentioning; namely, for unpolarized initial nucleons the outgoing nucleon beam should be unpolarized. The detection of polarization effects is, however, not an easy matter experimentally.