

TEST OF CP CONSERVATION IN NEUTRAL K -MESON DECAY*

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The time dependence of charge asymmetry effects in leptonic decay of neutral K mesons, observed and reported on in the foregoing Letter by Ely et al.,¹ is a source of valuable information on certain crucial aspects of the weak interactions.²

There is a general class of weak-interaction theories³ picturing them as being generated, either directly or possibly through intermediate bosons, by the interaction with itself of a weak current having vector and axial vector transformation properties. In setting up their theory, Feynman and Gell-Mann³ presumed that the strangeness change $\Delta S=2$ was forbidden and were thereby led to impose on the strangeness-changing part of the weak current the selection rule

$$\Delta S = \Delta Q, \quad (1)$$

where ΔQ is the change in electric charge. The supposition that $\Delta S=2$ is forbidden may be tested⁴ and evidently has been confirmed^{5,6} by measurements of the $K_1 - K_2$ mass difference. On the other hand, the results reported by Ely et al.,¹ taken together with present evidence on the $K_1 - K_2$ mass difference, suggest that the reaction $K^0 \rightarrow \pi^+ + e^- + \bar{\nu}$, for which

$$\Delta S = -\Delta Q, \quad (2)$$

in fact takes place. Thus there appears to be a direct violation of the selection rule Eq. (1). It is worth noting that in the case of leptonic decay, Eq. (2) implies $\Delta I_3 = \frac{3}{2}$; hence the $\Delta I = \frac{1}{2}$ rule is also violated by the leptonic decays.

If these results are borne out by further experiment, we must conclude that the above-mentioned class of theories is not capable of describing the strangeness-changing leptonic decays. In addition, the possibility would open up for testing CP invariance of the weak interactions in new ways. It is this aspect of leptonic decay of neutral K mesons which will concern us here.

The data¹ in hand at the present time are too limited to allow any reliable tests of the question we shall raise here; and the main experimental concern has been to establish reliably whether or not the $\Delta S = \Delta Q$ rule is indeed violated. If, how-

ever, the rule does in fact break down, then, as further data accumulate, it should become possible to test for novel aspects of CP symmetry.

Let us then turn directly to the implications of CP noninvariance for leptonic decay of neutral K mesons. Consider the reaction $K^0 \rightarrow \pi^- + e^+ + \nu$, for which $\Delta S = \Delta Q$. The matrix element has a well-defined structure characterized by a sum of terms, each term being the product of an electron-neutrino function and a form factor depending on the invariant momentum transfer between K meson and π meson.⁷ To simplify the discussion, let us assume for the moment that only one such term contributes to the matrix element, as indeed would be the case if the weak Hamiltonian contained only terms of the type V and A . The corresponding form factor will be denoted by f . According to the CPT theorem, the form factor for the reaction $\bar{K}^0 \rightarrow \pi^+ + e^- + \bar{\nu}$ is then f^* (up to a possible irrelevant minus sign). Similarly for the $\Delta S = -\Delta Q$ reactions, $K^0 \rightarrow \pi^+ + e^- + \bar{\nu}$ and $\bar{K}^0 \rightarrow \pi^- + e^+ + \nu$, we have form factors g^* and g , respectively.

If CP invariance is valid for all weak interactions, the K_1 and K_2 states are defined by

$$K_1 = (K^0 + \bar{K}^0)/\sqrt{2},$$

$$K_2 = (K^0 - \bar{K}^0)/\sqrt{2}, \quad (3)$$

where $\bar{K}^0 = (CP)K^0$. In this situation: (i) $K_2^0 \rightarrow 2\pi$ is strictly forbidden; (ii) the reactions $K_2^0 \rightarrow \pi^- + e^+ + \nu$ and $K_2^0 \rightarrow \pi^+ + e^- + \bar{\nu}$ are equally probable; (iii) the form factors f and g are real.²

Let us now entertain the possibility that CP invariance is violated, but only for the strangeness-changing leptonic decays. (The evidence for CP invariance in the nonleptonic decays is rather compelling; see below.) Since the states K_1 and K_2 are determined mainly by the dominant nonleptonic process $K \rightarrow 2\pi$, the expressions Eq. (3) would remain true in good approximation. Hence the items (i) and (ii) above would continue to hold in good approximation,⁸ as they in fact do experimentally.⁹ But the leptonic decay form factors may now be complex. The relevant form factors

for K_1 and K_2 decay are given by

$$\begin{aligned} K_1 &\rightarrow \pi^- + e^+ + \nu, & (f+g)/\sqrt{2} &= a_1, \\ K_2 &\rightarrow \pi^- + e^+ + \nu, & (f-g)/\sqrt{2} &= a_2, \\ K_1 &\rightarrow \pi^+ + e^- + \bar{\nu}, & (g^*+f^*)/\sqrt{2} &= a_1^*, \\ K_2 &\rightarrow \pi^+ + e^- + \bar{\nu}, & (g^*-f^*)/\sqrt{2} &= -a_2^*. \end{aligned} \quad (4)$$

The K_{e3} decay spectra may be expressed in terms of certain amplitudes A_1, A_2 which are products of a_1, a_2 , respectively, and an energy- and spin-dependent factor arising from the electron-neutrino wave functions. Thus, in the standard fashion,² we find for the decay curve at time t for a beam which at $t=0$ was pure K^0 :

$$\begin{aligned} R(\pi^- + e^+ + \nu) &= \frac{1}{2}\{|A_1|^2 \exp(-\lambda_1 t) + |A_2|^2 \exp(-\lambda_2 t) + 2|A_1||A_2| \exp[-\frac{1}{2}(\lambda_1 + \lambda_2)t] \cos(\Delta m t + \varphi)\}, \\ R(\pi^+ + e^- + \bar{\nu}) &= \frac{1}{2}\{|A_1|^2 \exp(-\lambda_1 t) + |A_2|^2 \exp(-\lambda_2 t) - 2|A_1||A_2| \exp[-\frac{1}{2}(\lambda_1 + \lambda_2)t] \cos(\Delta m t - \varphi)\}. \end{aligned} \quad (5)$$

Here Δm is the $K_1 - K_2$ mass difference, λ_1 and λ_2 are the K_1 and K_2 decay rates, and φ is the phase difference between A_1 and A_2 .

The total K_{e3} decay curve, irrespective of charges, is

$$R_{e3} = |A_1|^2 \exp(-\lambda_1 t) + |A_2|^2 \exp(-\lambda_2 t) - 2|A_1||A_2| \sin\varphi \exp[-\frac{1}{2}(\lambda_1 + \lambda_2)t] \sin(\Delta m t), \quad (6)$$

while the K_{e3} charge asymmetry curve is

$$A_{e3} = R(\pi^- + e^+ + \nu) - R(\pi^+ + e^- + \bar{\nu}) = 2|A_1||A_2| \cos\varphi \exp[-\frac{1}{2}(\lambda_1 + \lambda_2)t] \cos(\Delta m t). \quad (7)$$

The generalization of these results in the case for which there is more than one form factor is straightforward. The amplitudes A_i are then linear combinations of the form factors a_i , the coefficients being functions of the energies and spins.⁷

It should also be noted here that observed intensities will usually involve a spin sum and a weighted integral over the spectrum. The appropriate sum and integral will be denoted by angular brackets, as in $\langle |A_1|^2 \rangle$, etc.

The $\Delta S = \Delta Q$ rule [$g=0$ in Eq. (4)] implies $A_1 = A_2$ and therefore, of course, $\varphi=0$. But the fit of the data to the theoretical curves Eq. (5) (with φ set equal to zero) presented in reference 1 indicates an experimental ratio

$$\langle |A_1|^2 \rangle / \langle |A_2|^2 \rangle \geq 6. \quad (8)$$

Taken at face value this is, of course, a result which implies the occurrence of $\Delta S = -\Delta Q$ reactions.

Moreover, if $\Delta S = \Delta Q$ and $\Delta S = -\Delta Q$ processes both occur, we have the possibility of testing whether the relative phase φ of A_1 and A_2 does actually vanish, i.e., whether CP invariance is valid here. There are two rather straightforward ways in which the test can be carried out. One consists of improving the determination of the total decay curve and charge asymmetry to the point that the data yield reliable separate values of the independent quantities $\langle |A_1|^2 \rangle / \langle |A_2|^2 \rangle$ and

$\langle A_{e3} \rangle / \langle R_{e3} \rangle$, the latter being proportional to $\langle |A_1||A_2| \cos\varphi \rangle / \langle |A_1|^2 + |A_2|^2 \rangle$ when $\lambda_1 t \ll 1$. In order to make maximum use of their data to resolve the $\Delta S = \Delta Q$ question, Ely *et al.*¹ have combined these independent quantities by assuming $\varphi=0$. In this connection it should be noted that an accurate determination of the ratio $\langle |A_1|^2 \rangle / \langle |A_2|^2 \rangle$ of K_1 to K_2 partial rates is itself sufficient to resolve the $\Delta S = \Delta Q$ question, whether or not CP is conserved in leptonic decays.

The other method for determining φ would consist of a search for the characteristic third term on the right-hand side of Eq. (6) by measuring accurately the total decay curve as a function of time.

Notice that according to Eq. (4) the possibility $\varphi \neq 0$ can arise only if the $\Delta S = \Delta Q$ and $\Delta S = -\Delta Q$ transitions both occur and if their respective amplitudes f and g are out of phase.

This is a unique situation. Aside from phenomena connected with neutral K -meson decay, weak-interaction effects are in practice always detected only in lowest order of the weak couplings. Thus various classes of coupling, nonleptonic, leptonic $\Delta S = \Delta Q$, leptonic $\Delta S = -\Delta Q$, etc., do not interfere with one another; and each test of CP invariance is internal to the class. Therefore, even if CP invariance is found to be valid in this sense, the question of over-all invariance of the weak interactions remains open; the coupling constants re-

ferring to different classes may have different phases. This possibility can be tested only by study of the decay of neutral K mesons, where the leptonic $\Delta S = \pm \Delta Q$ terms interfere. Moreover, here the states K_1 and K_2 are determined by second order transitions¹⁰ which again involves interference between classes.

Tests of CP invariance for $\Delta S \neq 0$ reactions have thus far been carried out only for nonleptonic processes.¹¹ The large magnitude of the asymmetry parameter in $\Lambda \rightarrow p + \pi^-$, $\Lambda \rightarrow n + \pi^0$, and $\Sigma^+ \rightarrow p + \pi^0$ decays¹² constitutes good evidence for CP invariance in the nonleptonic interactions. The small branching ratios ($< 0.3\%$)⁹ for the process $K_2 \rightarrow 2\pi$ is further evidence for this conclusion.¹³

However, as we have seen, any violation of CP invariance in the leptonic interaction would also result in the appearance of $K_2 \rightarrow 2\pi$ transitions and a charge asymmetry in K_2 decay [in violation of (i) and (ii) above] through interference effects, but the effects are then expected⁸ to be smaller than the presently observed upper limits.⁹

The relevant experiments to test for CP invariance in the strangeness-changing, leptonic interactions are the following:

(1) Independent determination of the charge asymmetry in K^0 decay and of the ratio of the $K_1 \rightarrow \pi^\pm + e^\mp + (\nu, \bar{\nu})$ partial rate to the $K_2 \rightarrow \pi^\pm + e^\mp + (\nu, \bar{\nu})$ partial rate.

(2) A detailed measurement of the total electron rate curve, Eq. (6). Concrete evidence for the interference term would establish violation of CP invariance with respect to the relative phases of $\Delta S = \pm \Delta Q$ couplings.

(3) Further effort to determine the $K_2 \rightarrow 2\pi$ branching ratio and the charge asymmetry in leptonic decay of the K_2 .

(4) Measurement of the muon polarization normal to the decay plane in the $K_{\mu 3}$ modes of the K^\pm or K_2^0 decay. The former has a bearing only on internal (to $\Delta S = \Delta Q$) CP invariance while the latter also involves the interference between $\Delta S = \pm \Delta Q$ transitions.

(5) Standard tests for CP invariance in leptonic decay of hyperons. These again are only "internal" tests.

Further, let us note that if both $\Delta S = \Delta Q$ and $\Delta S = -\Delta Q$ transitions occur, their respective amplitudes f and g could well behave differently with energy; indeed the coupling types (scalar, vector, tensor) could appear with different weights. In effect, the amplitudes A_1 and A_2 of Eqs. (6) and (7) could depend differently on energies and spins. Among the experiments which would bear on this

possibility, we mention the following:

(6) Measurement of the pion energy distribution in the K_{e3} and $K_{\mu 3}$ modes of the neutral K meson. Ultimately the time dependence of the spectra will be of great interest.

(7) Measurement of the longitudinal polarization of the leptons in the $K_{\mu 3}$ and K_{e3} modes of the neutral K .

Finally, of obviously great interest is the following:

(8) Performance of the charge asymmetry experiment for the $K_{\mu 3}$ mode of the neutral K meson.

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the sort considered here. This is, however, not necessarily so. If the $K \rightarrow 2\pi$ amplitude has an energy-dependent, CP -nonconserving phase, then virtual, off-the-energy-shell transitions $K^0 \rightarrow 2\pi \rightarrow \bar{K}^0$ would induce $K_2^0 \rightarrow 2\pi$ decay. This was not treated in the analysis given by S. Weinberg, Phys. Rev. 110, 782 (1958). Even aside from this, we do not expect the $\Delta I = \frac{1}{2}$ rule to be exact, as witness the occurrence of $K^+ \rightarrow \pi^+ + \pi^0$ decay. In neutral K -meson decay, if the amplitude for a final two-pion state of isotopic spin $I = 2$ is only $1/20$ that for $I = 0$ (a value suggested by $K^+ \rightarrow \pi^+ + \pi^0$ decay), and if the two amplitudes are not in phase, then the $K_2^0 \rightarrow 2\pi$ branching ratio would be of order unity. This latter point was called to our attention by Dr. B. Sakita.

TESTS OF THE SINGLE-PION EXCHANGE MODEL

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The single-pion exchange model (SPEM) of high-energy particle reactions provides an attractively simple picture of seemingly complex processes and has accordingly been much discussed in recent times.¹ The purpose of this note is to call attention to the possibility of subjecting the model to certain tests precisely in the domain where the model stands the best chance of making sense.

Consider a collision between particles p and k (labelled here by their four-momenta) which results in two groups of outgoing particles, (p_1', \dots, p_m') and (k_1', \dots, k_n') . We restrict ourselves to configurations where the outgoing particles, as viewed in the barycentric system, form two well-defined narrow cones and we partition the particles accordingly into the two groups $\{p_i'\}$ and $\{k_i'\}$. We suppose, in addition, that the selection rules permit the exchange of a single pion: $p + k \rightarrow \{p_i'\} + \pi + k \rightarrow \{p_i'\} + \{k_i'\}$. Define the invariant momentum transfer $\Delta = p - \sum p_i' = \sum k_i' - k$. Regarded as a function of Δ^2 , the transition amplitude has a pole at $\Delta^2 = -\mu^2$ ($\mu = \text{pion mass}$), corresponding to the diagram of Fig. 1. The residue involves a product of the amplitudes, $M(p + \pi \rightarrow \{p_i'\})$ and $M(k + \pi \rightarrow \{k_i'\})$, which describe, respectively, the indicated physical processes. The point $\Delta^2 = -\mu^2$ of course lies outside of the physical domain for the reaction $p + k \rightarrow \{p_i'\} + \{k_i'\}$. Nevertheless, in the SPEM pic-

ture one accepts the diagram of Fig. 1 as representing the dominant contribution at small enough physical Δ^2 . Indeed one hopes that the only configurations which are ever very probable are those in which there is some partition of final particles corresponding to not too large Δ^2 .

It is clear that, given information on the physical reactions $p + \pi \rightarrow \{p_i'\}$ and $k + \pi \rightarrow \{k_i'\}$, one is led on the basis of the diagram of Fig. 1 to quite definite, and testable, predictions. In less optimistic applications, however, one envisages allowing for at least some additional, unspecified dependence on the variable Δ^2 , to correct for off-the-mass-shell effects at the vertices and in the pion propagator.²

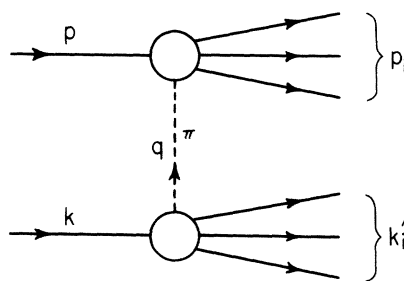


FIG. 1. Diagram for single-pion exchange.