ALTERNATIVE NUCLEON FORM FACTORS*

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The discovery of a resonance in a $J=1, T=1$, $\pi\pi$ state at 750 Mev,[1] and of 3π resonances $J=1$, $T=0$ at 785 Mev,[2] and of unidentified J at 550 Mev,[3] have led to renewed and detailed interest in Sakurai's vector theory of strong interactions (VTSI)[4,5] where these are identified, respectively, as ρ, ω, and η. These resonances must enter into the nucleon form factors, and analyses of these have been discussed.[6-8] It is the purpose of this Letter to point out that these analyses are considerably aided by expressing the electron-nucleon scattering in a different pair of invariants from that usually adopted.

The Rosenbluth formula can be expressed in the form[9,10,11]

$$\frac{d\sigma}{d\Omega}=\left(\frac{\alpha r_0 m}{2E\sin(\theta/2)}\right)^2\frac{E'}{E}$$

$$\times\left[\frac{\cot^2(\theta/2)}{1+q^2/4M^2}G_E{}^2+\frac{q^2}{4M^2}\left(2+\frac{\cot^2(\theta/2)}{1+q^2/4M^2}\right)G_M{}^2\right],$$

where $G_M=F_1+\mu F_2$, $G_E=F_1-(q^2/4M^2)\mu F_2$, $F_{1p}(0)=F_{2p}(0)=F_{2n}(0)=1, F_{1n}=0$; $\mu_p=1.79$, $\mu_n=-1.91$; and we can form the usual isotopic vector and scalar combinations,[6] $G_{MV}=(1/2)\times(G_{Mp}-G_{Mn})$, etc. The advantages of this parametrization are that G_E and G_M are more directly determined from experiment than F_1 and F_2, and complicating error correlations are reduced. Thus G_M is directly determined by 180° scattering, and $G_E{}^2+q^2G_M{}^2/4M^2$ by forward scattering. The derivation is therefore simpler than the method of intersecting ellipses often used.[8]

For inelastic scattering a similar separation occurs with the same angular dependence. The $q^2/4M^2$ in the denominator is replaced by $\Delta E^2/q^2$, where ΔE is the energy transferred to the system $=q^2/2M$ for elastic scattering. It can be shown[11] that this separation corresponds to the separation into longitudinal and transverse quanta, with $\Delta m=0$ and ±1, respectively. We can see, for example, from the calculation of final-state interactions in electron-deuteron scattering[12] that different final states appear in the two interactions: ${}^3S$, ${}^1P$, etc., for the longitudinal and ${}^1S$, ${}^3P$, etc., for the transverse. The corrections are therefore independent for $G_{Ep}{}^2+G_{En}{}^2$ and for $G_{Mp}{}^2+G_{Mn}{}^2$. For nuclear physics, the separation into G_M and G_E corresponds to excitation of transitions with and without spin flip. As $q\to0$, G_M gives the photoproduced processes. In particular, $0\to0$ transitions are excited only by the G_E term.

Dispersion theory suggests that we write the F's in the form,[6]

$$F=1-\alpha+\alpha q_R{}^2/(q^2+q_R{}^2).$$

There is, however, an important difference depending whether we regard F_1 and F_2 or G_E and G_M as fundamental. It is easy to see that G_E cannot be in this form unless $\alpha=1$ for F_2; we choose G_E and G_M as fundamental because of the convenience in expressing selection rules, and allow different constants α in the two cases.[13]

Figures 1 and 2 show the data provisionally analyzed in this way. The values are taken by computation from the F_1 and F_2 plots of Cornell and Stanford data.[7,8] Errors are assigned by

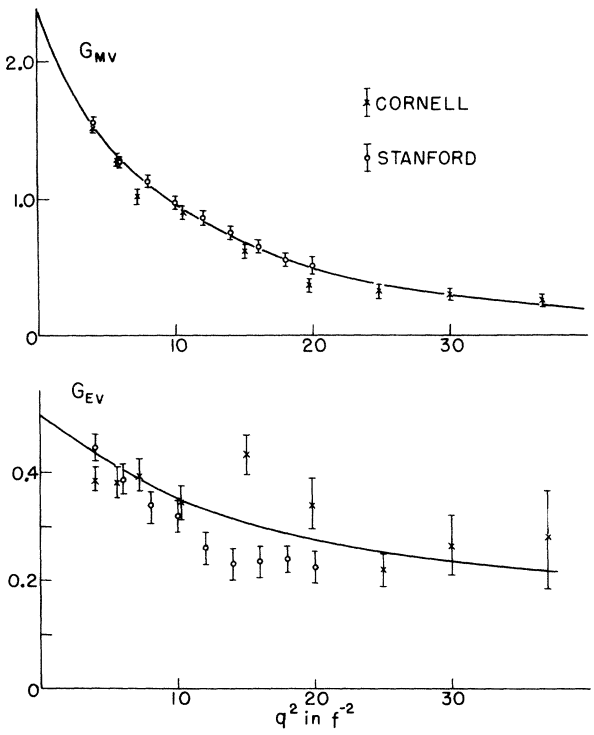

FIG. 1. Isotopic vector form factors.

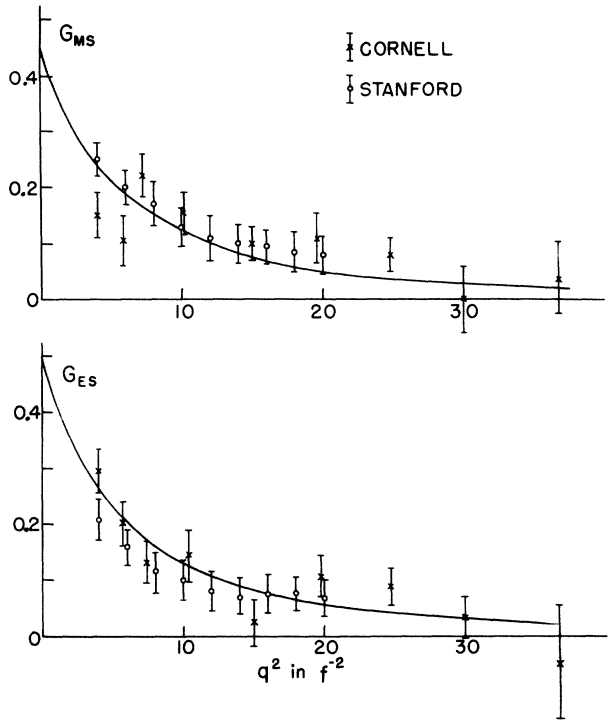

FIG. 2. Isotopic scalar form factors.

considering the errors on the respective cross sections. We see, for example, that the errors on G_M are determined primarily by the error on the backward angle cross sections; if $G_{Mn}{}^2 \approx \frac{1}{2} G_{Mp}{}^2$, as is the case over much of the range, the error on G_{MV} is determined almost entirely by the error in the inelastic e-d cross section.

We note at once the precision of the G_{MV} points. The apparent systematic difference between the Cornell and Stanford data for G_{MV} is probably a direct consequence of the systematically low values of the Cornell 135° cross sections. The curves show a fit to the form

$$G_{MV} = 5.2/(1+q^2/14.5) - 2.8/(1+q^2/26),$$

which fits the ρ-meson mass (750 Mev) and a "core" of the opposite sign which we have here taken with a mass close to the nucleon mass. There is flexibility in this latter choice including a constant core, leading to different values of the coupling constant.

The usual sign ambiguity for the neutron charge form factor is equivalent to interchange of the role of G_{EV} and G_{ES}. We wish to take the reverse of the usual choice here. There is slight experimental support for this choice from elastic electron-deuteron scattering[14] and from electron pion production.[11] The sign of G_{En} ($\simeq F_{1n}$) is uncertain at this time. With this choice of sign we see that G_{ES} and G_{MS} have similar shapes—fitted here by an η meson (550 Mev) and an ω^0 (785 Mev) with no core. The fits are

$$2G_{ES} = 2/(1+q^2/7.8) - 1/(1+q^2/15.8)$$

and

$$2G_{MS} = 1.76/(1+q^2/7.8) - 0.88(1+q^2/15.8).$$

This similarity of shape is startling and depends on taking negative G_{En}. We fit G_{EV} by a core alone.

The neutron-electron interaction experiments demand that $dG_{En}/dq^2 = d(G_{ES} - G_{EV})/dq^2$ be positive at low q^2 (equal to the Foldy term). This is harder to achieve with our choice of sign of G_{En}. Our fits to the data ignore the electron-neutron interaction and we suggest that 5-10% of a low-mass state ($\sim m_\pi$) in G_{ES} or G_{EV} can solve the discrepancy. $G_{ES} + G_{EV} = G_{Ep}$ is now well known, particularly at low q^2;[15] precise (2%) data on elastic electron-deuteron scattering should yield points which join smoothly onto the electron-neutron interaction prediction. No sharp bends, as are now suggested, are allowed unless a low-mass state contributes.

We have proposed an alternative set of form factors G_E and G_M to describe electron scattering data which have advantages in reducing error correlations. This set separates the scattering with spin flip from that without. The same separation occurs in other places in physics, and we believe that dispersion theory relates these directly.

Our fit to G_{MV} includes the ρ meson and a core and leads to a diffraction minimum in electron scattering between $q^2 = 40$ fermi^{-2} and 80 fermi^{-2}. It is at $q^2 = 40$ fermi^{-2} that suggestions have been made that the Rosenbluth formula might break down because the two-photon exchange becomes dominant. The recoil proton should then be polarized with the polarization changing sign at the minimum.

We wish to emphasize that we are here putting forward a point of view and only intend our analysis to be provisional.

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PRODUCTION OF A QUASI-MONOCHROMATIC γ-RAY BEAM FROM MULTI-Gev ELECTRON ACCELERATORS

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Experimental results concerning electron pair production and bremsstrahlung from high-energy photons and electrons in a silicon single crystal have been reported in two previous Letters.[1,2] These results appeared to be in qualitative agreement with Überall's calculation.[3]

After these experiments were reported, we carried out a set of measurements with better angular resolution. The most important result of these measurements was the discovery of a "fine structure" in the coherent bremsstrahlung from the crystal. We then proceeded to calculate the bremsstrahlung cross section taking into account the actual structure of the crystal lattice planes, which were assumed as continuous planes by Überall. Our calculation shows a "fine structure" in good agreement with our experimental results. This will be shown in detail in a future paper to be published.

We want to indicate in this Letter a consequence of this result which may be of importance for some new possibilities in experiments with multi-Gev electron accelerators.

In effect it is possible to obtain, by the use of a suitably oriented single crystal as a radiator, a bremsstrahlung γ-ray beam which reveals a "line" for a certain value k of the photon energy, the value k being in the low-energy region relative to the maximum energy. The most suitable crystal for this effect is diamond because of its small lattice spacing and high Debye temperature.

Figure 1 shows the quantity $I(k, \theta)$ which is proportional to the bremsstrahlung intensity and has been defined in a previous Letter.[2] We have calculated this quantity for an electron beam with $E_0 = 6$ Gev striking a single diamond crystal at room temperature. We have chosen the angle θ between the momentum $\vec{p}_0$ of the primary electron and the $\vec{a}_1$ crystal axis [110] equal to 0.29 mrad. We have also assumed the plane $(\vec{p}_0, \vec{a}_1)$ coincident with the plane $(\vec{a}_1, \vec{a}_2)$, where $\vec{a}_2$ is a crystal axis perpendicular to $\vec{a}_1$.

The spectrum of Fig. 1 effectively shows a set of discontinuities; one may see that the first discontinuity dominates the others.

The angle $\theta = 0.29$ mrad has been chosen in such a way that this maximum discontinuity falls at $k = 1$ Gev. At the value $I(k, \theta)$ which is half the maximum, the relative width of the "line" associated with this discontinuity is $\Delta k/k = 0.25$. This