Double-Layer Quantum Hall Antiferromagnetism at Filling Fraction $\nu = 2/m$ where m is an Odd Integer

S. Das Sarma,[1] Subir Sachdev,[2] and Lian Zheng[1]
[1]*Department of Physics, University of Maryland, College Park, Maryland 20742-4111*
[2]*Department of Physics, Yale University, P.O. Box 208120, New Haven, Connecticut 06520-8120*
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A low energy action for double-layer quantum Hall systems at filling fractions $\nu = 2/m$ (where m is an odd integer) is introduced. Interlayer antiferromagnetic exchange induces a phase with canted spin order, as well as a spin-singlet phase. Universal properties of zero and finite temperature transitions are obtained. We compute the critical temperature at which the canted order vanishes in a Kosterlitz-Thouless transition. Implications for recent light scattering experiments at $\nu = 2$ are noted. [S0031-9007(97)03742-3]

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There has been much recent work on double-layer quantum Hall systems, the majority of which has focused on the case where the electron tunneling rate between the two layers is small [1–3]. Then, the electron layer index plays the role of a pseudospin, and for the case where the total filling factor $\nu = 1/m$ (where m is an odd integer), very interesting new physics arises from long-range correlations in the pseudospin orientation. However, the tunneling term acts like a "magnetic field" in pseudospin space, and so spontaneous long-range pseudospin order, and the associated finite temperature (T) phase transition, is only possible when the tunneling is vanishingly small [4].

Stimulated by recent light scattering experiments [5] at $\nu = 2$, we present here a general low energy theory for double-layer systems at filling $\nu = 2/m$ in the presence of moderate interlayer tunneling. We find a rich phase diagram with interesting transitions both at $T = 0$ and $T > 0$. In contrast to the phases at $\nu = 1/m$, which are driven by ordering in pseudospin space, the phases at $\nu = 2/m$ are associated with ordering in the physical electronic spin space. Consequently, our order parameters are defined even in the presence of interlayer tunneling; indeed, moderate interlayer tunneling is required to stabilize some of our phases. We will use our results to interpret recent experiments [5], and argue that they show indirect evidence for our $T > 0$ phase transition.

It is useful to begin discussion of the physics at $\nu = 2/m$ by considering the case where the layer separation, d, is much larger than the magnetic length, l_o. Then the two layers (labeled 1,2) are approximately decoupled, and each separately has filling fraction $\nu_1 = \nu_2 = 1/m$. Their ground states will be the familiar Laughlin states for $m > 1$, or a fully filled lowest Landau level at $m = 1$, both of which have a large energy gap to all charged excitations [6]. These states are also fully spin polarized and there is significant intralayer ferromagnetic exchange [2,3,7]. The low-lying excitations in each layer are spin waves which have a small excitation gap given precisely by the Zeeman energy $g\mu_B H$ (the gyromagnetic ratio g and the Bohr magneton μ_B will henceforth be absorbed by a rescaling of the magnetic field H). For small g, a complete description [3,8] of the low energy excitations of each layer can be given in terms of an action for unit vector fields $\vec{n}_{1,2}$ ($\vec{n}_{1,2}^2 = 1$) representing the orientation of the ferromagnetic orders. Spin waves are small fluctuations of $\vec{n}_{1,2}$ about an ordered state, while charged quasiparticles are Skyrmion [37] textures of $\vec{n}_{1,2}$.

Now reduce the value of d and couple $\vec{n}_1$ and $\vec{n}_2$. The simplest allowed coupling between them is an *antiferromagnetic* exchange interaction. These considerations lead to the following imaginary-time (τ) effective action (in units with $\hbar = k_B = 1$)

$$S_0 = \int d^2x \int_0^{1/T} d\tau \, (\mathcal{L}_F[\vec{n}_1] + \mathcal{L}_F[\vec{n}_2] + J\vec{n}_1 \cdot \vec{n}_2), \tag{1}$$

$$\mathcal{L}_F[\vec{n}] \equiv iM_0\vec{A}(\vec{n}) \cdot \partial_\tau \vec{n} + \frac{\rho_s^0}{2}(\nabla_x \vec{n})^2 - M_0\vec{H} \cdot \vec{n}\,.$$

The intralayer ferromagnetic spin correlations [3,7,8] are controlled by $\mathcal{L}_F$: $M_0 = 1/4\pi m l_o^2$ is the magnetization density per layer, ρ_s^0 is the spin stiffness of each layer when they are well separated [for $m = 1$, we have [9] $\rho_s^0 = e^2/(16\sqrt{2\pi}\,\epsilon l_o)$], and $\vec{A}$ accounts for the Berry phase accumulated under time evolution of the spins [$\epsilon_{ijk}\partial A_k(n)/\partial n_j = n_i$]. The interlayer antiferromagnetic correlations are induced by the positive coupling $J \sim M_0\Delta_{\rm sas}^2/U$ where $\Delta_{\rm sas}$ is the tunneling matrix element between the layers, and $U \sim e^2/\epsilon l_o$ is the Coulomb interaction energy.

Two potentially important terms have been omitted from S_0 (and from S_1 below): the Hopf term which endows the Skyrmions with fractional statistics, and the long-range Coulomb interaction between the Skyrmions. There are simple physical arguments which justify this. As the layers are antiferromagnetically correlated, Skyrmions in one layer will be paired with anti-Skyrmions in the other, and their correlated motion will then produce opposite, canceling phases under the Hopf term. A similar

cancellation occurs in double-layer antiferromagnets [10], where the Berry phases of hedgehogs in one layer are neutralized by those of antihedgehogs in the other; in this case, numerical results for universal quantum-critical properties are in good agreement with predictions of theories assuming this cancellation [10]. The Coulomb potential of a Skyrmion is also neutralized by its anti-Skyrmion partner in the opposite layer. Further, the charge gap in well separated layers ensures that the Skyrmion/anti-Skyrmion pairs remain short-lived virtual fluctuations: we believe this charge gap persists through the quantum-critical points to be considered below. Note that no new term is necessary to induce charge transfer between the layers: a hedgehog/antihedgehog pair in the two layers corresponds to an event transferring Skyrmion number between them. Such spacetime singularities are absent in the strict continuum limit but appear when a short-distance regularization is introduced. Finally, for $m > 1$ and larger g, the spin 0 Laughlin quasiparticles become the lowest energy charged excitations, but these can be neglected for similar reasons.

Now we parametrize

$$\vec{n}_i = (-1)^i (1 - \vec{L}^2)^{1/2} \vec{n} + \vec{L}\,, \tag{2}$$

where the constraints $\vec{n}_{1,2}^2 = 1$ are now replaced by $\vec{n}^2 = 1$ and $\vec{L} \cdot \vec{n} = 0$. Because the layers are antiferromagnetically correlated we expect that $\vec{L}$ will not be too large. We insert (2) into (1), expand to quadratic order in $\vec{L}$, and then integrate out the $\vec{L}$ degrees of freedom. This yields the following effective action for the antiferromagnetic order parameter $\vec{n}$

$$S_1 = \frac{c}{2t} \int d^2x \times \int_0^{1/T} d\tau \left[(\nabla_x \vec{n})^2 + \frac{1}{c^2} \left(\frac{\partial \vec{n}}{\partial \tau} - i\vec{H} \times \vec{n} \right)^2 \right],$$

where $t = (J/2\rho_s^0 M_0^2)^{1/2}$ and $c = (2\rho_s^0 J/M_0^2)^{1/2}$. This is precisely the action of the $2 + 1$ dimensional quantum $O(3)$ nonlinear sigma model in a field H coupling to the conserved global $O(3)$ charge. It is expected to apply to double-layer quantum Hall systems with $\nu = 2/m$ at length scales larger than $\Lambda^{-1} \sim l_o$.

The $T = 0$ phase diagram [11,12] of S_1 is shown in Fig. 1. For $\nu = 2$ a topologically identical phase diagram was obtained by a Hartree-Fock (HF) analysis of a realistic, microscopic double-layer Hamiltonian [13], and is shown as an inset; this agreement provides further justification for the validity of $S_{0,1}$. The HF theory will be used to compute *renormalized* $T = 0$ energy scales which completely specify the correlators of S_1 at low T: in this manner we determine observables of the system with no free parameters (for $m > 1$ these energy scales remain as phenomenological parameters). We use the Hamiltonian $\mathcal{H} = \mathcal{H}_0 + \mathcal{H}_{\rm I}$ with

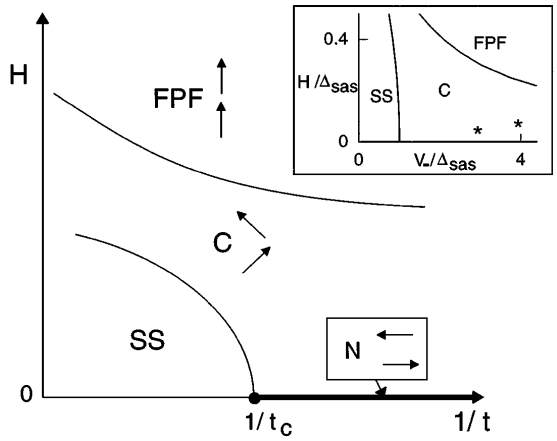

FIG. 1. $T = 0$ phase diagram of S_1. The phases are pictorially represented by the orientation of the spins in the two layers, with H pointing vertically upwards; C [N] has a broken $O(2)$ [$O(3)$] spin symmetry. There is a Kosterlitz-Thouless transition at $T = T_c > 0$ in C. The inset shows the phase diagram obtained from a microscopic HF calculation at $\nu = 2$ ($m = 1$), where the asterisks represent the two experimental samples of Ref. [5]. We argue in the text that the HF theory overestimates the stability of the C phase, and that experiments suggest that the actual SS region encloses the left sample in the inset.

$$\mathcal{H}_0 = -\frac{\Delta_{\rm sas}}{2} \sum_{\alpha\sigma} (C_{1\alpha\sigma}^\dagger C_{2\alpha\sigma} + \text{H.c.}) - \frac{H}{2} \sum_{i\alpha\sigma} \sigma C_{i\alpha\sigma}^\dagger C_{i\alpha\sigma}\,,$$

where $C_{i\alpha\sigma}$ annihilates an electron in the lowest Landau level in layer i ($i = 1, 2$) with spin σ ($\sigma = \pm 1$) in the z direction [we assume $\vec{H} = (0, 0, H)$] and with intra-Landau level index α. Interlayer tunneling induces the symmetric-antisymmetric energy separation $\Delta_{\rm sas}$. The Coulomb interaction part of $\mathcal{H}$ is

$$\mathcal{H}_{\rm I} = \frac{1}{2} \sum_{\sigma_1\sigma_2} \sum_{ij} \sum_{\alpha_1\alpha_2} \frac{1}{\Omega} \sum_{\mathbf{q}} V_{ij}(q) e^{-q^2 l_o^2/2} e^{iq_x(\alpha_1 - \alpha_2) l_o^2} \times C_{i\alpha_1 + q_y\sigma_1}^\dagger C_{j\alpha_2\sigma_2}^\dagger C_{j\alpha_2 + q_y\sigma_2} C_{i\alpha_1\sigma_1}\,, \tag{3}$$

where q is a wave vector, Ω is the area of the sample, and the interaction potentials are $V_{ij} = 2\pi e^2/\epsilon q$ for $i = j$ and $V_{ij} = (2\pi e^2/\epsilon q) e^{-qd}$ for $i \neq j$.

We now describe the phases in Fig. 1. The quantum phase transitions between these phases are continuous and are accompanied by the softening of the intersubband spin density excitations.

(I) Fully polarized ferromagnet (FPF).—In S_1 this is present for $H \gtrsim ct\Lambda^2$. This phase has $\langle n_{1z} \rangle = \langle n_{2z} \rangle = 1$. It is continuously connected to the large d limit discussed earlier.

(II) Canted (C).—We now have $\langle n_{1z} \rangle = \langle n_{2z} \rangle \neq 0$, and, for example, $\langle n_{1x} \rangle = -\langle n_{2x} \rangle \neq 0$. This phase has a broken spin rotational $O(2)$ symmetry in the x-y plane. For $m = 1$, the HF phase boundary between the FPF and C phases is $V_-/\Delta_{\rm sas} = (\Delta_{\rm sas}/H)[1 - (H/\Delta_{\rm sas})^2]$, and that between the C and SS phases is $V_-/\Delta_{\rm sas} = 1 - (H/\Delta_{\rm sas})^2$,

where $V_{\pm} = \frac{1}{\Omega}\sum_{\mathbf{q}} e^{-q^2 l_o^2/2}[V_{11}(q) \pm V_{12}(q)]$; a wave function for the C phase is obtained by the standard HF methods. For $m > 1$ a caricature of the wave function is two separate Laughlin states at $\nu_1 = \nu_2 = 1/m$ but polarized in the orientations shown.

(III) Neel (N).—This is the limiting case of C with $\langle n_{1z}\rangle = \langle n_{2z}\rangle = 0$ achieved at $H = 0$. Now an $O(3)$ spin rotation symmetry is broken.

(IV) Spin Singlet (SS). This corresponds to the quantum disordered phase of the $O(3)$ sigma model. The ground state is a spin singlet and is therefore unaffected by H: its wave function is the same as that at $H = 0$. For $m = 1$, in the independent electron HF picture, the electrons fill the layer-symmetric subband, with spin-up and spin-down levels equally populated. However, it is well known that HF theory overestimates the energy of a *nonmagnetic* phase like SS because correlations between opposite spin electrons, important for reducing the Coulomb energy, are now absent. It is likely, therefore, that the *real* SS phase is stable over a larger parameter region than that in our HF approximation, but the topology of the HF phase diagram in the inset of Fig. 1 (which is identical to that for S_1) should be correct. To build in charge correlations, one can use an approach similar to the Heitler-London picture of the hydrogen molecule, and consider pairs of electrons with their charge localized in opposite layers, while their spins form singlet bonds. Indeed, such a charge-localized picture was behind our introduction of the actions $S_{0,1}$. In such an approach, an alternative wave function for the SS phase (valid for $m = 1$ *and* $m > 1$) can be obtained in the $J \to \infty$ limit: pairs of electrons in opposite layers bind to form spin singlet, charge $2e$ bosons, which then condense into a boson Laughlin state at filling fraction $1/2m$, as demanded by the strength of the magnetic flux.

It is worth noting explicitly here that the HF computations at $m = 1$ allow us to assert that all the different phases of S_1 are the ground states in realistic parameter regimes. For $m > 1$, it remains an open question as to whether the phases of S_1 other than FPF are accessible, although we consider it a likely possibility that at least C will exist.

We now turn to the physics at $T > 0$. Only the N and C ground states have a broken spin rotation symmetry; the $O(3)$ symmetry of the former implies that the symmetry is restored at any $T > 0$, while the $O(2)$ symmetry of the latter implies a Kosterlitz-Thouless phase transition at a $T = T_c > 0$. We may characterize the order parameter fluctuations in both phases by a $T = 0$ spin stiffness $\rho_s(H)$ such that the energy cost of rotations of the order parameter by a slowly varying angle $\phi(r)$ is $E_\phi = [\rho_s(H)/2]\int d^2r|\nabla\phi(\mathbf{r})|^2$. A crude estimate [3,13] of T_c is $T_c \approx \rho_s(H)$, although this must fail as $H \to 0$. In the latter limit it is possible to obtain an exact leading asymptotic result [11] $T_c = 2\pi\rho_s(0)/\ln[\rho_s(0)/H]$ for $\ln[\rho_s(0)/H] \gg 1$. For $m = 1$ we computed $\rho_s(H)$ in the HF calculation and the results are shown in Fig. 2. We see that the T_c estimates are well in the experimentally accessible regimes for typical GaAs-based semiconductor samples. We emphasize that the Kosterlitz-Thouless transition at T_c is present even in the presence of interlayer tunneling, unlike the case for the pseudospin transition [2,3] at $\nu = 1/m$.

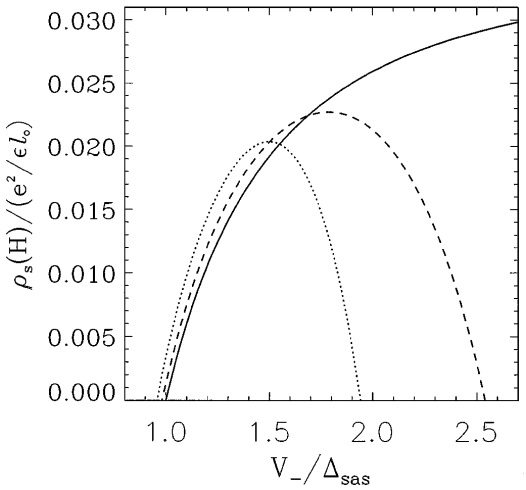

FIG. 2. Ground state spin stiffness $\rho_s(H)$ of a $\nu = 2$ double-layer system obtained from the microscopic HF calculations. It is nonzero only in the N and C phases. The solid line has $H = 0$, the dashed line $H = 0.05e^2/\epsilon l_o$, and the dotted line $H = 0.08e^2/\epsilon l_o$. The layer separation is $d = 1.0l_o$, and we have also included corrections from finite layer thickness $d_w = 0.8l_o$. In this figure, V_- is fixed to be $0.36e^2/\epsilon l_o$ for the given values of d and d_w, and $\rho_s(H)$ is shown as a function of Δ_{sas} for several values of H. (In typical GaAs-based samples, $e^2/\epsilon l_o$ is on the order of 50–100 K, which gives $\rho_s \sim 1$–2 K.)

A more precise approach to the $T > 0$ properties is to expand in the deviation from the $H = T = 0$ quantum-critical point between the N and SS phases at $t = t_c \sim \Lambda^{-1}$. This quantum-critical point is described by a renormalizable quantum field theory (with upper-critical spatial dimension $d = 3$), and so all thermodynamic properties are universal functions of energy scales characterizing "relevant" perturbations from this critical point; corrections due to irrelevant operators require additional energy scales and will be neglected here. Two of the relevant energy scales are the "bare" couplings T and H (there is no renormalization of the scale of H because it couples to a conserved charge [11]), and a third (the last) measures deviation of t from t_c. For $t > t_c$ we choose [14] this energy scale to be Δ, the energy gap of the SS state at $T = H = 0$, while for $t < t_c$ we choose the renormalized spin stiffness $\rho_s(0)$ of the N state also at $T = H = 0$. As t approaches t_c we have $\Delta \sim (t - t_c)^{\nu}$, while $\rho_s(0) \sim (t_c - t)^{(d-1)\nu}$, where ν is the correlation length exponent of the *classical* three-dimensional $O(3)$ ferromagnet. For $m = 1$, the microscopic HF calculation gives $\Delta = \Delta_{sas}\sqrt{1 - V_-/\Delta_{sas}}$, and $\rho_s(0) = \{[1 - (\Delta_{sas}/V_-)^2]/8\pi\Omega\}\sum_{\mathbf{q}}(ql_o)^2 e^{-q^2 l_o^2/2}V_{11}(q)$. Notice that these are consistent with the mean field exponent $\nu = 1/2$ in the upper-critical dimension $d = 3$.

One of our main results, which follows from the considerations above, is that the critical temperature T_c at which the ordering of the C phase disappears obeys, for $t > t_c$,

$$T_c = H\Psi_>(\Delta/H). \tag{4}$$

Here $\Psi_>(u)$ is a universal function of u with no arbitrary scale factors, and obeys the exact relation $\Psi_>(u \geq 1) = 0$ (because [11,13] the $T = 0$ boundary of the SS phase is given precisely by the condition $\Delta = H$). A similar scaling form holds for $t < t_c$ with $T_c = H\Psi_<(\rho_s(0)/H)$. We computed the functions $\Psi_{>,<}$ in an expansion in $\varepsilon = 3 - d$ using recently developed methods [15] and found to leading order

$$\Psi_>(u) = [33(1 - u^2)/(10\pi^2\varepsilon)]^{1/2}; \tag{5}$$

the structure of the subleading terms is quite complicated and is similar to that discussed elsewhere [15]. This result is valid for all u, except for u very close to 1; in that case we find, by a mapping to the dilute Bose gas problem, the exact asymptotic result [11,16] $\Psi_>(u \to 1) = y \ln(1/y)/[4 \ln \ln(1/y)]$ with $y = 1 - u$, which holds for $\ln(1/y) \gg 1$. For $t < t_c$ the ε expansion holds for $\Psi_<(u/\sqrt{\varepsilon})$ and we obtained

$$\Psi_<(u/\sqrt{\varepsilon}) = [(33 + 3u^2)/(10\pi^2\varepsilon)]^{1/2}. \tag{6}$$

Again this result is valid for all u, but now fails for $u \to \infty$ (which is $H \to 0$). While $T_c(H = 0) > 0$ for all $\varepsilon < 1$, we noted earlier that $T_c(H = 0) = 0$ for $\varepsilon = 1$; the latter property will not appear at any order in the ε expansion. Using results special to $d = 2$ for $H \to 0$ discussed earlier, we have instead the exact asymptotic form $\Psi_<(u \to \infty) = 2\pi u/\ln u$.

We draw attention to a particularly simple and striking limit of the above results. At $t = t_c$ we have $T_c = \mathcal{K}H$ where $\mathcal{K} = \Psi_>(0) = \Psi_<(0)$ is a universal number. Further, we do not expect any large or singular variation in T_c if t is close to but not exactly t_c. As both H and T_c are directly measurable energies, this relationship is amenable to a direct experimental test. On the theoretical side, while at present there is only the leading term in a ε expansion for the value of $\mathcal{K}$, it should be possible to obtain a reasonably precise result using quantum Monte Carlo simulations of double-layer lattice spin systems [10,17], which have been limited to $H = 0$ so far. Universality implies that these lattice models will have the same value of $\mathcal{K}$ as the quantum Hall system, and it appears to us that the simulation for $H \neq 0$ should also be free of the fermion sign problems.

We have also obtained results in the ε expansion for the crossovers of the dynamic spin susceptibility at frequency ω as universal functions of the energy ratios ω/T, H/T, and Δ/T $[\rho_s(0)/T]$: the methods are similar to those of Ref. [15], and results will be presented elsewhere.

We turn now to a comparison with recent light scattering experiments [5]. The high density sample (the right sample in the inset of Fig. 1) shows "mode-softening" consistent with a $T > 0$ phase transition which we identify with that above our C phase. Using input parameters from the HF calculation in (6), we obtain the prediction of $T_c \sim 0.5$ K, to be compared with the experimental value $T_c \sim 0.52$ K: the good agreement must be considered fortuitous until the accuracy of the ε expansion is better understood. The same sample also shows marked T dependence for $T > T_c$ in the light scattering spectrum at ω of order or greater than T: a natural explanation for this could be a crossover into the "high-T" region above the quantum-critical point [15]. The lower density sample (the left sample in the inset of Fig. 1) shows no mode softening and little T dependence in the light scattering spectrum: we suggest that this sample is in the SS phase. The HF computation puts this sample in the C phase, but as we discussed earlier, this could be in error because the HF theory overestimates the stability of the C phase.

Finally we note that we expect similar considerations to apply to all double-layer quantum Hall systems with $\nu = 2\nu_1$ where a single layer at filling ν_1 forms a fully polarized quantum Hall state with a charge gap.

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