# Motion of Neutron Vortices in the Inner Crust of a Neutron Star

P. B. Jones

*Department of Physics, Nuclear and Astrophysics Laboratory, University of Oxford, Keble Road, Oxford OX1 3RH, United Kingdom*

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The interaction of a macroscopic length of neutron vortex with a polycrystalline nuclear lattice can be simplified by a lattice potential density representation. Expressions are obtained for the weak-coupling time-averaged force on a moving vortex and, at very small velocities, for an approximate upper limit to the force for strong coupling between vortex and lattice. The force is too small to provide the minimum vortex pinning inferred from pulsar glitches. [S0031-9007(97)03750-2]

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The periodicity of the radio emission observed in pulsars is caused by the rotation of the charged components of magnetized neutron stars. The neutron superfluid rotates at an angular velocity $\Omega_n$ slightly greater than the observed angular velocity $\Omega$: its vortex array is sparse, the mean density being $2\Omega_n/\kappa \lesssim 10^5$ cm$^{-2}$, where $\kappa$ is the neutron superfluid quantum of circulation. It is believed that the angular velocity difference $\Omega_n - \Omega$ is normally maintained by vortex pinning to the Coulomb lattice of nuclei forming the solid crust of the star. The observed pulsar glitches are sudden increases in $\Omega$ which occur when large-scale vortex unpinning allows the transfer of angular momentum from the neutron superfluid to the charged components whose rotation is observed [1,2]. Postglitch relaxation of the spin-down rate $\dot{\Omega}$ is almost certainly determined by the dissipative motion of neutron vortices.

Neutron vortex pinning has certain differences from pinning in electronic type II superconductors. Cooperative effects are unimportant, except in large-scale unpinning, owing to the large intervortex spacing. The Coulomb lattice is homonuclear because proton shell closures determine the nuclear charge [3]. Because the internal cooling rate is slow, lattice defects are formed in local thermodynamic equilibrium and are those with low formation enthalpy. (Vortices in electronic type II superconductors pin to a dense microstructure of defects formed in the fabrication process. The density of pinning centers and the strength of the elementary pinning force are usually treated as phenomenological parameters.) The neutron superfluid coherence length $\xi$ is of the same order of magnitude as the bcc Coulomb lattice constant $a$ ($10^{-11}$–$10^{-12}$ cm). Therefore, the pinning mechanisms considered in this work are pinning by monovacancies and intrinsic pinning in a polycrystalline structure.

Energy transfer to Kelvin wave excitations is the most important dissipative process resulting from the vortex-nucleus interaction [4,5]. In this Letter, we observe that, for a vortex moving slowly with velocity $v_L$ through a polycrystalline structure, the instantaneous force effective in exciting long-wavelength Kelvin waves can be derived from a potential energy per unit length of vortex which has the translational properties of a lattice potential. We use the free-vortex Green function to obtain, for strong coupling and very small $v_L$, an approximate upper limit for the time-averaged force. Averaged over a macroscopic length, this force can be identified with the maximum pinning force acting on a deformable vortex through its interaction with a polycrystalline structure. An estimate, for reasonable superfluid parameters, shows that it is too small to provide the minimum vortex pinning required to explain the magnitude and frequency of pulsar glitches [6] and that other pinning mechanisms have to be considered.

The vortex-lattice interaction can be represented by a sum of potential functions of the instantaneous perpendicular distances $r_s$ between the vortex axis and the nuclei at lattice points $\mathbf{s}$. (Vortices can be assumed locally rectilinear over lengths of the order of the bcc lattice constant $a$.) The Cartesian coordinates adopted here are fixed in the rest frame of the undisturbed vortex whose axis coincides with the $z$ axis. Displacement of the vortex axis from its undisturbed position in the $x$-$y$ plane is denoted by the vector $\mathbf{u}(z,t)$. We consider constant motion of the lattice relative to the Cartesian coordinates oriented such that, for a lattice point $\mathbf{s}$ referred to the coordinates, $s_{1,3}$ are constant and $s_2 = -v_L(t - t_s)$. The vortex intersects a polycrystalline structure of single crystals $(i)$ with lengths $b_i$ and orientations, referred to the axes fixed in the vortex frame, specified by sets of reciprocal lattice vectors $\mathbf{g}_i$ (the subscript $i$ is suppressed in all formulas). The potential sum over any length of vortex smaller than $b_i$ but large compared with $a$ is replaced by an integral over a potential energy per unit length of vortex,

$$\sum_s V(r_s) \rightarrow \sum_{\mathbf{g}} \int U_{\mathbf{g}} e^{i\mathbf{g}\cdot\tilde{\mathbf{r}}}\, dz\,, \tag{1}$$

in which the potential energy density has the translational properties of a lattice potential and

$$\mathbf{g}\cdot\tilde{\mathbf{r}} = \mathbf{g}\cdot\mathbf{x}_0 + \mathbf{g}\cdot\mathbf{u} + g_2 v_L t + g_3 z\,, \tag{2}$$

where $\mathbf{x}_0$ is an arbitrary constant displacement. The dispersion relation for unperturbed Kelvin phonons [7] of

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angular frequency $\omega$ and wave number $p$ is $\omega = c_K p^2$, in which the parameter $c_K = -(\kappa/4\pi)\ln p\xi$ is only logarithmically dependent on $p$ and on the coherence length $\xi$ for the neutron superfluid. The basic angular frequency $\omega = g_2 v_L$ present in Eq. (2) corresponds, for $v_L \lesssim 10^2$ cm s$^{-1}$, with very small wave numbers $p \ll g$, and the dominant term in the potential density within a given single crystal is expected to be that for the reciprocal lattice vector $\mathbf{g}$ giving the smallest $|p - g_3|$. Although the replacement (1) is not generally correct unless specific model assumptions are made, we observe that it is always valid for a single dominant term with very small $g_3$ provided, as is confirmed *a posteriori*, that terms with $g_3$ of the order of $g$ can be neglected. The parameters $U_{\mathbf{g}}$ are obtained from $V(r_s)$ and are rapidly decreasing functions of the ratio of $\xi$ to the bcc lattice constant $a$: They also become small for reciprocal lattice vectors beyond the basic set.

The free-vortex Green function giving the displacement $\mathbf{u}(z,t)$ produced by unit impulse in each of the $\alpha = 1, 2$ directions is

$$G_{\alpha\beta}(z - z', t - t') = \frac{-1}{2\rho\kappa[\pi c_K(t - t')]^{1/2}} \times (\sin\chi + i\sigma_2\cos\chi), \quad (3)$$

where

$$\chi = \frac{(z - z')^2}{4c_K(t - t')} - \frac{\pi}{4}, \quad (4)$$

$\rho$ is the superfluid density, and $\sigma_2$ is the Pauli matrix. (The equations of motion are given in Ref. [7]: see also [8].) With the interaction in a given single crytsal limited to the dominant potential density term, the function of the displacement $\phi(z,t) = \mathbf{g}\cdot\mathbf{u}$ satisfies

$$\phi(z,t) = \sum_\alpha 2g_\alpha^2 U_{\mathbf{g}} \int_{-b/2}^{b/2} dz' \int_{-\infty}^{t} dt' \times G_{\alpha\alpha}(z - z', t - t') \times \sin[\mathbf{g}\cdot\mathbf{x}_0 + \omega t' + g_3 z' + \phi(z', t')]. \quad (5)$$

The force per unit length acting on the vortex can be expressed in terms of $\phi$. It is

$$f_\alpha(z) = 2g_\alpha U_{\mathbf{g}} \langle \sin(\mathbf{g}\cdot\mathbf{x}_0 + \omega t + g_3 z + \phi)\rangle, \quad (6)$$

where the angle brackets denote a time average.

The function $\phi$ can be expressed as a power series in the parameter $A = bU_{\mathbf{g}}(g_1^2 + g_2^2)/(\rho\kappa c_K p)$ and obtained to lowest order, for $|\phi| \ll 1$, by introducing the Fourier transform of $G_{\alpha\beta}$,

$$G_{\alpha\beta}(p', \omega') = -\frac{1}{2\rho\kappa}\left(1 - \frac{\omega'\sigma_2}{c_K p'^2}\right) \times \left(\frac{1}{\omega' - c_K p'^2 + i\eta} - \frac{1}{\omega' + c_K p'^2 + i\eta}\right), \quad (7)$$

in which $\eta$ is positive and infinitesimal. The displacement function, with the constant phase $\mathbf{g}\cdot\mathbf{x}_0$ suppressed, is

$$\phi(z,t) = \frac{A}{2b}\left\{\left(\frac{1}{g_3 - p} - \frac{1}{g_3 + p} + \frac{2p}{g_3^2 + p^2}\right)\sin(g_3 z + \omega t) - \frac{1}{g_3 - p}\sin\left((g_3 - p)\frac{b}{2} + pz + \omega t\right) - \frac{1}{g_3 + p}\sin\left((g_3 + p)\frac{b}{2} + pz - \omega t\right) - \frac{e^{p(z-b/2)}}{g_3^2 + p^2}\left[p\sin\left(g_3\frac{b}{2} + \omega t\right) + g_3\cos\left(g_3\frac{b}{2} + \omega t\right)\right] + \frac{e^{-p(z+b/2)}}{g_3^2 + p^2}\left[p\sin\left(g_3\frac{b}{2} - \omega t\right) + g_3\cos\left(g_3\frac{b}{2} - \omega t\right)\right]\right\}, \quad (8)$$

From Eqs. (6) and (8), the force per unit length at $z$ for which the exponential terms are negligible is linearly dependent on $\sin(g_3 \pm p)z$ and on $\cos(g_3 \pm p)z$. The total force acting on the sector is

$$F = -\frac{g_\alpha U_{\mathbf{g}} A}{2b}\left\{\frac{[1 - \cos(g_3 - p)b]}{(g_3 - p)^2} + \frac{[1 - \cos(g_3 + p)b]}{(g_3 + p)^2}\right\}, \quad (9)$$

confirming *a posteriori* the assumption that only a single reciprocal lattice vector need be considered. The weak-coupling Eqs. (8) and (9) are of interest as a guide to the strong-coupling case. The order of magnitude of the total force is independent of $b_i$ and, for displacement such that $|\phi| \approx 1$, is of the same order of magnitude $(g_\alpha U_{\mathbf{g}} g_3^{-1})$ as the instantaneous external force acting on the vortex. The $b_i$ independence of the force exists to any order in $A$. For $z$ such that the exponential terms in Eq. (8) are negligible, examination of Eqs. (5) and (7) shows that in order $|A|^k$, $\phi$ is linearly dependent on $\sin(\mathbf{g}\cdot\mathbf{x}_0 + \omega t + g_3 z + \phi)$ in order $k - 1$ and on $\sin(p_n z \pm n\omega t)$ and $\cos(p_n z \pm n\omega t)$ where the integers $n$ are $k, k - 2, \ldots$. The wave numbers $p_n$ are defined by $n\omega = c_K p_n^2$ and are not multiples of $g_3$. Systematic application of this result shows that $\cos\phi$ and $\sin\phi$ in order $k$ have, respectively, no terms linearly dependent on $\sin(\mathbf{g}\cdot\mathbf{x}_0 + \omega t + g_3 z)$ and $\cos(\mathbf{g}\cdot\mathbf{x}_0 + \omega t + g_3 z)$, so establishing that the time-averaged force per unit length given by Eq. (6) in order $k + 1$ is linearly dependent on sinusoidal functions of $z$, with wave numbers

given by linear combinations of the $p_n$ and $g_3$, as is the lowest-order force.

In the limit of very strong coupling ($|U_{\mathbf{g}}| \to \infty$) the form of Eq. (5) is such that the periodic component of $\phi$ for very small $v_L$ must be approximately equal to the sawtooth function $\phi^{\infty} = -\mathbf{g} \cdot \mathbf{x}_0 - \omega t - g_3 z$ with $-\pi < \phi^{\infty} < \pi$. The displacement components $u_{1,2}$ form a set of evenly spaced kinks (Fig. 1; see also Ref. [9]) each corresponding with movement of the vortex axis from one lattice plane to the adjacent. A kink moves with velocity $g_2 v_L / g_3$ parallel with the $z$ axis and has length of the order of $\ell = (2\pi\rho\kappa c_K / g^2 |U_{\mathbf{g}}|)^{1/2}$. The time averaged force acting on the sector at extremely small $v_L$ can be at most of the order of $\ell g |U_{\mathbf{g}}|$.

Application of the classical theory of homogeneous nucleation [10] indicates that the polycrystalline lattice should form, from the undercooled metastable state, with large $b_i$ owing to the high growth velocity of the crystalline phase and the low cooling rate of the star. (Structures with small $b_i$ were assumed in Ref. [5].) The single crystal reciprocal lattice vectors have a distribution of orientations with respect to the vortex so that in applying Eq. (6) to find the force on the $i$th sector, terms in $\phi$ arising from sectors $j \neq i$ make no contribution. The order of magnitude of the time-averaged force acting on a sector is $g_\alpha |U_{\mathbf{g}}| \min(g_3^{-1}, \ell)$. Summation over sectors with randomly oriented $\mathbf{g}_i$ and average linear dimension $\overline{b}_i$ gives a force per unit length of $f_R = 6(1 + \ln g\ell)|U_{\mathbf{g}}| / \overline{b}_i$ acting on a long length of vortex. Interaction of a vortex with the disordered planes forming single crystal boundaries gives a contribution about an order of magnitude smaller. Coupling with other degrees of freedom has been neglected, as has dissipation associated with transitions in a thermal population of Kelvin phonons [5].

The calculation of $U_{\mathbf{g}}$ assumes the bcc lattice constant of Negele and Vautherin [3] and a coherence length based on the neutron energy gap of Ainsworth, Wambach, and Pines [11]. The vortex-nucleus potential $V(r_s)$ has a component produced by the change in condensation energy at separations $r_s \lesssim \xi$ and a long-range component given by the change in kinetic energy of the circulating superfluid at $r_s \gtrsim \xi$. Our calculations of $U_{\mathbf{g}}$ show that the long-range term [12], compared with the condensation energy term of Alpar *et al.* [13], is the more important. The computed $U_{\mathbf{g}}$ for the basic reciprocal lattice vectors has a maximum $U_{\mathbf{g}} = 1.1 \times 10^5$ erg cm$^{-1}$ at a matter density of $3 \times 10^{13}$ g cm$^{-3}$ and decreases rapidly at higher densities ($U_{\mathbf{g}} = 6 \times 10^3$ at $8 \times 10^{13}$ g cm$^{-3}$) owing to the smoothing which occurs where the coherence length approaches the lattice constant.

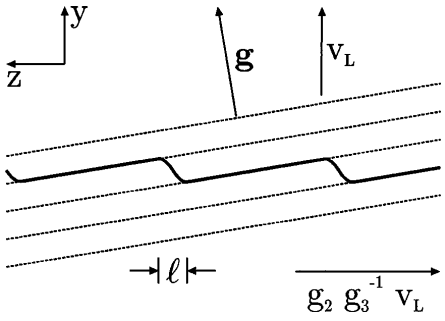


FIG. 1. The undisturbed vortex axis moves upward with a small velocity $v_L$; the kinks formed as the vortex passes between lattice planes have length $\ell$ and travel to the right with velocity $g_2 g_3^{-1} v_L$.

Models in which neutron vortices are pinned by interaction with the Coulomb lattice of nuclei in the crust are not consistent with the expression obtained for $f_R$. Analysis of the magnitude and frequency of Vela pulsar glitches [6] shows that the depinning threshold at matter density $8 \times 10^{13}$ g cm$^{-3}$ must exceed $10^{15}$ dyne cm$^{-1}$. The condition $f_R \gtrsim 10^{15}$ dyne cm$^{-1}$ would require $\overline{b}_i \lesssim 2.5 \times 10^{-10}$ cm, completely at variance with the kind of structure indicated by homogeneous nucleation theory [10]. However, it is possible that the $\mathbf{g}_i$ are not randomly oriented. Existing neutron energy gap calculations [14] are not sufficiently reliable to exclude the possibility that vortex formation precedes solidification. But even in the limit in which the whole crust forms as a specially oriented single crystal, it would be surprising if there were no disruption caused by evolution of the spin direction or of the internal magnetic flux distribution [15]. Pinning by a random distribution of monovacancies can be excluded by application of an approximate scaling theory [16], with the superfluid parameters of Ref. [13], which shows that unphysically large monovacancy fractional concentrations $c_v \gtrsim 0.02$ would be required to produce the necessary depinning threshold.

Separate phases of rod and slab nuclei have been predicted [17,18] at matter densities intermediate between those for the bcc lattice of spherical nuclei and for the liquid core. The time-averaged force calculations given here are unchanged in essence: The reciprocal lattice vectors are confined to a plane or to a fixed direction. Our conclusion that vortex pinning in the crust is improbable leaves the strong interaction between neutron and proton vortices in the liquid core [19–21] as a possible pinning mechanism. If this is the case, our entire picture of pulsar glitches, which appear to be a common phenomenon, may have to be revised.

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