Are Self-Similar States in Fibonacci Systems Transparent?

In a recent Letter [1] Maciá and Domínguez-Adame intended to address the physical nature of critical wave functions in a generalized Fibonacci system. Apart from several interesting results presented, one main conclusion reached is that "*self-similar wave functions are those exhibiting higher transmission coefficients in a finite Fibonacci system*." Although there exist extended or transparent states in many aperiodic systems under some special conditions, the conclusion itself is unfortunately incorrect and misleading, because it is based on a serious miscalculation of the transmission coefficient. In this Comment, we wish to clarify this point and present a correct calculation.

For the Hamiltonian considered in Ref. [1], four original transfer matrices (X, Y, Z, W) can be cast into two new matrices $R_A(\equiv ZYX)$ and $R_B(\equiv WX)$. R_A and R_B are arranged in a Fibonacci sequence. In the context of the formulation of Ref. [1], there always exists one energy E satisfying a relation

$$E = \alpha \frac{1 + \gamma^2}{1 - \gamma^2}, \tag{1}$$

where α represents the on-site energy, γ the transfer integral. For these energies, $[R_A, R_B] = 0$. Based on Eq. (1), the global transfer matrix $M(N)$ in Ref. [1] is obtained. Using the $M(N)$, the authors further derive the transmission coefficient as

$$\tau(N) = \frac{1}{1 + [(1 - \gamma^2)^2/(4 - E^2)\gamma^2] \sin^2(N\phi)}, \tag{2}$$

where ϕ is a function of E, α, and γ. Then they suggest that the transparent condition be fulfilled when $\sin(N\phi) = 0$. However, we must note that the condition $\sin(N\phi) = 0$ may not be consistent with Eq. (1), particularly in self-similar states. Therefore, under the condition $\sin(N\phi) = 0$ in the self-similar states, Eq. (2) cannot be used and thus $\tau(E) = 1$ should not be expected in the self-similar states. In the Letter, they take $N = F_{17}$, $\gamma = 2$, $\alpha = 0.1$, and $E = -\sqrt{\alpha^2 + 4\cos(1160\pi/N)} = 0.3348\ldots$ [correspond to $\sin(N\phi) = 0$] to plot Fig. 2 in Ref. [1], which statistically exhibits self-similar features and is claimed to be at a transparent state $\tau(E) = 1$. Unfortunately, the above three energy parameters (E, α, and γ) do not satisfy Eq. (1), so their claim that this self-similar state corresponds to a transparent state does not make sense.

Actually, if we consider E, α, and γ as three independent parameters and denote the global transfer matrix as $T(N)$ with matrix elements $t_{i,j}$s ($(i, j = 1, 2)$, the transmission coefficient can be obtained as

$$\tau(N) = \frac{4 - E^2}{[t_{21} - t_{12} + (t_{22} - t_{11})E/2]^2 + (t_{22} + t_{11})^2(1 - E^2/4)}, \tag{3}$$

where $t_{i,j}$ is a function of the three energy parameters. Note that Eq. (3) is more general and $T_N \neq M_N$ in a general case. Once $T(N) = M(N)$ (i.e., Eq. (1) holds), it is straightforward to show that Eq. (3) is equivalent to Eq. (2). From Eq. (3), we can check that when Eq. (1) is satisfied, (i) if $\gamma = 2$, $\alpha = 0.75$, $E = -1.25$, then $\tau(F_{16}) = 0.5909\ldots$; (ii) if $\gamma = 2$, $\alpha = 0.5$, $E = -5/6$, then $\tau = 0.7425\ldots$, as obtained in Ref. [1]; the corresponding states are not self-similar. However, if we substitute $\gamma = 2$, $\alpha = 0.1$, $E = -\sqrt{0.1^2 + 4\cos^2(1160\pi/F_{17})}$ (corresponding to a self-similar state) into Eq. (3), $\tau(N) = 0.2298\ldots$, instead of the result $\tau = 1$ in Ref. [1].

On the other hand, when Eq. (1) is fulfilled, we can further show that $R_A = R_C^3$, $R_B = R_C^2$, and $M(N) = R_C^N$, where

$$R_C = \begin{pmatrix} \gamma^{-1}(E - \alpha) & -\gamma \\ \gamma^{-1} & 0 \end{pmatrix}. \tag{4}$$

Then in terms of Chebyshev polynomials of the second order, $M(N)$ can be obtained more easily.

Finally, we wish to pinpoint that if R_A and R_B commutate, which corresponds to a zero-invariant case, one may find a higher transmission coefficient more easily. For example, (i) $\gamma = 2$, $\alpha = 0.1$, $E = -1/6$, $\tau(F_{17}) = 0.9232\ldots$; (ii) $\gamma = 1.5$, $\alpha = 0.1$, $E = -13/50$, $\tau(F_{17}) = 0.9938\ldots$; (iii) $\gamma = 2$, $\alpha = 1.5$, and $E = -2.5$, $\tau = 1$. Notice that these states are not self-similar. It is well known that, in the on-site or in the transfer model, the self-similar states correspond to nonzero invariants [2]. In the mixing model, can the states with zero invariant be self-similar? From both analytical and numerical calculations, we find actually that if R_A and R_B commutate, there exist only extended states rather than *self-similar states*, in sharp contrary to the conclusion reached in Ref. [1].

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