Quantum Networking with Optical Fibres

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A scheme is proposed which allows for reliable transfer of quantum information between two atoms via an optical fibre in the presence of decoherence. The scheme is based on performing an adiabatic passage through two cavities which remain in their respective vacuum states during the whole operation. The scheme may be useful for networking several ion-trap quantum computers, thereby increasing the number of quantum bits involved in a computation. [S0031-9007(97)04819-9]

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The possibility of reliable spatial transport of quantum states is of crucial importance to quantum communication. During the past few years, rapid progress has been made in using optical fibres for quantum communication on the single photon level [1]. This is needed for quantum cryptography [1,2] and quantum teleportation [3]. However, the spatial exchange of quantum information between quantum registers that have undergone local quantum processing has not yet been demonstrated experimentally. Quantum communication between locally distinct nodes of a quantum network will be essential to overcome small-scale quantum computing [4].

A promising model for storage and local processing of quantum information is the ion-trap quantum computer, in which the quantum bits are stored in stable ground states or in long-lived metastable states [5]. First experimental results have already been reported [6]. However, technology imposes an upper limit on the number of ions, and thus quantum bits, which can be used in a single ion-trap quantum computer. To overcome this limit, a network of several ion-trap quantum computers could be set up. But while ions are superb for storing quantum information, it seems more feasible to mediate quantum states by photons carried by optical fibres.

This problem has been recently addressed (to the author's knowledge, for the first time) by Cirac *et al.* [7]. They propose a scheme to transmit quantum bits by tailoring *time-symmetric photon wave packets*. In the present Letter, a different approach is pursued which is based on an *adiabatic passage via photonic dark states*. A representation of the scheme is depicted in Fig. 1.

The quantum bit to be transferred is initially stored in atom A, and atom B is prepared in a predefined state. Atoms A and B may be part of ion-trap quantum computers. Initially, both cavities and the fibre are in the vacuum state. By appropriate design of laser pulses as described below, the states of atom A and B may be swapped. It is demonstrated below that (ideally) the two cavities will never have a nonzero photon number.

The scheme has two distinctive features. Firstly, it is insensitive to losses from the cavities into other than the fibre-modes. This is due to the fact that the cavities are never populated. As a result, the coupling between the cavities and the fibre-mode does not need to be perfect. Secondly, it does not require precise control of the pulse shape; duration and intensity of the manipulating laser pulses is not required as long as some "global" (namely, adiabaticity) conditions are met. This implies that the Rabi frequencies need not be known in order to perform the transfer successfully. The individual atom-cavity systems are described as follows. The atoms are modeled by three-level Λ systems with two ground states, $|a_0\rangle$, $|a_1\rangle$, and one excited state, $|b\rangle$, as depicted in Fig. 2(a).

The frequency of the transition $|a_i\rangle - |b\rangle$ is denoted by ω_i, where $i = 0, 1$. The excited state $|b\rangle$ spontaneously decays with a rate γ. The cavity is modeled by a single, quantized mode with frequency ω_{cav} coupled to the transition $|a_1\rangle - |b\rangle$ with coupling strength g. The cavity is coupled to an optical fibre as described below. In addition, a loss rate κ of the cavity is included. This decay rate includes any undesired loss mechanisms such as absorption of cavity photons in the mirrors and coupling to other than the fibre-modes. The transition $|a_0\rangle - |b\rangle$ is coupled to a laser described by a c-number coherent field with frequency ω_{las}. The corresponding time-dependent Rabi frequency is denoted by $\Omega(t)$. According to continuous measurement theory, the evolution of a system in the absence of quantum jumps is determined by an effective, non-Hermitian Hamiltonian [8]. In the present context the following Hamiltonian is found ($\hbar = 1$ here and in the remainder of the paper):

$$
\begin{aligned}
H_{\text{eff}} &= H_0 + H_{\text{int}}\,, \\
H_0 &= (-\Delta_g - i\gamma)\,|b\rangle\langle b| + \Delta_r|a_1\rangle\langle a_1| + \kappa c^\dagger c\,, \qquad (1)\\
H_{\text{int}} &= \Omega(t)\,|b\rangle\langle a_0| + ga|b\rangle\langle a_1| + \text{H.c.}
\end{aligned}
$$

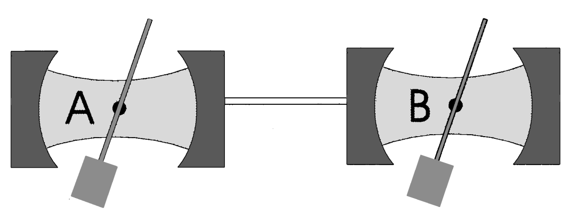

FIG. 1. Representation of the scheme. The atoms (quantum bits) are coupled to cavities and are manipulated by lasers. The cavities are connected via an optical fibre.

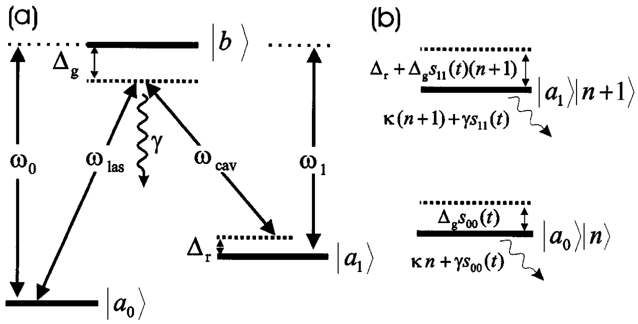

FIG. 2. (a) Atomic level scheme. See text for definition of the symbols. (b) Level scheme of the combined atom-cavity system after adiabatic elimination. The first parameter in the ket represents an atomic ground state, whereas the second parameter denotes a cavity-mode Fock state.

Equation (1) is written in a rotating frame. The symbols c and $c^\dagger$ denote the annihilation and creation operators of the cavity-mode. Here two detunings have been introduced, namely the global detuning $\Delta_g = \omega_{\text{las}} - \omega_0$ and the "Raman" detuning $\Delta_r = (\omega_{\text{cav}} - \omega_{\text{las}}) - (\omega_0 - \omega_1)$.

The scheme works in the low saturation regime, where the excited atomic states can be adiabatically eliminated. The conditions that are required to make this approximation meaningful are set out below. By applying standard quantum optical techniques [9], the adiabatically eliminated effective Hamiltonian reads

$$\begin{aligned}
\tilde{H}_{\text{eff}} &= \tilde{H}_0 + \tilde{H}_1 + \tilde{H}_{\text{int}} ,\\
\tilde{H}_0 &= [\Delta_g s_{00}(t) - i\gamma s_{00}(t)]\,|a_0\rangle\langle a_0| ,\\
\tilde{H}_1 &= [\Delta_r + \Delta_g s_{11}(t) c^\dagger c - i\kappa c^\dagger c - i\gamma s_{11}(t)]\\
&\quad \times |a_1\rangle\langle a_1| , \qquad\qquad (2)\\
\tilde{H}_{\text{int}} &= (\Delta_g - i\gamma) s_{01}(t) c |a_0\rangle\langle a_1|\\
&\quad + (\Delta_g - i\gamma) s_{10}(t) c^\dagger |a_1\rangle\langle a_o| .
\end{aligned}$$

Here the following saturation parameters have been introduced:

$$s_{00}(t) = \frac{|\Omega(t)|^2}{\Delta_g^2 + \gamma^2}, \qquad s_{11}(t) = \frac{|g|^2}{\Delta_g^2 + \gamma^2},$$
$$s_{01}(t) = s_{10}^*(t) = \frac{\Omega(t) g^*}{\Delta_g^2 + \gamma^2} . \qquad (3)$$

Adiabatic elimination is applicable if the following inequalities hold:

$$s_{ii}(t) \ll 1, \qquad i = 1, 2 .$$

The detunings, light shifts, and decay rates of the two ground states are shown in Fig. 2(b). In Eq. (2), $\tilde{H}_0$ and $\tilde{H}_1$ contain all the terms which are diagonal in the atomic basis. Both ground states undergo light shifts and damping. Later it will be shown that the light shift of state $|a_0\rangle$ gives rise to undesired effects which will make compensation necessary. $\tilde{H}_{\text{int}}$ describes Rabi oscillations between the ground states and contains dissipative terms as well. The decay terms are of course unwanted. However, it will be shown below how they can be avoided.

The next step is to consider two atom-cavity systems, as described above, connected by an optical fibre. The free Hamiltonian of the fibre and the interaction Hamiltonian with the two atom-cavity systems are assumed to take the following form:

$$\begin{aligned}
H_{\text{fib}} &= \sum_k \Delta_k f_k^\dagger f_k\\
&\quad + \left\{ \nu \sum_k [c_A + (-1)^k c_B] f_k^\dagger + \text{H.c.} \right\} . \qquad (4)
\end{aligned}$$

Equation (4) is written in a frame rotating at the cavity frequency. By f_k ($f_k^\dagger$) the annihilation (creation) operator of the kth fibre-mode is denoted. Δ_k is the frequency difference between the kth fibre-mode and the cavity-mode, and ν is the coupling strength between the fibre-modes and the cavity. The factors $(-1)^k$ model the phase difference of π between the electric fields on both ends of the fibre for every second fibre-mode. For simplicity, it is assumed that in the frequency range where the coupling is significant the coupling strength is constant. Here, and in the remainder of this Letter, the subscripts (or superscripts) A and B distinguish the two atom-cavity subsystems. Note that the fibre is assumed to be lossless, which is of course unrealistic for long transmission distances. However, the present scheme is designed for short distances for which this assumption seems appropriate. Note also that the scheme is sensitive to losses due to nonideal input coupling of the fibre into the cavities.

The goal is to achieve quantum state swapping. In this model, energy eigenstates of the system and logical values are identified as follows:

$$|i\rangle\,|j\rangle \equiv |a_i, 0\rangle_A |a_j, 0\rangle_B |\text{vac}\rangle , \qquad i = 0, 1 .$$

Here the first and second ket on the right-hand side refer to the atom-cavity subsystem A and B. The first parameter in each ket denotes the atomic ground state, while the second represents a Fock state of the respective cavity-modes. The third ket denotes the subsystem of the fibre modes, all being in the vacuum state. It is shown below how the following operation can be performed:

$$(\alpha|0\rangle + \beta|1\rangle)\,|1\rangle \rightarrow |1\rangle\,(\alpha|0\rangle + \beta|1\rangle) . \qquad (5)$$

Here α and β are arbitrary (in general, unknown) complex amplitudes. The result of the process by starting with the second quantum bit in state $|0\rangle$ will be undefined.

The primary objective is to perform the desired quantum state transfer with as little disturbing influence from dissipative processes as possible. In the present context these are spontaneous emissions from the atomic excited states (via optical pumping) and the decay of the cavity mode into other than the fibre-modes. These two decay mechanisms are dealt with as follows: (i) The optical pumping rate is the product of the saturation parameter $s_{ii}(t)$ and the spontaneous decay rate γ, whereas the effective Rabi frequency is the product of $s_{01}(t)$ and the detuning Δ_g. Therefore, by increasing the detuning and simultaneously increasing the Rabi frequency, the optical pumping rate can be made arbitrarily small while maintaining the effective Rabi frequency constant. (ii) Undesired loss of cavity photons is avoided by performing the process as an adiabatic passage through a dark state of both cavities [10,11]. In other words, the photon number of two cavity-modes will not differ significantly from zero throughout the whole process. This scheme will be explained in more detail in the following paragraphs.

Since spontaneous emission can be dealt with in the aforementioned way, γ is set to zero in the following. The present scheme is based on the fact that, for the Hamiltonian $H_{\text{part}} = \tilde{H}_1 + \tilde{H}_{\text{int}} + H_{\text{fib}}$ specified in Eqs. (2) and (4), dark states with respect to the two cavity-modes exist. This requires that a fibre-mode exists that is resonant with the cavity-mode, i.e., that a detuning $\Delta_k = 0$ for a certain k exists. Note that H_{part} is not the full Hamiltonian of the system since $\tilde{H}_0$ is missing. It turns out that $\tilde{H}_0$ deteriorates the dark state and thus the performance of the scheme. It will be shown below how this effect can be avoided. The two relevant dark states of the partial Hamiltonian H_{part} thus read:

$$\begin{aligned}|\Psi_0^D\rangle &\propto \nu\Delta s_{10}^B(t)\,|a_0,0\rangle_A|a_1,0\rangle_B|\text{vac}\rangle \\ &\quad - \Delta^2 s_{10}^A(t)s_{10}^B(t)\,|a_0,0\rangle_A|a_1,0\rangle_B|1_{k_0}\rangle \\ &\quad + \nu\Delta s_{10}^A(t)\,|a_0,0\rangle_A|a_1,0\rangle_B|\text{vac}\rangle\,, \\ |\Psi_1^D\rangle &= |a_0,0\rangle_A|a_1,0\rangle_B|\text{vac}\rangle\,.\end{aligned} \tag{6}$$

$|1_{k_0}\rangle$ denote the fibre state corresponding to fibre-mode k_0 being in a one photon Fock state and all the others in the vacuum state. The index k_0 corresponds to the fibre-mode for which $\Delta_{k_0} = 0$. These states are eigenstates of H_{part} and do not contain excited states of the cavity-mode. In the first dark state $|\Psi_0^D\rangle$ the cavity-modes are not populated due to destructive quantum interference, whereas the second dark state $|\Psi_1^D\rangle$ is decoupled from the laser interaction in a trivial way.

The central idea is to use this dark state for adiabatic passage. If the system is prepared in a superposition of the two dark states in Eq. (6) and the laser intensities are changed slowly (i.e., adiabatically) no other eigenstates of H_{part} will be populated. Therefore, throughout the whole process the two cavity-modes will not be populated. Initially, an unknown quantum superposition is prepared in subsystem A as given in Eq. (5). This initial state can in fact be written as a superposition of the two dark states $|\Psi_{0,1}^D\rangle$ provided that $s_{10}^B(t) \gg s_{10}^A(t)$:

$$(a|0\rangle + \beta|1\rangle)\,|1\rangle = \alpha|\Psi_0^D\rangle + \beta|\Psi_1^D\rangle\,.$$

In practice, this means that at the beginning of the transfer the laser which acts on the atom in subsystem B is switched on first. During the transfer the laser intensities are changed such that at the end the inequality $s_{10}^B(t) \ll s_{10}^A(t)$ holds. If the change is carried out slowly enough, the system at the end is still the above superposition of $|\Psi_0^D\rangle$ and $|\Psi_1^D\rangle$. However, this quantum state now corresponds to the desired final state of the transfer;

$$\alpha|\Psi_0^D\rangle + \beta|\Psi_1^D\rangle = |1\rangle(\alpha|0\rangle + \beta|0\rangle)\,.$$

An important feature of the scheme is that the details of the laser pulses are not important as long as the process is carried out adiabatically.

As mentioned above, the Hamiltonian H_{part} is not the total Hamiltonian of the system because it does not contain $\tilde{H}_0$ specified in Eq. (2). This Hamiltonian contains a light shift of the state $|a_0\rangle$ which will destroy the dark state Eq. (6). However, this light shift can be compensated for quite straightforwardly by using a second laser which couples the atomic level $|a_0\rangle$ nonresonantly with an additional level farther up in the atomic level scheme. The intensity of this laser is chosen such that the total light shift of $|a_0\rangle$ adds up to zero.

In the remainder of this Letter numerical results will be presented in order to evaluate the performance of the scheme. Note that the coupling strength ν will decrease when the length of the fibre is increased. Therefore, it is advantageous to introduce a coupling strength per square root of unit length defined as $\alpha = \nu\sqrt{L}$, where L is the length of the fibre. It will be shown below that with this definition α is independent of L. The order of magnitude of α in terms of known quantities can be estimated as follows. Suppose the fibre is infinitely long and the decay rate of the single cavity-mode into the continuum of fibre-modes is Γ. As a result of the quantum fluctuation-dissipation theorem the coupling strength G between the cavity-mode and the output field of the fibre-modes is $G = \sqrt{2\Gamma}$ [9]. Now the coupling of the cavity-mode to the individual fibre-modes is estimated, provided that L is finite. Let us first estimate the number of fibre-modes which are coupled to the cavity-mode. For simplicity it is assumed that the mode separation between neighboring fibre-modes is $4\pi c/L$, where c denotes the speed of light. This means that the number of fibre-modes which couple significantly to the cavity-mode is of the order of $N = \Gamma L/4\pi c$. The coupling of the cavity-mode to an individual fibre-mode can be estimated by multiplying G by a factor f which reflects the discretization of the spectrum. From the commutation relations of the field operators, it follows that this factor fulfills the following relation:

$$f \approx \sqrt{\Gamma/N}\,.$$

Thus the coupling of the cavity-mode to an individual fibre-mode is approximately

$$\nu = \alpha/\sqrt{L} \approx \sqrt{8\Gamma\pi c/L}\,.$$

As a concrete numerical example, let us assume that the decay rate of the cavity into the fibre is $\Gamma/2\pi = 0.5$ GHz. Thus it is found that $\alpha/2\pi \approx 0.8$ GHz m$^{1/2}$. Moreover, suppose the unwanted cavity loss rate is $\kappa/2\pi = 100$ MHz. In units of α, the cavity loss rate

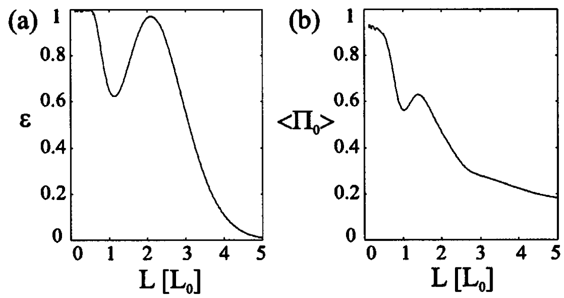

FIG. 3. (a) Population in the quantum state $|a_1, 0\rangle \times |a_0, 0\rangle|\text{vac}\rangle \equiv |1\rangle|0\rangle$ after the transfer (initial state $|a_0, 0\rangle|a_1, 0\rangle|\text{vac}\rangle \equiv |0\rangle|1\rangle$) against the length of the fibre in units of L_0. L_0 denotes an arbitrarily chosen reference length of the fibre. The total time of the transfer was chosen as $T = 300\alpha^{-1}L_0^{1/2}$. The pulse shape of the two lasers was assumed to be Gaussian: $\Delta_g s_{01}^{A,B}(t) = c\exp[-(t - t_{A,B})^2/w^2]$, with $c = 2\alpha L_0^{-1/2}$, $t_B - t_A = 0.2T$, and $w = 0.05T$. The scaled cavity loss rate and the separation between neighbor fibre-modes were chosen as $\kappa = 0.1\alpha L_0^{-1/2}$ and $\Delta = 0.1\alpha L_0^{-1/2}$, respectively. (b) The time average of the expectation value of the photon number in the resonant, dark fibre-mode divided by the time average of the expectation value of the total photon number in the fibre. The parameters are the same as in (a).

and the mode detuning are $\kappa \approx (0.13\ \text{m}^{1/2})\alpha$ and $\Delta_k \approx (k/L)(0.77\ \text{m}^{1/2})\alpha$. It will be shown later that reliable transfer can be achieved, in principle, for a transfer time of the order of $T \approx (300\ \text{m}^{1/2})\alpha^{-1} \approx 60$ ns.

In Fig. 3(a) the performance of the scheme is studied as a function of the length of the fibre. A good measure is the population ε in the quantum state $|\alpha_1, 0\rangle|\alpha_0, 0\rangle|\text{vac}\rangle \equiv |1\rangle|0\rangle$ after the transfer for an initial state $|\alpha_0, 0\rangle|\alpha_1, 0\rangle|\text{vac}\rangle \equiv |0\rangle|1\rangle$. As pointed out earlier, the state $|\alpha_0, 0\rangle|\alpha_1, 0\rangle|\text{vac}\rangle \equiv |1\rangle|1\rangle$ is decoupled from the laser interaction and thus remains unchanged. The length L of the fibre is plotted in units of L_0, which denotes an arbitrarily chosen reference length. For small values of L (relative to L_0) the transfer is almost perfect. As the length of the fibre is increased, the population ε decreases (after going through a local maximum) and thus the performance of the scheme deteriorates. This behavior is due to the fact that the distance between neighbor modes decreases with increasing fibre length. Therefore, more and more "grey" states (i.e., states which are not perfectly dark) in the neighborhood of the resonant dark state are involved in the transfer. Note that this asymptotic behavior is required for causality reasons. If all the modes besides the central mode are neglected, a transfer between the nodes could take place in constant time regardless of the length of the fibre.

Finally, Fig. 3(b) shows to what extent the dark state Eq. (6), is populated during the transfer. Here $\langle\Pi_0\rangle$ is plotted, the average population in the dark state divided by the average total photon number within the fibre. Surprisingly, rather good transfer can be found even in the case where $\langle\Pi_0\rangle$ is not close to one. This suggests that a significant portion of the population passes through nondark states in the neighborhood of the dark state and very good transfer can take place for parameters where the single mode approximation of the fibre is not valid.

In summary, a novel scheme has been proposed to transfer quantum states between distant nodes of a quantum network which is robust against important sources of decoherence. The scheme could be used to enable networking of several ion-trap quantum computers. In addition, error correction methods could be applied to further increase the stability of the scheme [12,13].

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