Cosmic Necklaces and Ultrahigh Energy Cosmic Rays

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Cosmic necklaces are hybrid topological defects consisting of monopoles and strings, with two strings attached to each monopole. We argue that the cosmological evolution of necklaces may significantly differ from that of cosmic strings. The typical velocity of necklaces can be much smaller than the speed of light, and the characteristic scale of the network much smaller than the horizon. We estimate the flux of high-energy protons produced by monopole annihilation in the decaying closed loops. For some reasonable values of the parameters it is comparable to the observed flux of ultrahigh-energy cosmic rays. [S0031-9007(97)04934-X]

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The observation of cosmic ray particles with energies higher than 10^{11} GeV [1] gives a serious challenge to the known mechanisms of acceleration. The shock acceleration in different astrophysical objects typically gives maximal energy of accelerated protons less than $(1-3) \times 10^{10}$ GeV [2]. Much attention has recently been given to acceleration by ultrarelativistic shocks [3,4]. The particles here can gain a tremendous increase in energy, equal to Γ^2, at a single reflection, where Γ is the Lorentz factor of the shock. However, it is known (see, e.g., the simulation for pulsar relativistic wind in [5]) that particles entering the shock region are captured there or at least have a small probability to escape.

Topological defects (for a review, see [6]) can naturally produce particles of ultrahigh energies (UHE) well in excess of those observed in cosmic rays (CR) [7]. In most cases the problem with topological defects is not the maximal energy, but the fluxes.

Cosmic strings can produce particles when two segments of string come into close contact, as in *cusp* events [8]. When the distance between two segments of the cusp becomes of the order of the string width, the cusp may "annihilate" turning into high-energy particles. However, the resulting cosmic ray flux is far too small [9].

Superconducting strings [10] appear to be much better suited for particle production. Moving through cosmic magnetic fields, such strings develop electric currents and copiously produce charged heavy particles when the current reaches a certain critical value. The CR flux produced by superconducting strings is affected by some model-dependent string parameters and by the history and spatial distribution of cosmic magnetic fields. Models considered so far failed to account for the observed flux [11].

Monopole-antimonopole pairs $(M\bar{M})$ can form bound states and eventually annihilate into UHE particles [12,13]. For an appropriate choice of the monopole density n_M, this model is consistent with observations; however, the required (low) value of n_M may be difficult to explain.

We shall consider here another potential source of UHE CR, the topological defects which have not been much studied so far: *cosmic necklaces*. Such defects can be formed in a sequence of symmetry breaking phase transitions $G \to H \times \mathrm{U}(1) \to H \times Z_2$. If the group G is semisimple, then the first phase transition produces monopoles, and at the second phase transition each monopole gets attached to two strings, with its magnetic flux channeled along the strings. The resulting necklaces resemble "ordinary" cosmic strings with monopoles playing the role of beads. "Realistic" particle physics models with necklaces can readily be constructed [14].

The evolution of necklaces is rather complicated, and its analysis would require high-resolution numerical simulations. Here we shall attempt to indicate only the relevant physical processes and to give very rough estimates for the efficiency of some of these processes.

The monopole mass m and the string tension μ are determined by the corresponding symmetry breaking scales, η_s and η_m $(\eta_m > \eta_s)$: $m \sim 4\pi\eta_m/e$, $\mu \sim 2\pi\eta_s^2$. Here, e is the gauge coupling. The mass per unit length of string is equal to its tension, μ. Each string attached to a monopole pulls it with a force $F = \mu$ in the direction of the string. The monopole radius δ_m and the string thickness δ_s are typically of the order $\delta_m \sim (e\eta_m)^{-1}$, $\delta_s \sim (e\eta_s)^{-1}$.

Monopoles are formed at a temperature $T_m \sim \eta_m$. Their initial average separation, d, can range from δ_m (for a second-order phase transition) to the horizon size (for a strongly first-order transition). The monopoles are diluted by the expansion of the Universe, so that d grows as $d \propto T^{-1}$. There is some additional decrease in the monopole density, and associated increase in d, due to $M\bar{M}$ annihilation. The latter process, however, is rather inefficient.

At the second phase transition, each monopole gets attached to two strings, resulting in the formation of necklaces. There will be infinite necklaces having the shape of random walks and a distribution of closed loops. The two strings attached to a monopole are pulling it with an equal force; hence, there is no tendency for a monopole to be captured by the nearest antimonopole, unless their separation is comparable to the string thickness, δ_s. (We assume that no unconfined magnetic fluxes are left after the string formation, so that there is no Coulombic magnetic force between the monopoles.)

An important quantity for the necklace evolution is the dimensionless ratio $r = m/\mu d$. The average mass per unit length of necklaces is $(r + 1)\mu$. The initial value of r can be large ($r \gg 1$) or small ($r \ll 1$), depending on the nature of the two phase transitions.

We expect the necklaces to evolve in a scaling regime. If ξ is the characteristic length scale of the network, equal to the typical separation of long strings and to their characteristic curvature radius, then the force per unit length of string is $f \sim \mu/\xi$, and the acceleration is $a \sim (r + 1)^{-1}\xi^{-1}$. We assume that ξ changes on a Hubble time scale $\sim t$. Then the typical distance traveled by long strings in time t should be $\sim \xi$, so that the strings have enough time to intercommute in a Hubble time. This gives $at^2 \sim \xi$, or

$$\xi \sim (r + 1)^{-1/2} t \,. \tag{1}$$

The typical string velocity is $v \sim (r + 1)^{-1/2}$.

For $r \ll 1$ the monopoles are subdominant, and the string evolution is essentially the same as that of ordinary strings without monopoles. The opposite case $r \gg 1$ is much different: the string motion is slow and their average separation is small. Like ordinary strings, cosmic necklaces can serve as seeds for structure formation. Significant quantitative changes in the corresponding scenario can be expected for $r \gg 1$.

Disregarding $M\bar{M}$ annihilation, the evolution of $r(t)$ can be analyzed using the energy balance equation $\dot{E} = -P\dot{V} - \dot{E}_g$. Here, E is the energy of long necklaces in a comoving volume V, P is the effective pressure, and $\dot{E}_g$ is the rate of energy loss by gravitational radiation from small-scale wiggles on long strings. If the scale of the wiggles is set by the gravitational back-reaction, then the strings radiate a substantial part of their energy in a Hubble time [15,16], and we can write $\dot{E}_g = \kappa_g Nm/rt$, where N is the number of monopoles in volume V and $\kappa_g \sim 1$. The effect of loop formation is not relevant for the evolution of $r(t)$ and has not been included in the energy balance equation.

For $r \ll 1$, the effect of monopoles on the string dynamics is negligible, and we can write $P = (Nm/3Vr)(2v^2 - 1)$, where v is the *rms* string velocity. Then, with a power-law expansion $a(t) \propto t^\nu$, we obtain the following equation for $r(t)$:

$$\dot{r}/r = -\kappa_s/t + \kappa_g/t \,, \tag{2}$$

where $\kappa_s = \nu(1 - 2v^2)$. The first term on the right-hand side of Eq. (2) describes the string stretching due to expansion of the Universe while the second term describes the competing effect of string shrinking due to gravitational radiation [17]. In this regime, we can use the values of v^2 from the string simulations [18]: $v^2 = 0.43$ in the radiation era and $v^2 = 0.37$ in the matter era. The corresponding values of κ_s are, respectively, 0.07 and 0.14. Our estimate for κ_g is $\kappa_g \sim 1$, so it seems reasonable to assume that $\kappa_g > \kappa_s$. The solution of Eq. (2) is $r(t) \propto t^{\kappa_g - \kappa_s}$, suggesting that if r is initially small, it will grow at least until it reaches values $r \sim 1$.

An equation similar to (2) can also be written for $r \gtrsim 1$, but in this case the results of numerical simulations [18] can no longer be used, and the relative magnitude of κ_s and κ_g cannot be assessed. Order-of-magnitude estimates suggest $\kappa_s \sim \kappa_g \sim 1$, and in this paper we shall assume that $\kappa_g > \kappa_s$, so that $r(t)$ is driven towards large values, $r \gg 1$. (An alternative possibility is an attractor with $r \sim 1$.)

As r grows and monopoles get closer together, $M\bar{M}$ annihilation should become important at some point. In any case, the growth of r should terminate at the value $r_{\max} \sim \mu/m\delta_s \sim \eta_m/\eta_s$, when the monopole separation is comparable to the string thickness δ_s. It is possible that annihilations will keep r at a much smaller value. For example, if monopoles develop appreciable relative velocities along the string, they may frequently run into one another and annihilate. Then it is conceivable that r will decrease as $r \propto t^{-\alpha}$ with $0 < \alpha < 1$. The terminal value of r cannot be determined without numerical simulations of network evolution; here we shall assume that $r \gg 1$.

It should be noted that for $r \gtrsim 10^6$, the characteristic velocity of the network will fall below the virial velocity, and the necklaces will be trapped by the gravitational clustering of matter. This may have a dramatic effect on the network evolution. (We are grateful to C. Thompson for pointing this out to us.) Here we assume for simplicity that $r < 10^6$.

Self-intersections of long necklaces result in copious production of closed loops. For $r \gtrsim 1$ the motion of loops is not periodic, so loop self-intersections should be frequent and their fragmentation into smaller loops very efficient. A loop of size ℓ typically disintegrates on a time scale $\tau \sim r^{-1/2}\ell$. All monopoles trapped in the loop must, of course, annihilate in the end.

Annihilating $M\bar{M}$ pairs decay into Higgs and gauge bosons, which we shall refer to collectively as X particles. The rate of X-particle production is easy to estimate if we note that infinite necklaces lose a substantial fraction of their length to closed loops in a Hubble time. The string length per unit volume is $\sim \xi^{-2}$, and the monopole rest energy released per unit volume per unit time is $r\mu/\xi^2 t$. Hence, we can write

$$\dot{n}_X \sim r^2\mu/t^3 m_X \,, \tag{3}$$

where $m_X \sim e\eta_m$ is the X-particle mass and we have used Eq. (1).

In the extreme case of $r \sim r_{\max} \sim \eta_m/\eta_s$, Eq. (3) gives the rate of X-particle production which does not depend on the string scale η_s. It is possible that the evolution of $r(t)$ is actually saturated in this regime.

X particles emitted by annihilating monopoles decay into hadrons, photons, and neutrinos, which contribute to the spectrum of cosmic ultrahigh energy radiations.

The diffuse flux of ultrahigh energy protons can be evaluated as

$$I_p(E) = (\dot{n}_X/4\pi m_X)\lambda_p(E)W_N(m_X, x), \tag{4}$$

where dn_X/dt is given by Eq. (3), $\lambda_p(E)$ is the attenuation length for ultrahigh energy protons due to their interaction with microwave photons, and $W_N(m_X, x)$ is the fragmentation function of X particle into nucleons of energy $E = xm_X$.

The fragmentation function is calculated using the decay of X particle into QCD partons (quark, gluons, and their supersymmetric partners) with the consequent development of the parton cascade. We have used the fragmentation function in the Gaussian form as obtained in the modified leading logarithm approximation in [19,20]. Additionally, we took into account the supersymmetric corrections to the coupling constant α_s at large Q^2. The explicit form of the fragmentation function at small x is found as

$$W_N(m_X, x) = \frac{K_N}{x} \exp\left(-\frac{\ln^2 x/x_m}{2\sigma^2}\right), \tag{5}$$

where $2\sigma^2 = (1/6)[\ln(m_X/\Lambda)]^{3/2}$, $x = E/m_X$, $x_m = (\Lambda/m_X)^{1/2}$, $\Lambda = 0.234$ GeV with the normalization constant K_N to be found from energy conservation assuming that about 10% of initial energy (m_X) is transferred to nucleons.

The attenuation lengths we took from the book [21].

Note that in our calculations the UHE proton flux is fully determined by only two parameters, $r^2\mu$ and m_X. The former is restricted by low energy diffuse gamma radiation. It results from e-m cascades initiated by high-energy photons and electrons produced in the decays of X particles.

The cascade energy density predicted in our model is

$$\omega_{\text{cas}} = \frac{1}{2} f_\pi r^2 \mu \int_0^{t_0} \frac{dt}{t^3} \frac{1}{(1+z)^4} = \frac{3}{4} f_\pi r^2 \frac{\mu}{t_0^2}, \tag{6}$$

where t_0 is the age of the Universe (here and below we use $h = 0.75$), z is the redshift, and $f_\pi \sim 1$ is the fraction of energy transferred to pions. In Eq. (6) we took into account that half of the energy of pions is transferred to photons and electrons. The observational bound on the cascade density, for the kind of sources we are considering here, is [22] $\omega_{\text{cas}} \lesssim 10^{-5}$ eV/cm^3. This gives a bound on the parameter $r^2\mu$.

In numerical calculations we used $r^2\mu = 0.8 \times 10^{28}$ GeV^2, which results in $\omega_{\text{cas}} = 4.5 \times 10^{-6}$ eV/cm^3, somewhat below the observational limit. Now we are left with one free parameter, m_X, which we fix at 1×10^{14} GeV. The maximum energy of protons is then $E_{\max} \sim 10^{13}$ GeV. Note that these values of the parameters correspond to $r = e\eta_m/\sqrt{2\pi}\,\eta_s \sim 0.1 r_{\max}$. The calculated proton flux is presented in Fig. 1, together with a summary of observational data. These data are usually interpreted as indicating the presence of a new component at energy higher than 1×10^{10} GeV.

Let us now turn to the calculations of UHE gamma-ray flux from the decays of X particles. The dominant channel is given by the decays of neutral pions. The flux can be readily calculated as

$$I_\gamma(E) = (\dot{n}_X/4\pi)\lambda_\gamma(E)N_\gamma(E), \tag{7}$$

where $\dot{n}_X$ is given by Eq. (3), $\lambda_\gamma(E)$ is the absorption length of a photon with energy E due to e^+e^- pair production on background radiation, and $N_\gamma(E)$ is the number of photons with energy E produced per one decay of X particle. The latter is given by

$$N_\gamma(E) = (2K_{\pi^0}/m_X)\int_{E/m_X}^1 (dx/x)W_N(m_X, x)/K_N. \tag{8}$$

The normalization constant K_{π^0} is again found from the condition that neutral pions take away $f_\pi/3$ fraction of the total energy m_X.

At energy $E > 1 \times 10^{10}$ GeV the dominant contribution to the absorption length λ_γ comes from the radio background. The significance of this process was first noticed in [23]. New calculations for this absorption were recently done [24]. We have used the absorption lengths from this work. When evaluating the flux (7) at $E > 1 \times 10^{10}$ GeV we neglected cascading of a primary photon, because of dominant energy losses of produced electron and positron on the radiophotons (cascading is further suppressed by a magnetic field $B \gtrsim 10^{-10}$ G).

The calculated flux of gamma radiation is presented in Fig. 1 by the curve labeled γ. One can see that at

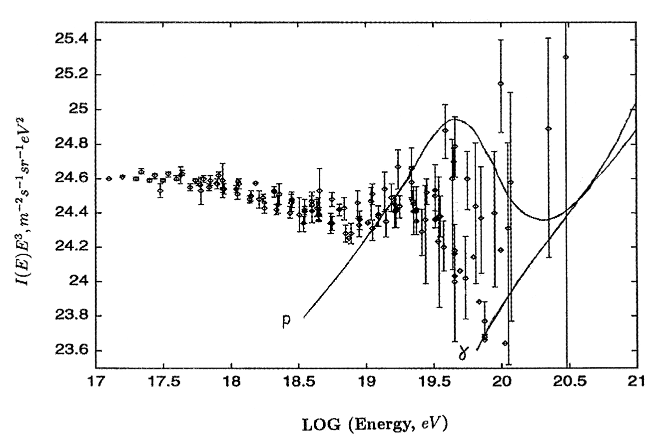

FIG. 1. Predicted proton and gamma-ray fluxes from necklaces. The data points are fluxes from the compilation by AGASA group [26].

$E \sim 1 \times 10^{11}$ GeV the gamma-ray flux is considerably lower than that of protons. This is mainly due to the difference in the attenuation lengths for protons (110 Mpc) and photons (2.6 Mpc [24] and 2.2 Mpc [23]). At higher energy the attenuation length for protons dramatically decreases (13.4 Mpc at $E = 1 \times 10^{12}$ GeV) and the fluxes of protons and photons become comparable. This conclusion agrees with recent calculations of Ref. [25], but both are in conflict with the last reference in [7]. This discrepancy may be caused by a different radio flux used in the calculations.

A requirement for the models explaining the observed UHE events is that the distance between sources must be smaller than the attenuation length. Otherwise the flux at the corresponding energy would be exponentially suppressed. This imposes a severe constraint on the possible sources. For example, in the case of protons with energy $E \sim (2-3) \times 10^{11}$ GeV the proton attenuation length is 19 Mpc. If protons propagate rectilinearly, there should be several sources inside this radius; otherwise all particles would arrive from the same direction. If particles are strongly deflected in extragalactic magnetic fields, the distance to the source should be even smaller. Therefore, the sources of the observed events at the highest energy must be at a distance $R \lesssim 15$ Mpc in the case of protons.

In our model the distance between sources, given by Eq. (1), satisfies this condition for $r > 3 \times 10^4$. This is in contrast to other potential sources, including supeconducting cosmic strings and powerful astronomical sources such as AGN, for which this condition imposes severe restrictions.

The difficulty is even more pronounced in the case of UHE photons. These particles propagate rectilinearly and their absorption length is shorter: 2–4 Mpc at $E \sim 3 \times 10^{11}$ GeV. It is rather unrealistic to expect several powerful astronomical sources at such short distances. This condition is very restrictive for topological defects as well. The necklace model we introduced here is rather exceptional.

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