Purifying Two-Bit Quantum Gates and Joint Measurements in Cavity QED

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(Received 18 August 1997)

Using a cavity QED setup we show how to implement a particular joint measurement on two atoms in a fault tolerant way. Based on this scheme, we illustrate how to realize quantum communication over a noisy channel when local operations are subject to errors. We also present a scheme to perform and purify a fundamental two-bit gate. [S0031-9007(97)04830-8]

PACS numbers: 03.65.Bz, 42.50.–p

One of the most intriguing features of quantum mechanics is the possibility of entangling physical systems, which has both practical and fundamental implications. On one hand, Bell's theorem [1] states that quantum mechanics and any local realist theory are incompatible based on the peculiar properties of entanglement. On the other hand, quantum communication and computation exploit these properties to guarantee secure communication and to construct algorithms that allow fast computations [2].

In a series of remarkable experiments the first steps towards these lofty goals have been taken [3–5]. In particular, it seems that quantum communication will have several practical applications in the near future. For example, quantum cryptography has been tested experimentally over long distances using standard telecommunication fibers [6]. This, combined with recent proposals [7,8] for exchanging quantum information between atoms and photons based on cavity QED, suggests that a full quantum network including local processing and transmission of quantum data is possible. Since practical uses of quantum networks require a high degree of entanglement, one might think that this is not feasible due to the presence of errors and decoherence. However, the recent discovery of quantum error correction protocols and purification schemes [9–11] shows that this is not a fundamental obstacle. In a quantum network one can classify the errors in two categories: *transmission errors*, i.e., those occurring during the transfer of quantum information between nodes, and *local errors,* i.e., those occurring during local processing and measurements. Since transmission errors are much more likely than local errors, one usually assumes that the latter are absent. With this assumption, noisy channels have been defined and protocols have been devised to achieve ideal transmission of quantum information [12]. Most proposals allow for quite general types of noise and require unbounded resources to achieve this goal. In contrast, based on a specific model for quantum communication, we have proposed a protocol that requires only finite resources [8] and corrects for the physically relevant errors. Therefore, in that physical scenario, the only remaining problem is local errors. Although one could in principle use standard error correction schemes to solve this problem, this would again require infinite resources.

In this Letter we will give a physical implementation that allows us to perform local operations and measurements ideally using finite resources. The scheme is based on cavity QED and therefore can be easily connected to the previous proposal for quantum communication [7,8]. We will assume that operations acting on a single atom are error free, whereas any other operation is not. This is motivated by the experimental fact that single-bit operations are much simpler than multiple-bit operations [5]. First we will show how to perform a particularly useful joint measurement which is fault tolerant [13] in the sense that it operates even in the presence of errors occurring during this measurement. An essential element for this measurement is the introduction of a "red light atom" R [10] which reveals the occurrence of errors. We will also show how to implement a fundamental two-bit operation [14] which also involves measurements that indicate whether an error took place or not. In the former case, one has to start the procedure again, whereas in the latter case, one knows one has succeeded. Our schemes can be regarded as purification protocols [11] since with certain probability they are successful, while sometimes the information is lost. We emphasize that in applications in quantum communication the loss of information is not central, whereas the knowledge that one has reliably transmitted the quantum information is indispensable.

We start by discussing the physical details of our setup. We consider two atoms, 1 and 2, inside a single cavity. The internal structure of the atoms is displayed in Fig. 1; the qubit is stored in the states $|0\rangle$ and $|1\rangle$, and there is an auxiliary state $|r\rangle$. The states $|1\rangle$ and $|r\rangle$ are coupled by a far-off-resonance Raman transition induced by an external

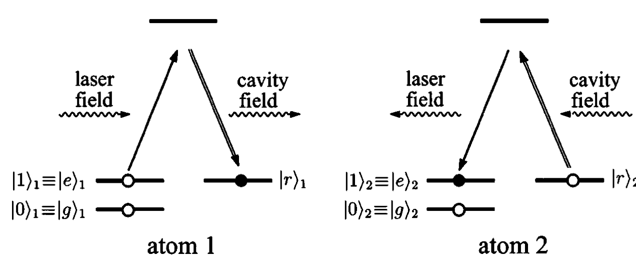

FIG. 1. Level structure of atoms and couplings induced by laser and cavity fields.

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laser field and the cavity mode, whereas the state $|0\rangle$ is not coupled by either the laser or the cavity field. The Hamiltonian describing the interaction between the atoms and the cavity mode is given, in a rotating frame at the cavity mode frequency, by

$$H = \frac{g_1}{2}|1\rangle_{11}\langle r|a + \frac{g_2}{2}|1\rangle_{22}\langle r|a + \text{H.c.}, \qquad (1)$$

where a is the annihilation operator for the cavity mode and $g_{1,2}$ are the effective coupling constants of the Raman transition. In the following, we will consider that a laser pulse of duration $\Delta t_1 = \pi/g_1$ is applied to atom 1 and then another laser pulse of duration $\Delta t_2 = \pi/g_2$ is applied to atom 2 [15]. Denoting by $|0\rangle_{\text{cav}}$ and $|1\rangle_{\text{cav}}$ the cavity state of zero and one photons, respectively, this gives under ideal conditions

$$|0\rangle_1|0\rangle_2|0\rangle_{\text{cav}} \mapsto |0\rangle_1|0\rangle_2|0\rangle_{\text{cav}}, \qquad (2a)$$

$$|0\rangle_1|r\rangle_2|0\rangle_{\text{cav}} \mapsto |0\rangle_1|r\rangle_2|0\rangle_{\text{cav}}, \qquad (2b)$$

$$|1\rangle_1|0\rangle_2|0\rangle_{\text{cav}} \mapsto -i|r\rangle_1|0\rangle_2|1\rangle_{\text{cav}}, \qquad (2c)$$

$$|1\rangle_1|r\rangle_2|0\rangle_{\text{cav}} \mapsto -|r\rangle_1|1\rangle_2|0\rangle_{\text{cav}}, \qquad (2d)$$

where we have considered only the cases in which the first atom is in $|0\rangle_1$ or $|1\rangle_1$ and the second atom is in $|0\rangle_2$ or $|r\rangle_2$, since this will be sufficient for our purposes. Note that if the first atom is in the state $|0\rangle_1$ nothing will change. However, if it is in $|1\rangle_1$, then it will be transferred to $-i|r\rangle_1$. Then if the second atom is in $|r\rangle_2$, it will be transferred to the state $-i|1\rangle_2$, whereas if it is in $|0\rangle_2$, it will not change its state and a cavity photon will remain in the cavity. In reality there will be errors. Since we are considering a far-off resonance Raman transition, the most important ones will be photon losses either at the mirrors or by leaking out of the cavity. As in our previous Letter [8] we will also consider systematic errors in the detuning, timing, laser pulses, phase shifts, etc. It is straightfoward to account for these errors in Eq. (2) by including the state of the environment and different operators acting on it, as well as adding new terms in the last two lines which describe the effect of photon loss (see below). On the other hand, we will also need single-atom operations involving the three atomic levels. As mentioned in the introduction, we will concentrate here on errors occurring in processes involving two bits.

In the first part of this Letter, we will be interested in the following situation: atom 2 is initially in state $|0\rangle_2$ and is transferred to state $|r\rangle_2$; then the process (2) takes place, followed by two single-atom operations, namely, $-|r\rangle_1 \leftrightarrow |1\rangle_1$ and $|r\rangle_2 \leftrightarrow |0\rangle_2$ in the first and second atoms, respectively. Hence, ideally we have

$$|0\rangle_1|0\rangle_2 \mapsto |0\rangle_1|0\rangle_2, \qquad |1\rangle_1|0\rangle_2 \mapsto |1\rangle_1|1\rangle_2, \qquad (3)$$

which corresponds to a controlled-NOT gate. In the presence of the errors mentioned above,

$$|0\rangle_1|0\rangle_2|E\rangle \mapsto |0\rangle_1|0\rangle_2\mathcal{L}_0|1\rangle, \qquad (4a)$$

$$|1\rangle_1|0\rangle_2|E\rangle \mapsto |1\rangle_1|1\rangle_2\mathcal{L}_1|E\rangle + |1\rangle_1|0\rangle_2\mathcal{L}_a|E\rangle, \qquad (4b)$$

where $|E\rangle$ denotes the initial state of the environment (including the cavity mode) and the operators $\mathcal{L}$ act on this state. We used that one can optically pump the state $|r\rangle_1$ to the state $|1\rangle_1$ after the whole procedure. Note that with this notation this process is formally equivalent to the *photonic channel* introduced in Ref. [16].

In the following we will assume the environment operators $\mathcal{L}_{0,1}$ fulfill the *stationary property* for two consecutive operations

$$\mathcal{L}_1^{(2)}\mathcal{L}_0^{(1)}|E\rangle = \mathcal{L}_0^{(2)}\mathcal{L}_1^{(1)}|E\rangle, \qquad (5)$$

starting at times $t_{1,2}$ of duration $\Delta t_{1,2}$, respectively. Here we have used the short-hand notation $\mathcal{L}_i^{(j)} \equiv \mathcal{L}_i(t_j, \Delta t_j)$, where $i = 0,1$ and $j = 1,2$. In Ref. [8], the validity of (5) has been demonstrated for the present model using the quantum trajectories approach. Here, as a simple example, we illustrate this stationarity property in the context of photon absorption: We consider a cavity mode coupled to a bath of oscillators in the vacuum state $|E\rangle \equiv |\mathbf{0}\rangle$ (i.e., at zero temperature). We assume a linear coupling Hamiltonian

$$H = \omega a^\dagger a + \sum_k \omega_k b_k^\dagger b_k + \sum_k g_k(a^\dagger b_k + \text{H.c.}), \qquad (6)$$

where $b_k, b_k^\dagger$ are creation and annihilation operators for the bath oscillators and ω_k and g_k are the corresponding frequencies and coupling constants. Denoting by t the initial time, after a time Δt we will have

$$\begin{aligned}|0\rangle_{\text{cav}}|E\rangle &\rightarrow |0\rangle_{\text{cav}}|E\rangle \\ &\equiv |0\rangle_{\text{cav}}\mathcal{L}_0(t,\Delta t)|E\rangle, \\ |1\rangle_{\text{cav}}|E\rangle &\rightarrow c(\Delta t)|1\rangle_{\text{cav}}|E\rangle + |0\rangle_{\text{cav}}\sum_k c_k(\Delta t)b_k^\dagger|E\rangle \\ &\equiv |1\rangle_{\text{cav}}\mathcal{L}_1(t,\Delta t)|E\rangle + |0\rangle_{\text{cav}}\mathcal{L}_a(t,\Delta t)|E\rangle,\end{aligned}$$

where c and c_k are c numbers. Note that $\mathcal{L}_{0,1}$ only depend on Δt but not on the initial time t. Moreover, they commute and therefore they satisfy (5). The stationary property is related to the zero temperature of the reservoir, which for optical frequencies is a good approximation even at room temperature. On the other hand, one can verify that systematic errors also fulfill (5), since the corresponding $\mathcal{L}_{0,1}$ will be c numbers only depending on Δt but not on t.

Our goal is to use (4) to perform ideal joint measurements and entanglement operations as are required in quantum communication via a photonic channel [8,16]. In this scheme, one has to perform a local joint measurement on two atoms to check whether they are in the state $|0\rangle|0\rangle$ or not. It must be implemented such that an error occurring during this measurement will be detected by the measurement itself. To be specific, let us consider two atoms in a state $|\Psi\rangle = |\Psi_c\rangle|E_c\rangle + |0\rangle_1|0\rangle_2|E_a\rangle$, where $|E_{c,a}\rangle$ denote unnormalized states of the environment and $|\Psi_c\rangle = \alpha|0\rangle_1|1\rangle_2 + \beta|1\rangle_1|0\rangle_2$ with α and β arbitrary coefficients. The goal is to make a filtering measurement of the state $|0\rangle_1|0\rangle_2$, so that with certain probability the state of the atoms is projected onto the $|\Psi_c\rangle$, which is the one we want to keep intact. In order to perform the joint measurement we need the red light atom R initially prepared in the state $|0\rangle_R$. We use (4) between atoms 1 and R, and then between atoms 2 and R [see Fig. 2(a)]. This gives the transformation

$$\begin{aligned}
|0\rangle_1|1\rangle_2|0\rangle_R &\mapsto |0\rangle_1|1\rangle_2|1\rangle_R \mathcal{L}_1^{(2)}\mathcal{L}_0^{(1)} \\
&\quad + |0\rangle_1|1\rangle_2|0\rangle_R \mathcal{L}_a^{(2)}\mathcal{L}_0^{(1)}, \\
|1\rangle_1|0\rangle_2|0\rangle_R &\mapsto |1\rangle_1|0\rangle_2|1\rangle_R \mathcal{L}_0^{(2)}\mathcal{L}_1^{(1)} \\
&\quad + |1\rangle_1|0\rangle_2|0\rangle_R \mathcal{L}_0^{(2)}\mathcal{L}_a^{(1)}, \\
|0\rangle_1|0\rangle_2|0\rangle_R &\mapsto |0\rangle_1|0\rangle_2|0\rangle_R \mathcal{L}_0^{(2)}\mathcal{L}_0^{(1)}, \qquad (7)
\end{aligned}$$

where we have left out the state of the environment. Now a single-atom measurement on atom R in the state $|1\rangle_R$ or $|0\rangle_R$ reveals whether the joint state of atoms 1 and 2 was in the subspace spanned by $|0\rangle_1|1\rangle_2$ and $|1\rangle_1|0\rangle_2$, or a photon loss took place, respectively. In the first case, the state after the measurement will become

$$\begin{aligned}
|\Psi\rangle &\mapsto (\alpha|0\rangle_1|1\rangle_2\mathcal{L}_1^{(2)}\mathcal{L}_0^{(1)} + \beta|1\rangle_1|0\rangle_2\mathcal{L}_0^{(2)}\mathcal{L}_1^{(1)})|E_c\rangle \\
&= |\Psi_c\rangle\mathcal{L}_1^{(2)}\mathcal{L}_0^{(1)}|E_c\rangle, \qquad (8)
\end{aligned}$$

where we have used (5). We emphasize that the errors that may occur during the joint measurement either factor out (operators $\mathcal{L}_1$ and $\mathcal{L}_0$) or are projected out (terms containing $\mathcal{L}_a$).

Let us now show how this measurement can be used in the implementation for quantum communication proposed in [7,8]. In that case one needs the same three-level atoms, and the transmission between atom 1 in the first node (cavity) and atom 2 in the second node is performed by using an appropriate laser pulse to transfer $|1\rangle_1 \mapsto |r\rangle_1$, producing one cavity photon. This photon then travels to the second cavity, where it can induce the inverse transition in a second atom, $|r\rangle_2 \mapsto |1\rangle_2$ to which the time inverse laser pulse is applied. Finally, we transfer $|r\rangle_1 \mapsto |1\rangle_1$ in atom 1. Levels $|0\rangle_1$ and $|0\rangle_2$ are not coupled by the laser field. Using the same notation as before, this transmission can then be summarized as (4) but with local operators $\mathcal{L}$ replaced by the corresponding transmission operators $\mathcal{T}$

$$|0\rangle_1|0\rangle_2 \mapsto |0\rangle_1|0\rangle_2\mathcal{T}_0, \qquad (9a)$$

$$|1\rangle_1|0\rangle_2 \mapsto |1\rangle_1|1\rangle_2\mathcal{T}_1 + |1\rangle_1|0\rangle_2\mathcal{T}_a. \qquad (9b)$$

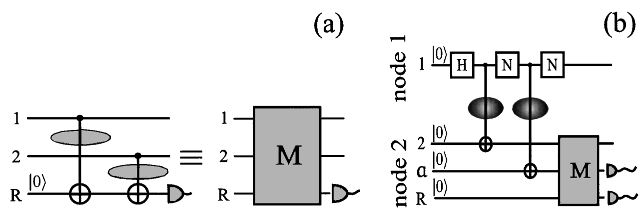

FIG. 2. Diagrammatic representation of (a) joint measurement and (b) establishing an EPR pair. H and N denote the Hadamard and NOT transformations, respectively.

We expect that in any realistic situation $||\mathcal{T}_a|| > ||\mathcal{L}_a||$, i.e., transmission results in more of these types of errors than local operations. In [8] we showed how, using this channel, one can send quantum information perfectly *provided local operations and measurements are perfect*. Here we will show how to accomplish the same goal using noisy local operations and the joint measurement described above. We will concentrate on producing a distant EPR pair entangling two atoms in different nodes [see Fig. 2(b)]. We consider one atom (1) in the first cavity and two atoms (2 and a) in the second cavity. Starting from state $|0\rangle_1 + |1\rangle_1$ [17], we use the channel (9) between atoms 1 and 2; then we interchange $|0\rangle_1 \leftrightarrow |1\rangle_1$ in atom 1; then we use again the channel (9) between atom 1 and atom a; finally, we reverse $|0\rangle_1 \leftrightarrow |1\rangle_1$ in atom 1. Using this procedure we obtain the map [8]

$$\begin{aligned}
(|0\rangle_1 + |1\rangle_1)|0\rangle_2|0\rangle_a \mapsto\ & (|0\rangle_1|0\rangle_2|1\rangle_a + |1\rangle_1|1\rangle_2|0\rangle_a) \\
& \times \mathcal{T}_1^{(2)}\mathcal{T}_0^{(1)} + |0\rangle_1|0\rangle_2|0\rangle_a \\
& \times \mathcal{T}_a^{(2)}\mathcal{T}_0^{(1)} \\
& + |1\rangle_1|0\rangle_2|0\rangle_a\mathcal{T}_0^{(2)}\mathcal{T}_a^{(1)}, \qquad (10)
\end{aligned}$$

where, as before, we have used the stationary property (see Ref. [8]),

$$\mathcal{T}_1^{(2)}\mathcal{T}_0^{(1)}|E_c\rangle = \mathcal{T}_0^{(2)}\mathcal{T}_1^{(1)}|E_c\rangle. \qquad (11)$$

The last two terms in (10) arise from photon loss errors, and can be detected by performing a *joint measurement* on atoms 2 and a, namely, checking whether they are in the state $|0\rangle_2|0\rangle_a$. In case they are not found in this state, a single-ion measurement on atom a (in the basis $|0\rangle \pm |1\rangle$) leaves atoms 1 and 2 in a maximally entangled state. The joint measurement requires entanglement and therefore is susceptible to errors. However, we can use instead our implementation of this joint measurement using the red light ion in cavity 2 [see Fig. 2(b)]. Repeating the transmission (10) and the subsequent measurement (7) until no photon loss was detected (the red light ion is found in the state $|1\rangle_R$), yields, after having measured atom a in the basis $|0\rangle_a \pm |1\rangle_a$, the state $|\psi\rangle_{12} = |0\rangle_1|0\rangle_2 \pm |1\rangle_1|1\rangle_2$. With this EPR state one can already distribute a random secret key using the Ekert protocol [18] for quantum cryptography [19].

For certain applications in quantum communication and quantum computing a two-bit fundamental gate is required, since when combined with one-bit operations this is sufficient for any unitary operation [2]. This gate cannot be implemented using Eq. (4) since there the state $|1\rangle|1\rangle$ is absent as input state, whereas in the gate this state has to be present. We show now how to perform the fundamental gate

$$|0\rangle_1|0\rangle_2 \mapsto |0\rangle_1|0\rangle_2;\ |1\rangle_1|0\rangle_2 \mapsto -|1\rangle_1|0\rangle_2; \qquad (12a)$$

$$|0\rangle_1|1\rangle_2 \mapsto |0\rangle_1|1\rangle_2;\ |1\rangle_1|1\rangle_2 \mapsto |1\rangle_1|1\rangle_2, \qquad (12b)$$

with the present implementation in the presence of errors. The gate consists of three steps: (i) A single atom operation on atom 2 exchanges $|1\rangle_2 \leftrightarrow |r\rangle_2$ while leaving the state $|0\rangle_2$ unchanged; (ii) we perform a conditional operation using the cavity mode such that the state $|1\rangle_1|0\rangle_2 \mapsto -|1\rangle_1|0\rangle_2$ by applying (2) twice; (iii) we apply the inverse of step (i). Note that, according to the evolution given by (1), if the initial state is $|1\rangle_1|0\rangle_2$ the cavity photon produced the first time will be absorbed again by atom 1 the second time, yielding a minus sign, as desired.

In reality there will be errors due to photon losses, phase shifts of the states involved, and imperfect state transfer. After applying the gate one obtains, including these errors,

$$|0\rangle_1|0\rangle_2 \mapsto |0\rangle_1|0\rangle_2\mathcal{L}_{00}\,, \tag{13a}$$

$$|0\rangle_1|1\rangle_2 \mapsto |0\rangle_1|1\rangle_2\mathcal{L}_{01}\,, \tag{13b}$$

$$|1\rangle_1|0\rangle_2 \mapsto -|1\rangle_1|0\rangle_2\mathcal{L}_{10} + |r\rangle_1|0\rangle_2\mathcal{L}_{r0}\,, \tag{13c}$$

$$|1\rangle_1|1\rangle_2 \mapsto |1\rangle_1|1\rangle_2\mathcal{L}_{11} + |r\rangle_1|1\rangle_2\mathcal{L}_{r1} + |r\rangle_1|r\rangle_2\mathcal{L}_{rr}\,. \tag{13d}$$

The "photon loss" errors $\mathcal{L}_{r0,r1,rr}$ can be detected by measuring if the first atom is in state $|r\rangle_1$. In order to perform the gate in the presence of all these errors we apply (13) four times but changing $|0\rangle \leftrightarrow |1\rangle$ first in atom 1, then in atom 2, and again in atom 1, after subsequent applications. Moreover, in the last one we change the phase of the laser field acting on atom 2 by π in the second part of step (ii) so that no extra minus sign is added to the state $|1\rangle_1|0\rangle_2$ [therefore, this fourth application performs just the (noisy) identity operation in order to symmetrize the errors]. If no error is found during the whole procedure (i.e., population in state $|r\rangle_1$) we obtain

$$|0\rangle|0\rangle \mapsto |0\rangle|0\rangle\mathcal{L}_{01}^{(4)}\mathcal{L}_{11}^{(3)}\mathcal{L}_{10}^{(2)}\mathcal{L}_{00}^{(1)}, \tag{14a}$$

$$|0\rangle|1\rangle \mapsto |0\rangle|1\rangle\mathcal{L}_{00}^{(4)}\mathcal{L}_{10}^{(3)}\mathcal{L}_{11}^{(2)}\mathcal{L}_{01}^{(1)}, \tag{14b}$$

$$|1\rangle|0\rangle \mapsto -|1\rangle|0\rangle\mathcal{L}_{11}^{(4)}\mathcal{L}_{01}^{(3)}\mathcal{L}_{00}^{(2)}\mathcal{L}_{10}^{(1)}, \tag{14c}$$

$$|1\rangle|1\rangle \mapsto |1\rangle|1\rangle\mathcal{L}_{10}^{(4)}\mathcal{L}_{00}^{(3)}\mathcal{L}_{01}^{(2)}\mathcal{L}_{11}^{(1)}. \tag{14d}$$

Using the same arguments as in (5), one can check that all these operators are identical. Thus, once no error was found the gate worked perfectly.

So far, we used the stationary properties (11) and (5) for transmission and local operations. It is important to realize that, even if the former one (11) does not hold, one can still establish a perfect EPR pair, since we have shown here how to purify all local operations (including the gate) needed for the procedure developed in [16]. On the other hand, if also (5) would not hold, one can establish an entangled state whose degree of entanglement is limited by the degree to which (5) is satisfied.

In summary, we have shown how to perform joint measurements in the presence of errors in a cavity QED implementation. The scheme works even if errors occur during the measurement itself. We have shown how to apply this proposal in quantum communication to achieve perfect transmission over a noisy channel including local errors. Using the same implementation, we have also presented a fundamental two-bit gate that operates perfectly in the presence of errors.

This work was supported in part by the TMR network ERB-FMRX-CT96-0087 and by the Austrian Science Foundation.

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