Theory of Self-Trapped Spatially Incoherent Light Beams

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We present a modal theory of self-trapping spatially *incoherent* light beams in *any* general nonlinear media. We find that a self-trapped incoherent beam induces a multimode waveguide which guides the beam itself by multiply populating the guided modes. The self-trapping process alters the statistics of the incoherent beam, rendering it localized. We find the conditions for self-trapping ("existence region" in parameter space) and the correlation function of the incoherent self-trapped beam.
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Optical spatial solitons have been extensively studied during the last three decades. Self-trapping of optical beams occurs when diffraction is exactly balanced by self-focusing due to an optical nonlinearity [1]. Self-trapping has been studied in Kerr-type [2], in photorefractive [3], in quadratic [4], and in resonant atomic [5] nonlinear media. All of these studies have investigated self-trapping of spatially coherent light beams only.

Recently, self-trapping of a quasimonochromatic partially spatially incoherent light beam [6] and of a "white" light beam which is both spectrally and spatially incoherent [7] has been demonstrated, using the nonlinearity associated with photorefractive screening solitons [8–10]. In a recent paper [11] we have examined theoretically the possibility of self-trapping spatially incoherent light beams. We have shown [11] that the process of incoherent self-focusing can be described by an infinite set of coupled nonlinear Schrodinger-like equations, which are initially weighted according to the angular power spectrum of the input beam. We were able to reproduce the experimental results of Ref. [6]. However, this dynamic approach [11] is better suited to describing the *dynamic* evolution of incoherent beams in nonlinear media, and does *not* lend itself readily toward identifying static (self-trapped) solutions and the conditions necessary for their existence. To do that, a more analytic approach should be pursued.

Here we present a semianalytic modal theory describing self-trapping of spatially incoherent light beams. We find the conditions that allow self-trapping in the form of an "existence range," and derive the statistical properties of the self-trapped incoherent beams.

We first state the three principles that enable the self-trapping of spatially incoherent beams. First, the nonlinearity must be noninstantaneous with a response time that is much longer than the phase fluctuation time across the incoherent beam. Such a nonlinearity responds to the time-averaged envelope and not to the instantaneous "speckles" that constitute the incoherent beam. In contrast, an instantaneous nonlinearity which responds to individual speckles will generate multiple filaments that randomly intersect and fragment the beam, prohibiting self-trapping of the beam envelope. The second principle is that the multimode (speckled) beam should be able to induce a multimode waveguide via the nonlinearity. This is easily satisfied with any saturable nonlinearity, in which even a single-mode (i.e., spatially coherent) beam induces a multimode waveguide in the saturated regime [12]. Finally, self-trapping requires self-consistency: The multimode beam must be able to guide itself in its own induced waveguide [12].

We solve the coupled nonlinear equations for the modal constituents of the self-trapped beam self-consistently in an iterative manner. We start by examining the guided modes of a waveguide and observe whether or not it is possible that a certain composition of them will give an intensity profile that recreates the waveguide itself via the nonlinearity. When this happens, the solution is self-consistent and self-trapping occurs. The simplest case is of bright coherent solitons which form when the first guided mode of the self-induced waveguide induces the index profile of the waveguide [12]. If a higher mode is additionally populated and if the modes are coherent with each other, then the modes interfere and cause evolution of the intensity profile due to differing modal propagation velocities. The subsequent induced index profile thus evolves with propagation and the beam does not maintain a constant profile [13]. On the other hand, if the modes of the self-induced waveguide do not interfere with each other, it is possible to have a self-trapped beam consisting of multiply populated modes that has a nonevolving intensity profile since the total intensity is simply the sum of the individual intensities of each mode. *This is the case of a self-trapped spatially incoherent light beam.* It is a multicomponent generalized version of the previously studied bipolarization temporal [14] and bimodal spatial [13] vector solitons, where the modes mutually self-trap in their common self-induced waveguide [13].

Here we show that *multiple* modes with the same polarization can be self-trapped if the modes are made to be incoherent with each other. This allows self-trapping of a beam consisting of many modes. We find that for such a multimode beam, there is a parameter range within which nondiffracting solutions exist. This allows for self-trapping of beams with widely varying beam profiles. When the input light is multimode in space and randomly varying in time (as a spatially incoherent light beam is) and if the nonlinearity is noninstantaneous, then the nonlinear medium responds to the intensity superposition only, and the contribution of the interference cross-terms to the refractive index averages out to zero.

How does this correspond to self-trapping of a spatially incoherent beam? When an initially partially spatially incoherent beam is launched into a multimode waveguide, the intensity created by the modes exhibits interference at any instant of time, but the time averaged interference is zero [15]. When this waveguide is self-induced via the nonlinearity, *self-consistency is satisfied,* and the resultant waveguide has a nonevolving index profile. For this to occur, the input light beam (which is continually exciting the modes at the waveguides input) must be changing in time such that the relative initial phase differences between guided modes vary randomly in time between 0 and 2π. For example, consider two modes of a waveguide, u_1 and u_2, with phases θ_1 and θ_2 that vary randomly in time. The time-averaged intensity is given as

$$\begin{aligned} I(x,z) &= \langle |u_1(x)e^{i[\delta_1 z+\theta_1(t)]} + u_2(x)e^{i[\delta_2 z+\theta_2(t)]}|^2\rangle \\ &= |u_1(x)|^2 + |u_2(x)|^2 + 2|u_1(x)|\,|u_2(x)| \\ &\quad \times \langle \cos[(\delta_1-\delta_2)z + \theta_1(t) - \theta_2(t)]\rangle . \end{aligned}$$

Therefore, $I(x) = |u_1(x)|^2 + |u_2(x)|^2$. *It is thus necessary that the nonlinearity responds slower than the rate of change of the initial phases of the waveguide modes.* A spatially incoherent beam that induces its own waveguide via such a noninstantaneous nonlinearity, effectively populates the guided modes *incoherently*.

Having satisfied the first two conditions, we now proceed to seek self-consistent multimode solutions. Each such "allowed" solution represents a spatially incoherent beam that possesses a different mutual coherence function, that is, both the intensity profile and the correlation statistics are fully defined having found an "allowed" modal composition. Obviously, *the characteristics of the solutions strongly depend on the specific nonlinearity, nonetheless, the general idea holds for any noninstantaneous nonlinear medium.* We study the properties of 1D self-trapped incoherent beams in biased photorefractive media, for which the explicit form of nonlinearity is well established [8,9] and confirmed in many experiments.

To solve for the self-trapped beam profiles we need to solve the following coupled nonlinear Schrodinger-like equations [8,9]:

$$\frac{d^2u_i(x)}{dx^2} = -\left(\delta_i - \frac{1}{1+I(x)}\right)u_i(x)\,,$$

where $I(x) = \langle \sum_i c_i u_i(x) \sum_j c_j^* u_j^*(x)\rangle = \sum_i |u_i(x)|^2 \times \langle |c_i|^2\rangle = \sum_i |u_i(x)|^2 d_i^2$ and d_i is the averaged modal amplitude of mode $u_i(x)$. Note that these modal amplitudes d_i are *not* normalized. Also notice that the time average of the cross terms between different modes is zero, i.e., $\langle c_i c_j^*\rangle = 0$ for $i \neq j$. This is *not* due to different propagation constants δ_i, but, as explained above, due to the fact that relative phases between modes vary randomly in time much faster than the nonlinear medium can respond. To solve these equations self-consistently, we start with an arbitrary initial index profile, solve for the guided modes, weigh the modes, and sum them incoherently to get the corresponding intensity profile $I(x)$, and finally use $I(x)$ to calculate the induced index. We repeat this until the solutions are stationary. This methodology is depicted in the inset in Fig. 1. We employ the shooting method to find the modes under a given index profile.

A self-consistent solution has a given modal amplitude distribution $(d_1, d_2, \ldots)$ as an initial condition, and one needs to determine which distributions allow for self-trapping of a beam. Whether or not the equations can be solved self-consistently depends upon the modal amplitude function used as the initial condition. When a solution is found, we have the profile and propagation constant of each mode. Of course, if we set the modal amplitude profile of the modes such that the first mode has an amplitude u_0 and the other modes are zero, we retrieve the existence curve of fundamental bright screening soliton [8], which describes the relation between u_0, the maximum change in the refractive index, the optical wavelength in the medium, and the soliton width. For bimodal solutions, allowing only the first and second modes to be populated, the existence curve turns into a

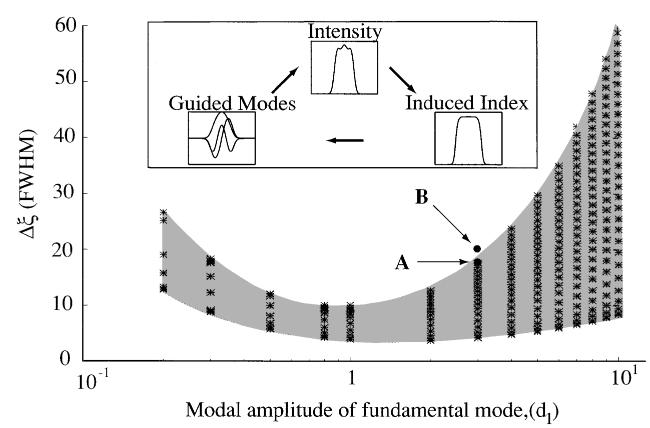

FIG. 1. Existence range for a beam composed of weighted combinations of the first and second guided modes. Positions *A* and *B* represent points in parameter space that are trappable and nontrappable, respectively. Inset: schematic of self-consistency between intensity, its induced index change, and the population of the guided modes of the induced waveguide.

range. The existence range is shown in Fig. 1, where the horizontal axis indicates the modal amplitude of the first guided mode d_1 and the vertical axis is the normalized full width half maximum (FWHM) of the total intensity made up from all the modes ($\Delta\xi = \Delta x k n_b^2 \sqrt{r_{\rm eff} V/\ell}$, as in Ref. [8]). The bottom edge of the curve corresponds to the case where only the first mode is populated (fundamental soliton solutions) [8]. Other points correspond to adding some contribution of the second mode. The upper edge of the curve is where the second mode attains its maximum allowed value, which also gives the widest possible bimodal (first + second modes) self-trapped solution. Notice that the second mode can vary from 0 to a maximum value that depends on the modal amplitude of the first mode. If the second mode is populated above the maximum value, self-consistency is not observed. *Thus, a self-trapped bimodal solution exists only in the existence range defined by the shaded area in Fig. 1.* To confirm this, we study the evolution of bimodal solutions using standard split-step Fourier transform beam propagation methods. For example, consider two adjacent points *A* and *B*, located on either side of the top of the existence range at $d_1 = 3$. Points *A* and *B* represent two different points in modal-amplitude-space, where point *A* lies within the existence range and point *B* lies outside it. Point *A* has the second mode d_2 weighted to 5.4. The evolution of this solution with and without the nonlinearity is shown in Figs. 2(a) and 2(b), respectively. For this point, it is evident that the nonlinearity supports stationary self-trapping, as expected from a self-consistent solution (note that this solution and the induced index profile that supports it are both *double humped*). On the other hand, point *B* is outside the existence range, i.e., a self-consistent solution for that point does not exist. Yet, since points *A* and *B* are close to each other, we can take the solution from point *A* and add slightly more weight to the second mode such that $d_2 = 5.7$. The evolution of this profile with and without the nonlinearity is shown in Figs. 2(c) and 2(d). It is obvious that although the input intensity profiles of both beams are very similar, beam *A* is self-trapped whereas the two intensity peaks of beam *B* diverge away from one another. The divergence angle of this "forbidden" solution in the presence of nonlinearity [Fig. 2(d)] is smaller than its natural diffraction [Fig. 2(c)]. We find that *the transition between self-trapped solutions and nontrapped (diverging) solutions is abrupt.* This is surprising, since coherent solitons "breathe" (oscillate) when the input beam slightly deviates from the soliton wave form (it sheds off excess power while evolving into a soliton), rather than diverge or disintegrate.

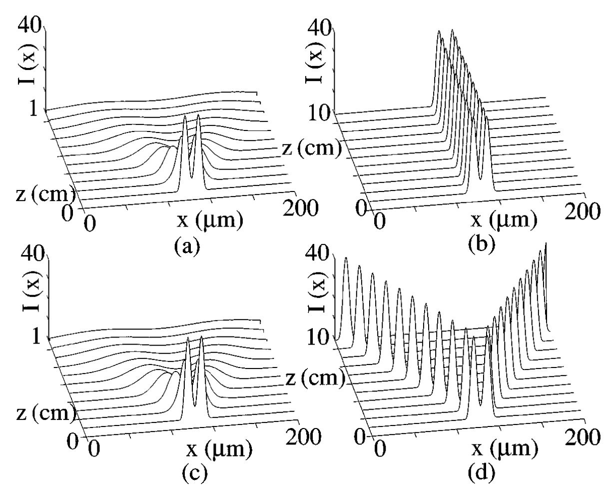

FIG. 2. Evolution of a bimodal beam: (a) Diffraction of the beam of point *A* in Fig. 1; (b) self-trapping of the beam (point *A*) with application of 870 V/5 mm; (c) diffraction of beam of point *B* in Fig. 1; (d) non-self-trapping evolution of beam (point *B*) with application of voltage.

When more modes are populated, the existence range becomes multidimensional with the dimensionality equal to the number of modes. For example, the existence range for three modes is shown in Fig. 3, where we plot the FWHM of the intensity profile vs total *power* in the beam. Examples of four different states profiles are shown in Fig. 3. The induced index profiles that support examples I, II, III, and IV are single, triple, double, and triple humped, respectively.

Having found the parameter range that supports a self-trapped incoherent beam, we extract the coherence statistics in the form of the mutual coherence function. In thermal light sources, the statistics are nonlocalized, i.e., the correlation function depends only upon the separation of two points and not on their position. The self-trapping of a beam from such a source alters the statistics and renders it localized, that is, the coherence function depends upon *both* the distance between two points *and* their location within the self-trapped beam. The spatial coherence function is given by $\gamma(x_1, x_2) = \langle E(x_1)E^*(x_2)\rangle / \sqrt{I(x_1)I(x_2)}$. The optical (electric) field $E(x)$ consists of all the guided modes, yet the time-averaged cross (interference) terms average out to zero. For example, consider a self-trapped beam with the first six modes excited with a modal amplitude distribution starting at

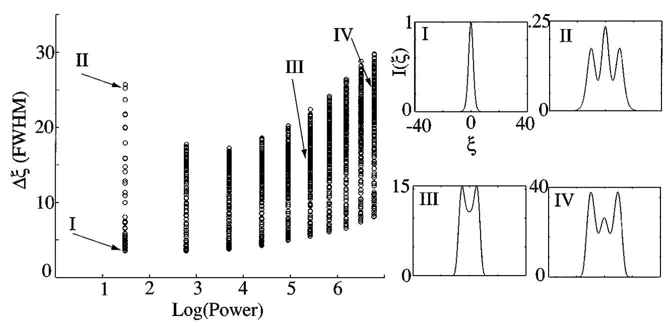

FIG. 3. Existence range for a beam composed of weighted combinations of the first three modes. The width vs total power is shown with four examples of trappable beam profiles.

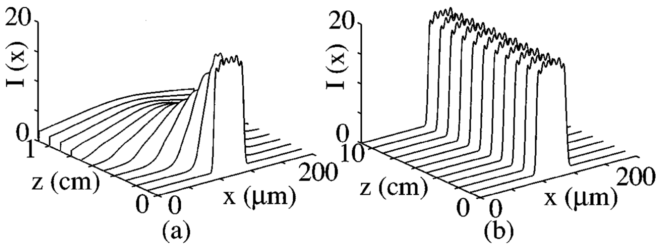

FIG. 4. Evolution of beam composed of six weighted modes. (a) Ordinary diffraction; (b) self trapping with application of 870 V/5 mm.

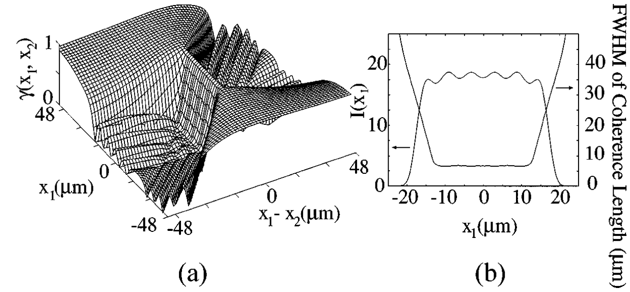

FIG. 5. Coherence properties of a six-mode self-trapped beam: (a) Spatial coherence function $\gamma(x_1, x_2)$; (b) intensity profile and coherence length as a function of position.

one and decreasing by 0.1 until the last mode with a value of 0.5. The diffraction of this beam is shown in Fig. 4(a), where a 38.4 μm wide (FWHM) input beam diffracts to 196.6 μm in 1 cm of propagation. This beam is self-trapped with the application of the nonlinearity as shown in Fig. 4(b) [16]. Numerical parameters are $n_e = 2.3$, $r_{33} = 1022$ pm/V, $\lambda = 488$ nm. The spatial coherence function of the guided beam is found to be asymmetric with respect to x_1-x_2, as shown in Fig. 5(a). The correlation distance as a function of coordinate is shown in Fig. 5(b), and it varies from 8 μm in the center of the beam, increasing to 30 μm when the intensity decreases to half its maximum value, and further increasing when the intensity diminishes at the beams' margins. The explanation for this is intuitive: around the center of the beam many modes contribute, thus the coherence is reduced because the modes are incoherent with each other. On the other hand, far away from the center only the higher modes contribute and the coherence increases. At large distances from the center the remaining optical field is only that of the highest mode, which is fully coherent with itself. Choosing the set of parameters that allows for self-trapping provides control over the correlation statistics of the self-trapped incoherent beam.

Finally, we consider how a nonideal self-trappable incoherent beam evolves into a self-trapped beam. It is important to note that the modes available for population in a given waveguide do not form a complete basis to represent an arbitrary excitation. Thus, part of the light does not couple into the modes of the self-induced waveguide, but escapes to radiation. The more modes available in the self-induced waveguide, the more complete the set, and less power escapes and more power "couples" into the self-trapped beam. We notice that when the excess noncoupled light is small, this light does not diffract away but stays with the beam and causes it to oscillate slightly during propagation. On the other hand, if the excess noncoupled light is large, the beam may not self-trap and instead diverge. Yet, this amount of light tends to enlarge the induced waveguide and possibly increase the number of modes, which then reduces the amount of uncoupled light, leading to "breathing" propagation.

In conclusion, we have presented a general modal theory of self-trapping spatially incoherent light beams (incoherent wave packets) in any noninstantaneous nonlinear media.

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[16] To illustrate the concept, a "multihump" solution is shown in Fig. 4. We also find more "conventional" solutions with the experimental parameters of Ref. [6].