

## Directed Quantum Chaos

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Quantum disordered problems with a direction (imaginary vector potential) are discussed and mapped onto a supermatrix  $\sigma$  model. It is argued that the 0D version of the  $\sigma$  model may describe a broad class of phenomena that can be called directed quantum chaos. It is demonstrated by explicit calculations that these problems are equivalent to those of random asymmetric or non-Hermitian matrices. A joint probability of complex eigenvalues is obtained. The fraction of states with real eigenvalues proves to be always finite for time reversal invariant systems. [S0031-9007(97)03575-8]

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New very interesting phenomena may occur in systems with non-Hermitian quantum Hamiltonians. Although the Hamiltonian of a closed system in equilibrium must be Hermitian, non-Hermitian models can describe nonequilibrium processes [1], open systems connected to reservoirs [2], dynamics of neural networks [3], and have many other interesting applications (see, e.g., [4–6]).

In a recent remarkable work [7], Hatano and Nelson considered a model of particles described by a random Schrödinger equation with an imaginary vector potential. This model arises as a result of mapping flux lines in a  $(d + 1)$ -dimensional superconductor to the world lines of  $d$ -dimensional bosons. Columnar defects introduced experimentally in order to pin the flux lines [8] lead to the random potential in the boson system, whereas the component of the magnetic field perpendicular to the defects results in the constant imaginary vector potential  $i\mathbf{h}$  [9]. The prediction made in Ref. [7] is that already a one-dimensional chain of the bosons has to undergo a localization-delocalization transition as a function of disorder. This effect is new and very unusual for physics of disordered systems, which clearly shows that random models with the imaginary vector potentials deserved further discussion.

The Hamiltonian  $H$  of the simplest model of noninteracting particles with a disorder and the imaginary vector potential  $i\mathbf{h}$  has the form

$$H = (\hat{\mathbf{p}} + i\mathbf{h})^2/2m + U(\mathbf{r}), \quad (1)$$

where  $\hat{\mathbf{p}} = -i\nabla$  and  $U(\mathbf{r})$  is a random potential.

The non-Hermitian Hamiltonian, Eq. (1), differs from Hamiltonians for open dots [2]. It is real and contains the vector  $\mathbf{h}$  that introduces a direction. Average physical quantities depend on the direction of  $\mathbf{h}$ , but the time-reversal symmetry is not broken. To understand better the physical meaning of the vector  $\mathbf{h}$ , it is instructive to write a lattice Hamiltonian  $H_L$  corresponding to  $H$

$$H_L = -\frac{t}{2} \sum_{\mathbf{r}} \sum_{\nu=1}^d (e^{\mathbf{h}\cdot\mathbf{e}_\nu} c_{\mathbf{r}+\mathbf{e}_\nu}^+ c_{\mathbf{r}} + e^{-\mathbf{h}\cdot\mathbf{e}_\nu} c_{\mathbf{r}}^+ c_{\mathbf{r}+\mathbf{e}_\nu}) + \sum_{\mathbf{r}} U(\mathbf{r}) c_{\mathbf{r}}^+ c_{\mathbf{r}}, \quad (2)$$

where  $c^+$  and  $c$  are creation and annihilation operators, and  $\{\mathbf{e}_\nu\}$  are the unit lattice vectors. Considering the one-particle Hamiltonian, we do not need to specify statistics of the particles. [Equation (2) was used in Ref. [7] for numerical calculations.]

In Eq. (2), the hopping probability along  $\mathbf{h}$  is higher than in the opposite direction. In other words, the Hamiltonian  $H_L$  describes a *directed* hopping in a random potential. This model can be considered as a quantum counterpart of a directed percolation model introduced by Obukhov [10]. Such models may describe properties of disordered systems without a center of inversion. The finite  $\mathbf{h}$  in Eq. (2) can be due to, e.g., the possibility of a virtual tunneling into a dielectric in an electric field. This would not lead to an imaginary contribution to the chemical potential as in open quantum dots, but might make hopping in different directions nonequivalent.

It is interesting to note that Green functions used in Ref. [10] contained as the energy spectrum the combinations  $(\mathbf{p} + i\mathbf{a})^2$  with a constant vector  $\mathbf{a}$ , which corresponds to the continuum version of the Hamiltonian, Eq. (1). The non-Hermitian operators described by Eqs. (1) and (2) enter also Fokker-Planck equations derived for models of random walks in random media with constrained drift forces [4–6]. The so-called noisy Burgers equation studied recently in a number of works [11–13] can be reduced by the Cole-Hopf transformation to a linear equation containing similar non-Hermitian operators with a direction [14].

“Conventional” (nondirected) disordered systems exhibit a variety of different phenomena. Depending on the geometry of the sample and strength of disorder, one can have conduction, localization, or, e.g., quantum chaos. One can guess that the models described by Eqs. (1) and (2) also contain these effects although the effects may be peculiar. In analogy with the directed percolation, it is reasonable to call the corresponding phenomena *directed quantum chaos*, *directed localization*, etc.

The main results of Ref. [7] were derived for 1D and 2D samples from numerical computations, although some qualitative conclusions were made considering a

one-impurity model. At the same time, very well developed analytical methods of study of conventional disordered systems exist and it is highly desirable to develop analogous schemes for the directed disorder problems. This would help to understand the localization-delocalization transitions, but also consider new phenomena like directed quantum chaos.

In this Letter it is shown that problems of directed disorder described by Eqs. (1) and (2) can be reduced to calculations within a nonlinear supermatrix  $\sigma$  model. This approach proved to be very useful for a broad variety of different problems (for a review see [15]). The  $\sigma$  model derived below is applicable in any dimension and differs from previous ones by a new  $\mathbf{h}$ -dependent term. Problems of quantum chaos correspond to the zero-dimensional version and are most simple for calculations. Some new results are obtained for this case classified below as *directed quantum chaos*.

Recently a "ballistic"  $\sigma$  model was derived by averaging over either rare impurities or energy [16]. Apparently, proper  $\mathbf{h}$  terms can be written for this case, too.

Although the imaginary vector potential  $\mathbf{h}$  enters Eqs. (1) and (2) almost in the same way as the physical vector potential  $\mathbf{A}$ , the presence of  $i$  changes considerably the derivation of the  $\sigma$  model. A simple replacement  $(e/c)\mathbf{A} \rightarrow i\mathbf{h}$  in the  $\sigma$  model of Refs. [15,17], would

lead to the absence of the ground state. This is because eigenenergies of the Hamiltonians, Eq. (1) or Eq. (2), are not necessarily real, and retarded  $G_\epsilon^R$  and  $G_\epsilon^A$  Green functions can have now poles everywhere in the complex plane of  $\epsilon$ . As a result, these functions cannot be written in the usual form of convergent Gaussian integrals, which was a necessary step in derivation of the  $\sigma$  model.

First of all, one should choose a proper quantity to calculate. According to a discussion of Ref. [7] important information can be obtained by studying a joint probability  $P(\epsilon, y)$  of real  $\epsilon'_k$  and imaginary  $\epsilon''_k$  parts of eigenenergies or, in other words, the density of complex eigenenergies. This function is introduced as

$$P(\epsilon, y) = V^{-1} \left\langle \sum_k \delta(\epsilon - \epsilon'_k) \delta(y - \epsilon''_k) \right\rangle, \quad (3)$$

where  $V$  is the volume of the system, the sum is taken over all eigenstates, and  $\langle \dots \rangle$  stands for averaging over the disorder. In contrast to the average density of states in conventional disordered systems, the function  $P(\epsilon, y)$  at  $\mathbf{h} \neq 0$  distinguishes between localized and extended states. For the localized states it is proportional to  $\delta(y)$  but is a nontrivial function for extended ones.

It is convenient to rewrite Eq. (3) as

$$P(\epsilon, y) = \frac{1}{\pi V} \lim_{\gamma \rightarrow 0} \left\langle \sum_k \frac{\gamma^2}{[(\epsilon - \epsilon'_k)^2 + (y - \epsilon''_k)^2 + \gamma^2]^2} \right\rangle, \quad (4)$$

which allows one to express the function  $P(\epsilon, y)$  in terms of a Gaussian integral over supervectors  $\psi_k$ . This can be done using the fact that the ratio in the sum in Eq. (4) can be composed of the elements of the matrix  $M_k^{-1}$ , where

$$M_k = \begin{pmatrix} i\gamma - (\epsilon - \epsilon'_k) & -i(y - \epsilon''_k) \\ i(y - \epsilon''_k) & i\gamma + (\epsilon - \epsilon'_k) \end{pmatrix}.$$

In order to replace the integrals over all  $\psi_k$  by integrals over supervector fields  $\psi(\mathbf{r})$ , one should use vectors  $u_k(\mathbf{r})$  and  $v_k(\mathbf{r})$  and their conjugates  $\bar{u}_k(\mathbf{r})$  and  $\bar{v}_k(\mathbf{r})$

$$u_k = \frac{1}{2} \begin{pmatrix} \phi_k(\mathbf{r}) + \bar{\phi}_k^*(\mathbf{r}) \\ \phi_k(\mathbf{r}) - \bar{\phi}_k^*(\mathbf{r}) \end{pmatrix},$$

$$v_k = \frac{1}{2} \begin{pmatrix} \phi_k(\mathbf{r}) - \bar{\phi}_k^*(\mathbf{r}) \\ \phi_k(\mathbf{r}) + \bar{\phi}_k^*(\mathbf{r}) \end{pmatrix},$$

where  $\phi_k(\mathbf{r})$  and  $\bar{\phi}_k(\mathbf{r})$  are right and left eigenfunctions of the Hamiltonian  $H$ , Eq. (1) [or  $H_L$ , Eq. (2)]; the symbol  $*$  is a complex conjugation. With the vectors  $u_k(\mathbf{r})$  and  $v_k(\mathbf{r})$  one can make Fourier expansion for any 2-component vector field.

Using 8-component supervectors  $\psi(\mathbf{r})$  with exactly the same structure as those in Refs. [15,17], one can rewrite Eq. (4) in terms of a functional integral with the Lagrangian  $\mathcal{L}$

$$\mathcal{L} = -i \int \bar{\psi}(\mathbf{r}) \mathcal{H} \psi(\mathbf{r}) d\mathbf{r}, \quad (5)$$

where the  $8 \times 8$  matrix operator  $\mathcal{H}$  has the form

$$\mathcal{H} = H' - \epsilon + i\gamma\Lambda + H''\Lambda_1 + iy\Lambda_1\tau_3. \quad (6)$$

In Eq. (6),  $H' = \frac{1}{2}(H + H^+)$  and  $H'' = \frac{1}{2}(H - H^+)$  are Hermitian and anti-Hermitian parts of the Hamiltonian, respectively. In the continuum version  $H'' = \mathbf{h}\nabla/m$ . The matrices  $\Lambda$  and  $\tau_3$  are the same as in Refs. [15,17], and the matrix  $\Lambda_1$  anticommutes with  $\Lambda$  being equal to

$$\Lambda_1 = \begin{pmatrix} 0 & \mathbf{1} \\ \mathbf{1} & 0 \end{pmatrix}, \quad (7)$$

where  $\mathbf{1}$  is the  $4 \times 4$  unit matrix. A proper preexponential term is rather lengthy and is not written here.

Further steps of derivation of the  $\sigma$  model are standard. One averages over the random potential and decouples the effective interaction by integration over  $8 \times 8$  supermatrices  $Q$ . The integral over the eigenvalues of  $Q$  is calculated using the saddle-point approximation. The  $\sigma$  model is finally obtained by expansion over  $\nabla Q$  and  $\mathbf{h}$ . As a result, the function  $P(\epsilon, y)$  takes the form

$$P(\epsilon, y) = - \lim_{\gamma \rightarrow 0} \frac{\pi \nu^2}{4V} \int A[Q] \exp(-F[Q]) DQ, \quad (8)$$

where the free energy functional  $F[Q]$  is written as

$$F[Q] = \frac{\pi\nu}{8} \int \text{STr}[D_0(\nabla Q + \mathbf{h}[Q, \Lambda_1])^2 - 4Q(\gamma\Lambda + y\Lambda_1\tau_3)]d\mathbf{r}. \quad (9)$$

In Eq. (9),  $D_0$  is the classical diffusion coefficient,  $[\cdot, \cdot]$  is the commutator,  $\text{STr}$  is the supertrace, and  $\nu$  is the density of states of the system without disorder at  $\mathbf{h} = 0$ . The preexponential functional  $A[Q]$  equals

$$A[Q] = \int \{ [Q_{42}^{11}(\mathbf{r}) + Q_{42}^{22}(\mathbf{r})][Q_{24}^{11}(\mathbf{r}') + Q_{24}^{22}(\mathbf{r}')] - [Q_{42}^{21}(\mathbf{r}) + Q_{42}^{12}(\mathbf{r})][Q_{24}^{21}(\mathbf{r}') + Q_{24}^{12}(\mathbf{r}')] \} \times d\mathbf{r}d\mathbf{r}'. \quad (10)$$

Numeration of matrix elements in Eq. (10) is the same as in Refs. [15,17]; Eqs. (8)–(10) can be used in any dimension. The supermatrix  $Q$  has the same symmetry as  $Q$  for the orthogonal ensemble in Refs. [15,17]. The free energy functional  $F[Q]$ , Eq. (9), has two additional terms with respect to the functional used for “conventional” disorder problems. These terms contain the matrix  $\Lambda_1$ , which leads to new effective “external fields” in the free energy. We see that the replacement of the physical vector potential  $\mathbf{A}$  by the imaginary quantity  $i\mathbf{h}$  changes the symmetry of  $F[Q]$ . This reflects the fact that  $\mathbf{A}$  and  $i\mathbf{h}$  violate different physical symmetries.

Of course, one can include in Eqs. (1) and (2) the vector potential  $\mathbf{A}$ , which would result in a standard term in Eq. (9) describing a magnetic field. If this field is strong enough, one can use Eqs. (8)–(10) as before, but in this case  $Q$  is a supermatrix with the structure corresponding to the unitary ensemble of Refs. [15,17]. This is a system with the broken time reversal invariance.

Calculation of the function  $P(\epsilon, y)$  can be carried out using methods presented in Ref. [15]. One can try to use renormalization group methods for study of the 2D case (corresponding to the 3D case for flux lines), transfer-matrix method for the 1D case (thick films with parallel line defects and magnetic field), or calculate definite integrals over  $Q$  for the 0D case (flux lines in long cylinders). Leaving 1D and 2D cases for future study, let us consider now the 0D case classified here as directed quantum chaos.

In this limit one should integrate over  $Q$ , assuming that this variable does not depend on coordinates. Then, the gradient terms in  $F[Q]$  in Eq. (9) can be omitted and integration over  $\mathbf{r}, \mathbf{r}'$  in Eq. (10) easily performed. The 0D form of  $F[Q]$  obtained in this way allows one to make a very interesting conclusion even without starting explicit computation. This can be done comparing Eq. (9) with results of a recent work [18] in which the supersymmetry technique was used to study density of complex eigenvalues of “almost-Hermitian” random matrices  $X$ . These matrices were written in the form

$$X = A + i\alpha N^{-1/2}B \quad (11)$$

with  $N \times N$  statistically independent Hermitian matrices  $A$  and  $B$ , and a number  $\alpha$  of the order of unity. Due to the

presence of the factor  $N^{-1/2}$ , the ensemble described by Eq. (11) differs from ensembles of matrices with arbitrary complex elements studied previously [19]. Fyodorov *et al.* [18] calculated the joint probability of the real and imaginary parts of eigenvalues of the matrices  $X$  corresponding to the function  $P(\epsilon, y)$ , Eq. (3).

Reducing in a standard way integration over the matrices  $A$  and  $B$  to integration over supervectors and then to supermatrices, the authors of Ref. [18] arrived in the limit  $N \rightarrow \infty$  at the 0D version of the free energy  $F[Q]$ , Eq. (9), with  $\mathbf{h}^2 \sim \alpha^2$ . The symmetry of  $Q$  corresponded to the supermatrices  $Q$  of Refs. [15,17] for the unitary ensemble, which is due to the hermicity of the matrices  $A$  and  $B$ . Apparently, if they used real symmetric matrices  $A$ , antisymmetric matrices  $B$ , and imaginary  $\alpha$  they would obtain Eq. (9) with the supermatrices  $Q$  corresponding to the orthogonal ensemble (below these cases are called simply orthogonal and unitary). This demonstrates that the models of disorder, Eqs. (1) and (2), in a limited volume are equivalent to the ensembles of weakly nonsymmetric (or non-Hermitian if the time reversal symmetry is broken) random matrices.

For explicit calculations, Fyodorov *et al.* [18] used the parametrization of Ref. [17]. However, due to the presence of the new terms with  $\mathbf{h}$  and  $y$  in Eq. (9), the calculations with this parametrization are very difficult. So, the final result was obtained for the unitary ensemble only. As concerns the orthogonal case, computations with this parametrization do not seem to be possible at all. At the same time, study of the orthogonal ensemble can be very important because it describes the vortices in superconductors and, as will be seen later, the function  $P(\epsilon, y)$  for the orthogonal ensemble at small  $h$  is qualitatively different from that for the unitary one.

Fortunately, one can circumvent the difficulties using a new parametrization. Leaving details for another publication, I want to present here only a general structure. The supermatrix  $Q$  is written as

$$Q = ZQ_0\bar{Z} \quad (12)$$

with supermatrices  $Z$  satisfying the conditions  $Z\bar{Z} = 1$  and  $[Z, \Lambda_1] = 0$ . The central part  $Q_0$  is chosen as

$$Q_0 = \begin{pmatrix} \cos \hat{\phi} & -\tau_3 \sin \hat{\phi} \\ -\tau_3 \sin \hat{\phi} & -\cos \hat{\phi} \end{pmatrix}, \\ \hat{\phi} = \begin{pmatrix} \phi & 0 \\ 0 & i\chi \end{pmatrix}.$$

The most lengthy part is calculation of the Jacobian, but then the computation is of no difficulty and the density of complex eigenenergies  $P(\epsilon, y)$  for the unitary ensemble takes the form

$$P(\epsilon, y) = \frac{\nu\sqrt{\pi}}{a\Delta} \exp\left(-\frac{x^2}{4a^2}\right) \times \int_0^1 \cosh xt \exp(-a^2 t^2) dt, \quad (13)$$

where  $x = 2\pi y/\Delta$ ,  $a^2 = 2\pi D_0 h^2/\Delta$ , and  $\Delta$  is the mean level spacing.

Equation (13) is exactly the result obtained in Ref. [18]. For the models of weak disorder, Eqs. (1) and (2),  $P(\epsilon, y)$  depends only on the variable  $y$  because  $\nu$  is constant. For the random matrix model, Eq. (11),  $\nu$  depends on  $\epsilon$  and obeys the Wigner semicircle law. In the limit  $a \gg 1$  one obtains

$$P(\epsilon, y) \approx \frac{\pi\nu}{2a^2\Delta}, \quad |x| \leq 2a^2, \quad (14)$$

which means that the function  $P(\epsilon, y)$  is practically constant within the interval  $|x| \leq 2a^2$ . Beyond this interval the function exponentially decays. For the random matrix model, this behavior corresponds to the well-known ‘‘elliptic law,’’ Ref. [19]. For any  $a$  the function  $P(\epsilon, y)$  is smooth, which means, in particular, that the probability to find real eigenvalues is negligible.

The situation drastically changes if the time-reversal symmetry is not broken. Although now the computation of  $P(\epsilon, y)$  is somewhat more lengthy, one can finally express this function in a rather simple form

$$P(\epsilon, y) = P_r + P_c, \\ P_r = \nu \delta(y) \int_0^1 \exp(-a^2 t^2) dt, \quad (15) \\ P_c = \frac{\pi\nu}{\Delta} \Phi\left(\frac{|x|}{2a}\right) \int_0^1 \exp(-a^2 t^2) \sinh(|x|t) t dt,$$

where  $\Phi(v) = \frac{2}{\sqrt{\pi}} \int_v^\infty \exp(-u^2) du$ .

We see that the function  $P(\epsilon, y)$  contains the part  $P_r$  proportional to  $\delta(y)$ . This means that a finite fraction of eigenstates has real eigenenergies for any finite  $a$ . The distribution of complex eigenenergies is described by the smooth function  $P_c$ . The fraction of the states with real eigenvalues vanishes only in the limit  $a \rightarrow \infty$ . In this limit one comes for  $|x| \gg a$  to Eq. (14), which again results for the random matrix model in the elliptic law.

The result about the finiteness of the fraction of eigenstates with real eigenenergies as well as Eq. (15) itself is new, although indications of a peculiar behavior of the probability of real eigenvalues for asymmetric real matrices can be found in Ref. [19]. In Ref. [7] a mixture of states with real and complex eigenvalues was found numerically near the band center for the 2D model, Eq. (2). Perhaps the localization length in that region of parameters exceeded the sample size. This would correspond to the 0D situation rather than 2D. If it is so, the analytical result agrees with the numerical observation.

Study of the directed disorder problems in higher dimensions, although more difficult, is definitely of interest. Solution of the  $\sigma$  model, Eqs. (14) and (15), in one dimension seems to be possible using the transfer-matrix method. This case corresponds to a thick film in a parallel magnetic field in the problem of the vortices in a superconductor. It would be interesting to obtain

a localization-delocalization transition analytically. The one-dimensional  $\sigma$  model is also known to describe the quantum kicked rotor problem [20]. Maybe finite  $h$  corresponds to a dissipation in this problem and one can find a transition here, too.

In summary, quantum disorder problems with a direction (imaginary vector potential) are considered. It is demonstrated that they can be mapped onto a new supermatrix  $\sigma$  model. The 0D version of the model describes a class of phenomena that can be called directed quantum chaos. Using the 0D  $\sigma$  model, the equivalence of this class of systems to ensembles of weakly asymmetric or non-Hermitian random matrices is established. The joint probability of complex eigenvalues is computed explicitly, and it is discovered that the fraction of real eigenenergies for time reversal invariant systems is always finite.

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