A Different View of the Quantum Hall Plateau-to-Plateau Transitions

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We demonstrate experimentally that the transitions between adjacent integer quantum Hall (QH) states are equivalent to a QH-to-insulator transition occurring in the top Landau level, in the presence of an inert background of the other completely filled Landau levels, each contributing a single unit of quantum conductance, e^2/h, to the total Hall conductance of the system. The equivalence holds for numerical parameters describing the transition, as well as for the recently discovered reflection symmetry of the resistivity. [S0031-9007(97)03595-3]

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The study of the transition regions separating adjacent quantum Hall (QH) states is an active topic of research in the field of two dimensional electron systems [1–6]. These transition regions were considered mainly in the framework of scaling [6,7], extending to the high magnetic field (B) QH regime a methodology developed for the study of the $B = 0$ metal-insulator transition [8].

An experimental study by Wei and his collaborators [1] revealed some of the fascinating physics of these transitions. By focusing on the diagonal resistivity (ρ_{xx}) peaks separating adjacent QH minima, and on the Hall resistivity (ρ_{xy}) steps accompanying them, they demonstrated scaling behavior in InGaAs/InP samples characterized by a temperature-dependent conductivity that is governed by a single exponent, κ, independent of Landau level (LL). This apparent *universality*, which is an expected signature of a quantum phase transition [6], is not always observed, e.g., see the study of GaAs/AlGaAs samples by Koch *et al.* [3].

More recently, another class of transitions in the quantum Hall regime were studied [9–12]. These B-driven transitions are not between adjacent QH states but between QH states and the insulating phase that terminates the QH series. Since in the insulator $\rho_{xx} \to \infty$, these transitions are not characterized by a ρ_{xx} peak and experimentally they appear, on first sight, to be different from the inter-QH, plateau-to-plateau, transitions. It was soon realized, however, that a critical B exists for these transitions as well and results supporting the existence of scaling behavior in their vicinity were also reported [12,13], with a critical exponent that is consistent with the κ obtained for the inter-QH transitions. For these QH-to-insulator transitions, there is significant experimental evidence in support of the existence of the theoretically expected [14] universal critical amplitude, with the ρ_{xx} value at the transition point close to h/e^2 [11,12]. This should be contrasted with the inter-QH transitions, for which most researchers report a critical amplitude that is not only significantly (40–80)% smaller than the theoretically expected value, but in many cases is also T dependent [15], in conflict with the scaling framework.

Theoretically, the transitions to the insulator are similar to the inter-QH transitions; both occur as the Fermi energy crosses the extended states in the center of a LL. They differ, however, in that the inter-QH transitions take place in the presence of a number of filled LLs, separated by a gap from the top LL where the action takes place. If one assumes that the only contribution of the background LLs is to the Hall conductance, each filled LL adding e^2/h to the value of σ_{xy}, it may be possible, by numerically removing that contribution from the experimental data, to test this equivalence. There is a similar scenario involving fractional QH states, in which the transitions take the form of a QH-to-insulator transition of a set of quasiparticles in parallel with a background (parent) QH liquid. This insight has been systematized theoretically by Kivelson *et al.* [16], and their unified treatment of the transitions in the integer and fractional regimes has been supported by recent experiments [11,12]. In this paper we will only be concerned with integer QH transitions.

The purpose of this Letter is to report on an experimental test of the conjecture that the inter-QH and the QH-to-insulator transitions are similar. For simplicity, we focus on the transition from the $\nu = 2$ to $\nu = 1$ integer QH states (dubbed 2-1), which occurs at the top spin-split first LL, and compare it to the $\nu = 1$-to-insulator (1-0) transition in the same sample, which takes place near the center of the lower spin-split first LL. To implement this comparison, we utilized a straightforward scheme, closely akin to that used by McEuen *et al.* [17] to disentangle different edge-state contributions to the resistivity. Our method is as follows. First, we obtain traces of

ρ_{xx} and ρ_{xy} for the 2-1 transition using a Hall-bar shaped sample etched in a high-density ($n = 2.27 \times 10^{11}$ cm^{-2}), low-mobility ($\mu = 10.8 \times 10^3$ cm^2/V sec), MBE grown GaAs/AlGaAs wafer. These resistivity traces, taken at several T's, are plotted in Fig. 1(a). The transition is typified by a ρ_{xx} peak that widens with T and by the accompanying step in ρ_{xy}. Next, we convert the ρ's to σ's using the standard matrix conversion,

$$\sigma_{xx(xy)} = \frac{\rho_{xx(yx)}}{\rho_{xx}^2 + \rho_{xy}^2}, \tag{1}$$

and plot the σ traces in Fig. 1(b). We then obtain the conductivity of the topmost LL by subtracting from the conductivity data the contribution of the lowest, full LL,

$$\sigma_{xx}^t = \sigma_{xx}, \tag{2}$$

$$\sigma_{xy}^t = \sigma_{xy} - e^2/h, \tag{3}$$

assuming, as mentioned, that the only contribution of the lowest LL is e^2/h to the Hall conductivity (throughout this paper, the index t refers to the contribution of the topmost LL to the transport coefficients). Next we convert σ_{xx}^t and σ_{xy}^t to new resistivities, ρ_{xx}^t and ρ_{xy}^t, which are the resistivities of the topmost LL. This allows comparison with the data obtained from the 1-0 transition in the same sample.

The comparison is made in Fig. 2, where we plot ρ_{xx}^t (solid lines) and ρ_{xy}^t (short-dashed lines) as a function of ν for the 2-1 transition [Fig. 2(a)], and traces of ρ_{xx} and ρ_{xy} vs ν obtained from the same sample near the 1-0 transition terminating the QH series, in Fig. 2(b) (here, of course, $\rho^t = \rho$). While for the ρ_{xx} traces in both graphs of Fig. 2 we present data at our lowest T range ($T < 150$ mK), the ρ_{xy} traces shown were taken at an elevated T ($\approx$320 mK) for which reliable data can be obtained. The difficulties with the Hall component data at lower T's will be discussed below.

A central point that can be observed in Fig. 2 is the clear similarity of the overall appearance of the traces in the two graphs. In particular, both sets of ρ_{xx} traces are characterized by a T-independent crossing point of the traces taken at different T's which, for the 1-0 transition, has been identified as the QHE-to-insulator transition point [10,11]. It is thus natural to associate the 2-1 transition ν, ν_c, with the crossing point of the ρ_{xx}^t traces observed in Fig. 2(a). Adopting this identification of ν_c, we now proceed to explore its consequences in the ρ and σ traces of Fig. 1.

It is immediately obvious that ν_c (dashed line in Fig. 1) is *not* at the ρ_{xx} peak. In fact, the position of the ρ_{xx}

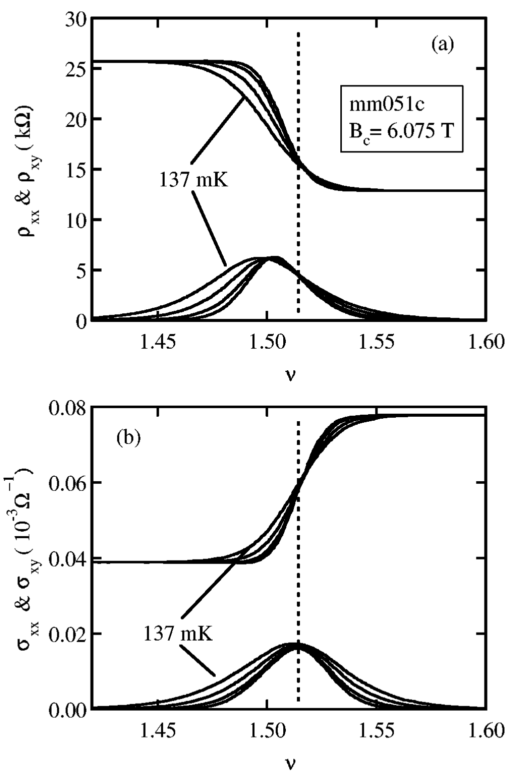

FIG. 1. (a) ρ_{xx} (lower curves) and ρ_{xy} vs ν taken in the vicinity of the $\nu = 2$ to 1 transition, at $T = 42$, 70, 101, and 137 mK. Note the narrowing of the transitions as T is lowered. (b) σ_{xx} and σ_{xy} vs ν, calculated from (a). The dashed lines in both (a) and (b) indicate ν_c, inferred from the data in Fig. 2(a) (see text).

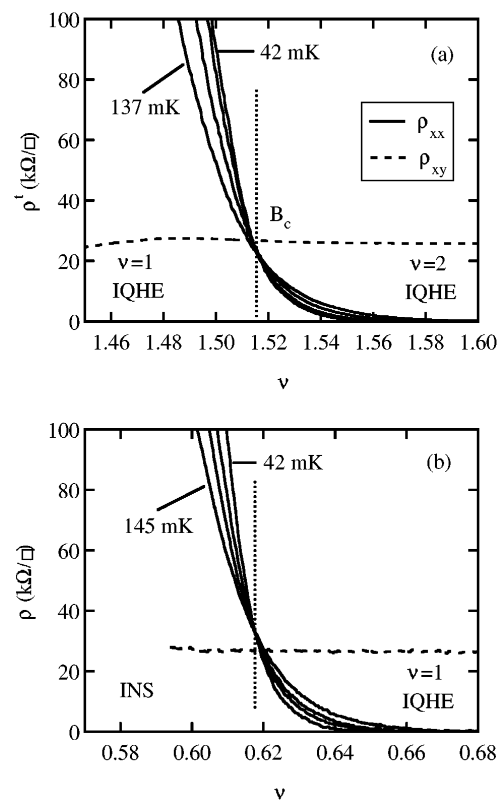

FIG. 2. (a) ρ_{xx}^t (solid lines) and ρ_{xy}^t (long-dashed line) for the 2-1 transition, calculated from the data in Fig. 1(a). The T for the ρ_{xx}^t data are 42, 70, 101, and 137 mK, and for the ρ_{xy}^t trace $T = 330$ mK. (b) Measured ρ_{xx} (solid lines) and ρ_{xy} (long-dashed line) for the 1-0 transition. The T for the ρ_{xx} data are 42, 84, 106, and 145 mK, and for the ρ_{xy} trace $T = 323$ mK. Dashed line in both (a) and (b) indicates the transition ν inferred from the common crossing point of the ρ_{xx}^t (or ρ_{xx}) traces.

peak is clearly T dependent, at the same T range where the crossing point in ρ^t_{xx} [see Fig. 2(b)] is not. Instead, a T-independent point can be seen at the high-ν shoulder of the ρ_{xx} peaks. It is this point, rather than the peak's center, that coincides with the transition as inferred from the ρ^t_{xx} traces. On the other hand, inspecting the σ_{xx} traces in Fig. 1(b) reveals that the σ_{xx} peak is centered around ν_c, and its position is much less T dependent. One can immediately draw the following conclusions: (a) associated with the 2-1 transition is a σ_{xx} peak, whose position is T independent (at low T), and is centered around ν_c, (b) a peak in ρ_{xx} also exists in the vicinity of the transition, but its center is offset by a T-dependent amount from ν_c, and (c) on the ρ_{xx} peak, the transition point is characterized by a T-independent point located at its shoulder. The fact that only one of either the ρ_{xx} or the σ_{xx} peak is symmetric is hardly surprising because obtaining one from the other involves the distinctly asymmetric Hall component. Here, we demonstrate that at low T, the σ_{xx} peak is the symmetric one.

We now turn to the Hall component of the transport. First, by inspecting Fig. 1, one can identify the existence of crossing points in both σ_{xy} and ρ_{xy}. For the σ_{xy} traces, the value at the crossing point is close to the midvalue between the two QHE plateaus [14], but for the ρ_{xy} traces it is not so, with the transition point clearly at a much lower value. The ν position of these crossing points coincides with ν_c, as it is identified from the crossing points of the ρ_{xx} traces.

Second, as reported previously, ρ_{xy} can remain constant and quantized across the transition, into the insulating phase, near both the 1-0 transition [18] and the $1/3$-fractional QH-to-insulator transition [19]. As seen in Fig. 2(a), ρ^t_{xy} indeed remains constant across the 2-1 transition, providing further evidence to the equivalence of the transitions. The intriguing possibility that ρ_{xy} remains quantized in the insulating regime is actually implicit in a recent theoretical work of Ruzin and his co-workers [20]. Their result is cast in the form of a "semicircle" law, which imposes a relation between the elements of the conductivity tensor for the 2-1 transition,

$$(\sigma^t_{xx})^2 + (\sigma^t_{xy})^2 = \frac{e^2}{h}\,\sigma^t_{xy}\,. \tag{4}$$

To test this we plot, in Fig. 3, σ^t_{xy} vs $(\sigma^t_{xx})^2 + (\sigma^t_{xy})^2$ for traces obtained at several T's from 26–222 mK. Overall, the data follow the expected straight line at higher T's, with systematic deviations at low σ_{xy} that becomes more significant as T is lowered. At present, we cannot delineate the source of these deviations [18] or decide whether the semicircle law is exact as $T \longrightarrow 0$, or just an approximation applicable at higher T's.

So far we discussed the similarity of the 2-1 and 1-0 transitions on a qualitative level. One way of extracting quantitative parameters from the QH-insulator transition is by conducting a scaling analysis of the data. In Fig. 4 we present such an analysis of our transitions. Using a scaling procedure similar to that used by Wong *et al.* [12] we plot ρ_{xx} for the 1-0 transition [Fig. 2(a)] and ρ^t_{xx} for the 2-1 transition [Fig. 2(b)] versus the scaling argument, $(\nu - \nu_c)/T^\kappa$. While ν_c can be directly obtained from the data, we vary the value of κ until we obtain the optimal "collapse" of the ρ_{xx} traces obtained at different T's. We emphasize that the data in Fig. 4 do not represent a significant test of scaling given its limited range. The sole intention of our presentation of the scaling analysis is to provide a quantitative measure of the similarity of the transitions which is the main

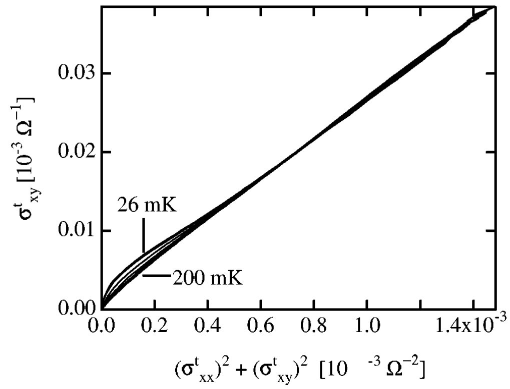

FIG. 3. A plot of σ^t_{xy} vs $(\sigma^t_{xx})^2 + (\sigma^t_{xy})^2$ for the 2-1 transition. A straight line indicates a semicircle relation between the conductivity components.

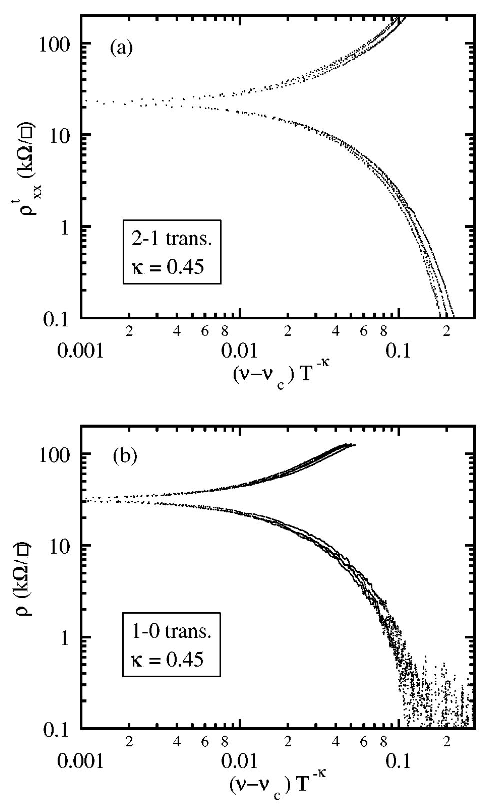

FIG. 4. A scaling analysis of the ρ^t_{xx} (a) and ρ_{xx} (b) data in Figs. 2(a) and 2(b), respectively, intended to quantify the similarity observed in Fig. 2 (see text). The scaling exponent κ, is determined to within 20%.

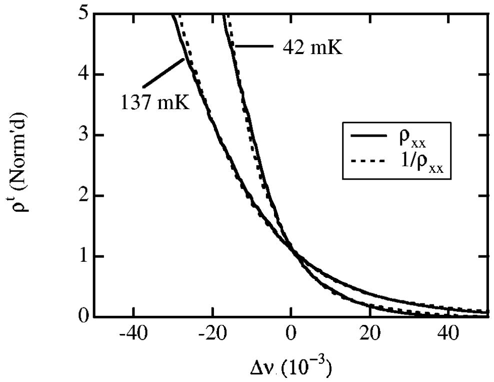

FIG. 5. ρ_{xx}^t and $1/\rho_{xx}^t$ vs $\Delta\nu = \nu - \nu_c$ at two temperatures. Note the symmetry between $\rho_{xx}(\Delta\nu)$ and $1/\rho_{xx}(-\Delta\nu)$.

topic of this Letter. The resulting values of the critical exponent, $\kappa = 0.45 \pm 0.05$, are indeed the same for both transitions. Incidentally, they are also in good agreement with results obtained in some of the studies of QHE transitions [1,12,13].

Another quantitative aspect of the transitions is their critical resistivity. Both transitions depicted in Fig. 2 have a value of the critical resistivity at B_c close to h/e^2, again in good agreement with studies of the 1-0 transitions [11]. As mentioned before, the issue of the universal critical resistivities for the inter-QHE is still under debate. One may argue that the process of eliminating the contribution of the lowest LL from the 2-1 data may result in an incorrect critical resistivity value for the ρ_{xx}^t obtained for that transition. To alleviate this concern we carefully inspect the raw ρ_{xx} data presented in Fig. 1(a). We recall that the scaling theory of the QH transitions predicts not one, but two, distinct universal critical amplitudes for the 2-1 transition. The first is ρ_{xx}^c, the value of ρ_{xx} at B_c, and the second is ρ_{xx}^p, the peak value of ρ_{xx} which, as discussed above, is distinct from ρ_{xx}^c. As can be seen, both are T independent at the low-T range shown, with $\rho_{xx}^c = 4.4$ kΩ and $\rho_{xx}^p = 6.25$ kΩ. Both these values are within 20% of the expected [21] values of $h/5e^2$ and $h/4e^2$, respectively, lending support to the universal character of the transitions.

Finally, we take advantage of our analysis to check whether the recently observed reflection symmetry of ρ_{xx} near the QH-to-insulator transitions also holds for ρ_{xx}^t. In Fig. 5 we replot ρ_{xx}^t, normalized to its transition value (20 kΩ), vs $\Delta\nu = \nu - \nu_c$ at two temperatures (solid curves), along with its inverse (dashed curves), plotted against $(-\Delta\nu)$. Clearly, the reflection symmetry is evident for this transition as well.

To summarize, in this work we have demonstrated the equivalence of the inter-QH, plateau-to-plateau transitions, to the QH-to-insulator transition that terminates the QH series, and tested the proposed semicircle relation between the longitudinal and Hall components of the conductivity tensor near the transitions.

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Note added.—After this paper was submitted, we carried out an alternative analysis of the data discussed here outside the framework of quantum critical scaling. This does not affect the central feature of our data that is of interest in this paper—the close connection between the 1-0 and 2-1 transitions—but specialists in quantum critical phenomena may wish to peruse a forthcoming publication [22].

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