Spin Squeezing in an Ensemble of Atoms Illuminated with Squeezed Light

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We propose an experimentally feasible way to generate spin squeezing in an ensemble of V-type atoms. The proposal involves absorption of a squeezed light beam, and it does not require a large solid angle to be occupied by squeezed light. 50% of the amount of squeezing of the optical field can be transferred onto spin squeezing of the excited atomic states. The analogy with the input-output theory for quantum fields is used to elucidate this result. An experimental setup for generation and detection of spin squeezing within magnetic or hf manifolds is outlined. [S0031-9007(97)04808-4]

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The extensive research activity devoted during the last decade to generation and applications of nonclassical squeezed states of the electromagnetic field [1] has been paralleled by the quest for similar states in other physical systems, in particular, spin squeezed states (SSS) of atoms [2,3]. Just as squeezed states of light provide a reduction in the measurement uncertainty due to quantum fluctuations of light, SSS of atoms and ions hold the promise to overcome uncertainties introduced by quantum fluctuations in atomic systems. Atomic noise has been observed in a variety of experiments [4–7], therefore, its reduction becomes a practical problem for sensitive measurements. More specifically, possible applications of SSS include improved sensitivity of atom interferometry, Ramsey separated oscillatory fields spectroscopy, and atomic spin polarization measurements [3,7,8].

For a system of two level atoms the spin components are defined as

$$F_x = \frac{1}{2}(F_{12} + F_{21}),$$
$$F_y = \frac{-i}{2}(F_{12} - F_{21}), \qquad (1)$$
$$F_z = \frac{1}{2}(F_{11} - F_{22}),$$

with collective operators $F_{ij} = \sum |i\rangle\langle j|, \quad i,j = 1,2$ summed over all atoms in the sample.

We apply the definition of spin squeezing of [2,9] where the collective spin of a system of spin-1/2 particles is considered squeezed if the variance of a component of the total spin perpendicular to the direction of the mean spin is less than the standard quantum limit defined for a coherent spin state, $\Delta F_i < \sqrt{F/2}$. For trapped ions it has been suggested to use the coupling via their translational degrees of freedom to impose correlation between the individual spins and in this way to obtain a spin squeezed state [3,9]. Oscillatory exchange of squeezing between nondecaying atoms and squeezed cavity modes in the framework of the Jaynes-Cummings model has been discussed in [9,10].

Spin squeezing via interaction of neutral two-level atoms with squeezed vacuum in free space has been considered in [11]. However, this proposal requires squeezed light to occupy a major part of the full solid angle of vacuum modes which match the atomic dipole pattern. Although considerable experimental progress has been achieved in the generation of frequency tunable squeezed light and its applications in atomic physics experiments [12,13], the existing sources generate a Gaussian beam of squeezed light which is difficult to mode match to the atomic dipole pattern. This "4π" problem is common for a variety of theoretical proposals concerning the dynamics of an atom driven by nonclassical light [14].

In this Letter we consider the effect of *driving* V atoms with a normal Gaussian beam of squeezed light, which leads to spin squeezing within the pair of excited states with no optically driven transition between them (see inset in Fig. 1). The light absorption process creates an entanglement of the field and the individual atoms, and when all the light is absorbed in the sample we realize a steady state multiparticle entanglement, which as we shall

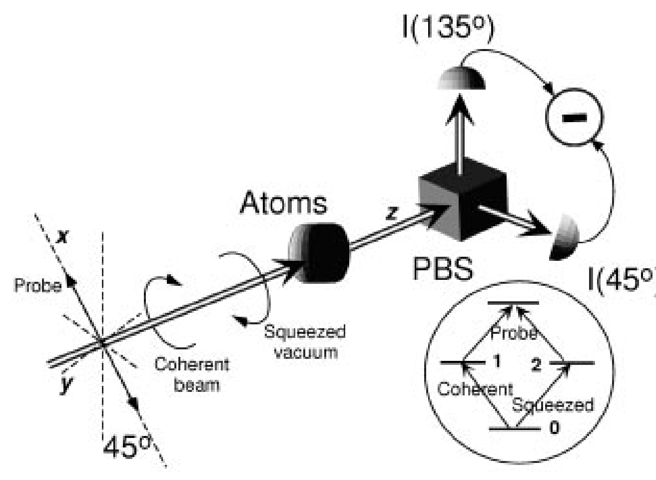

FIG. 1. Detection of spin squeezing in a polarization experiment. σ_- coherent and σ_+ squeezed beams fully absorbed at 0-1, 2 transitions create a squeezed spin state within the atomic substates $1(m = -1)$ and $2(m = 1)$. For the correct choice of the phase between the coherent and squeezed beams the F_y spin component is squeezed. Quantum noise of the probe difference photocurrent $I_- = I(45°) - I(135°)$ is then reduced beyond the limit set by the coherent spin state fluctuations, thus demonstrating SSS of atoms (more details in the text).

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show is only partly deteriorated (50%) by the effect of atomic spontaneous emission. We stress that considering spin squeezing within the final states of the transition is crucial because the large initial state population of atoms provides a dominant nonsqueezed contribution to the collective optical atomic coherence. Spin squeezing within a close pair of atomic states is, in fact, the situation relevant for precision spin measurements in frequency standards, magnetometers, etc.

Interaction of an ensemble of atoms with free space quantized field modes.—We consider first the interaction of a field mode propagating along the z axis through an ensemble with number density ρ of two-level atoms with the lower state $|0\rangle$ and upper state $|1\rangle$. The field is described by the continuous annihilation operator $a(z,t)$, and the atoms are characterized by the atomic operators $\sigma_{ij} = |i\rangle\langle j|$, $i,j = 0,1$. To describe the interaction of the field with the atoms we use the approach of [15], where continuous atomic operators are introduced:

$$\sigma_{ij}(\vec{r},t) = \frac{1}{\rho\Delta V}\sum_{\mu}\exp\left[i\,\frac{\omega_{ij}}{c}\,(z - z^{\mu})\right]\sigma_{ij}^{\mu} \quad (2)$$

and where the sum is performed over atoms, enumerated by the superscript μ, enclosed in the volume ΔV around the position $\vec{r}$. The phase factor on each term in the sum, where z^{μ} is the z coordinate of the μth atom, ensures that the volume ΔV may extend over many optical wavelengths, being still "infinitesimal" on the length scale of field changes due to absorption and dispersion. ω_{ij} is the Bohr frequency between the atomic levels i and j. We separate the transverse mode function $u(x,y)$ assumed not to depend on z over the extent of the atomic sample. In the slowly varying amplitude and phase approximation the evolution of the continuous field operator is then described by the equation

$$\left(\frac{1}{c}\,\frac{\partial}{\partial t} + \frac{\partial}{\partial z}\right)a(z,t) = g\rho\int\int dx\,dy\,u(x,y)\sigma_{01}(\vec{r},t)\,. \quad (3)$$

We chose a definition of $a(z,t)$ so that $a^{+}(z,t)a(z,t)$ provides the flux of photons and the relation between continuous field operator and the operator of the incident field is simply $a(z=0,t) = a^{\mathrm{in}}(t)$.

The Heisenberg equation of motion for the continuous atomic dipole operator is readily obtained from the usual single-atom equations [16],

$$\begin{aligned}\dot{\sigma}_{01}(\vec{r},t) = &-\frac{1}{2}\,\gamma\sigma_{01} - i\omega_{01}\sigma_{01}\\ &+ gu(x,y)a(\sigma_{11} - \sigma_{00})\\ &+ \sqrt{\gamma}\,(\sigma_{11} - \sigma_{00})b^{\mathrm{in}},\end{aligned} \quad (4)$$

where the interaction constant g and the atomic decay rate γ are given by the atomic dipole moment and the transition frequency ω_{01}, and where we have introduced continuous operators of atomic noise $b^{\mathrm{in}}(\vec{r},t), b^{\mathrm{in}\,+}(\vec{r},t)$ in the same manner as the atomic operators in (2). Supposing the noise operators $b^{\mu}(t)$ for different atoms to be independent we have the commutation relations

$$\rho[b^{\mathrm{in}}(\vec{r},t), b^{\mathrm{in}\,+}(\vec{r}_1,t_1)] = \delta(\vec{r} - \vec{r}_1)\delta(t - t_1)\,. \quad (5)$$

We now suppose that the field is so weak that in the equation for $\sigma_{01}(z,t)$ we can omit the terms involving the upper-state populations and make the replacement $\sigma_{00}(\vec{r},t) - \sigma_{11}(\vec{r},t) = 1$. The equation (4) then becomes linear and this equation and Eq. (3) can be solved by introducing the Fourier transform of $\sigma_{01}, b^{\mathrm{in}}, a$,

$$\sigma_{01}(\vec{r},\omega) = -\frac{gu(x,y)a(z,\omega) + \sqrt{\gamma}\,b^{\mathrm{in}}(\vec{r},\omega)}{\frac{1}{2}\,\gamma - i(\omega - \omega_{01})}\,, \quad (6)$$

$$\begin{aligned}a(z,\omega) = &\ a(0,\omega)e^{ik(\omega)z}\\ &- \int_0^z dz_1\,\frac{\sqrt{\gamma}\,g\rho\,e^{ik(\omega)(z-z_1)}}{\frac{1}{2}\,\gamma - i(\omega - \omega_{01})}\,b^{\mathrm{in}}(z_1,\omega)\,,\end{aligned} \quad (7)$$

where we used the normalization of the transverse mode function $\int\int dx\,dy\,u^2(x,y) = 1$ and redefined the continuous noise operators as $b^{\mathrm{in}}(z,\omega) = \int\int dx\,dy\,u(x,y)b^{\mathrm{in}}(\vec{r},\omega)$ with commutation relation $\rho[b^{\mathrm{in}}(z,\omega), b^{\mathrm{in}\,+}(z_1,\omega_1)] = \delta(z - z_1)\delta(\omega - \omega_1)$. In Eq. (7) we have introduced $ik(\omega) = i\omega/c - g^2\rho/[\frac{1}{2}\,\gamma - i(\omega - \omega_{01})]$.

The expectation values of the expressions in Eq. (7) reveal the usual expression for the dispersion and damping of a classical field in a gas of two-level atoms. When the result is substituted back in Eq. (6) we obtain the position dependent dipole and after integration over space the collective atomic dipole of the sample. Although the collective dipole F_{01} acquires some of the noise properties of the incident field, we cannot relate this to the definition of SSS given in the introduction because the main fraction of atoms stays in the ground state and their fluctuations are not affected by the light.

Instead, the above results will be used in the discussion of a V system, where spin squeezing in a conventional sense can be introduced consistently. We consider a V transition with each arm $0 \leftrightarrow 1(2)$ interacting with a separate quantum field $a_{1(2)}$ and we focus on the collective atomic operators

$$F_{ij} = \sum_{\mu}\sigma_{ij}^{\mu} = \rho\int d\mathbf{r}\,\sigma_{ij}(\mathbf{r}), \qquad i,j = 1,2\,. \quad (8)$$

We restrict ourselves to the case where the 0-2 transition interacts with a squeezed vacuum and the 0-1 transition interacts with a coherent field. To simplify our analysis, we assume that the two modes have identical transverse mode functions $u(x,y)$ and that the coherent amplitude is much stronger than the squeezed vacuum fluctuations so that the coherent part of σ_{01}, with $\langle\sigma_{10}(\vec{r},t)\rangle = \langle\sigma_{10}(\vec{r},t=0)\rangle e^{-i\omega_1 t}$, is much greater than the fluctuating quantity σ_{02}. Linearizing the operator products contributing to $\sigma_{12}(\vec{r},t)$ then yields

$$\sigma_{12}(\vec{r},t) = \langle\sigma_{10}(\vec{r},t)\rangle\sigma_{02}(\vec{r},t)\,,$$
$$\sigma_{12}(\vec{r},\Delta = \omega - \omega_1) = \langle\sigma_{10}(\vec{r},t=0)\rangle\sigma_{02}(\vec{r},\omega)\,. \quad (9)$$

Equations (6) and (7)) applied to each of the two transitions in the V system now yield

$$\sigma_{12}(\vec{r},\Delta) = \frac{gu(x,y)\rho(\alpha e^{ik(\omega_1)z})^*}{(\frac{\gamma}{2} + i(\omega_1 - \omega_{01}))(\frac{\gamma}{2} - i(\omega - \omega_{02}))} \times \left[gu(x,y)a_2^{\rm in}(\omega_2)e^{ik(\omega)z} + \sqrt{\gamma}\,b_2^{\rm in}(\vec{r},\omega) - \frac{g^2u(x,y)\rho\sqrt{\gamma}}{\frac{\gamma}{2} - i(\omega - \omega_{02})}\int_0^z dz_1 b_2^{\rm in}(z_1,\omega)e^{ik(\omega)(z-z_1)}\right]. \quad (10)$$

We assume complete absorption of the light fields in the atomic gas so that the z integral can be extended to infinity when Eq. (10) is inserted into Eq. (8). It is convenient to change the order of integration of the ensuing double integral $\int_0^\infty dz\int_0^z dz_1\cdot \to \int_0^\infty dz_1\int_{z_1}^\infty dz\cdot$ to perform the z integration of c-number functions first and to subsequently replace the symbol z_1 by z. Assuming for simplicity that $\omega_1 = \omega_{01} = \omega_{02}$ we thus obtain

$$F_{12}(\Delta) = \int_0^\infty dz\,\frac{g\rho(\alpha e^{ik(\omega_1)z})^*}{\frac{\gamma}{2}(\frac{\gamma}{2} - i\Delta)} \times \left[ga_2^{\rm in}(\omega)e^{ik(\omega)z} + \sqrt{\gamma}\,b_2^{\rm in}(z,\omega)\left(1 + \frac{g^2\rho}{\frac{\gamma}{2} - i\Delta}\,\frac{1}{\frac{i\Delta}{c} - \frac{g^2\rho}{\gamma/2} - \frac{g^2\rho}{\gamma/2 - i\Delta}}\right)\right]. \quad (11)$$

The noise operators $b_2^{\rm in}(z,\omega), b_2^{\rm in+}(z,\omega)$ are δ correlated in frequency and space. Thus any spatial integral of these operators will be δ correlated in frequency, and the noise operator defined as $d_2^{\rm in}(\omega) = \sqrt{2\,{\rm Im}[k(\omega_1)]\rho}\int_0^\infty dz(e^{ik(\omega_1)z})^* b_2^{\rm in}(z,\omega)$ is readily shown to satisfy the commutator relation $[d_2^{\rm in}(\omega), d_2^{\rm in+}(\omega')] = \delta(\omega - \omega')$. For practical cases of interest, $g^2\rho \gg \gamma\Delta/c$, implying that the absorption is complete and that light travels through the sample fast as compared to the atomic spontaneous decay rate. Under this assumption we finally obtain

$$F_{12}(\Delta) \simeq \frac{\alpha^*}{\gamma - i\Delta}\,a_2^{\rm in}(\omega_1 + \Delta) + \frac{\alpha^*}{\gamma - i\Delta}\,d_2^{\rm in}(\omega_1 + \Delta)\,. \quad (12)$$

Equation (12) is the main result of the paper. It shows that the collective atomic operator F_{12} obtains even noise contributions from a white noise reservoir, $d_2^{\rm in}$, and from the input beam $a_2^{\rm in}$. It is hence clear that any noise reduction in $a_2^{\rm in}$ compared to, e.g., a coherent field or a vacuum field will manifest itself in the spin noise as measured by F_{12}. Equation (12) is in fact identical to the expression for the intracavity field in the resonator input/output theory [17], and the variances of the pseudospin components are therefore simply linked to the quadrature phase variances $X_{+,-}^2$ of the input field a_2:

$$\langle F_{x,y}^2\rangle = \tfrac{1}{4}\langle F_z\rangle(4X_{+,-}^2 + 1)\,. \quad (13)$$

For the vacuum input field a_2 the variances $4X_{+,-}^2 = 1$ and $\langle F_{x,y}^2\rangle = \frac{1}{2}\langle F_z\rangle = \frac{1}{2}\langle F\rangle$ as they should be for the coherent spin state. With a broadband squeezed vacuum as an input field a_2, when, e.g., $X_+^2 = 0$, we obtain the spin squeezed state with 50% squeezing: $\langle F_x^2\rangle = \frac{1}{4}\langle F\rangle$. The analogy with the resonator goes beyond the resemblance of equations. In the resonator case, the 50% reduction is due to the intracavity field being coupled both to the incident and the reflected field modes at the input mirror. In the atom case, the atomic dipoles are similarly coupled to both the incident field and the scattered field modes (spontaneous emission). Like a resonator, the atomic medium interacts with the squeezed light in a frequency selective way, and a bandwidth of squeezing of a few γ is effectively "broad" and still in accordance with our assumption of complete absorption.

Discussion and an experimental proposal.—In our proposal SSS are generated for atoms in the final states of the transitions driven by quantum correlated excitation (states 1, 2 of the previous section). To observe it we need to address only these atoms in our measurement procedure. One way to do this is to perform the measurement of the quantum noise of a transmitted light probe quasiresonant with the transitions for which 1, 2 are the lower states. The layout for SSS observation in such an experiment is shown in Fig. 1. Levels 1 and 2 are specified to be magnetic sublevels with $m = -1, 1$ of an excited state with angular momentum $J = 1$. The coherent and the squeezed components of the exciting fields are σ_- and σ_+ polarized, respectively. The spin components $F_{x,y}$ (1) now physically correspond to certain components of the alignment tensor in the $J = 1$ excited state, $F_x \propto T_{-2}^2 + T_2^2$ and $F_y \propto T_{-2}^2 - T_2^2$ [18]. To measure, e.g., F_y, a probe linearly polarized along x is analyzed with a polarizing beam splitter oriented at 45° to the x axis rendering the intensities $I(45°)$ and $I(135°)$

[19]. For the resonant probe $I_- = I(45°) - I(135°) = \alpha I_0 \gamma' F_y$, where α is a constant proportional to the optical depth of the medium, I_0 is the probe intensity and γ' is the width of the transition. Obviously, quantum noise of I_- is determined by the quantum noise of F_y. When F_y is squeezed the noise of I_- drops below the level set by the coherent spin state fluctuations (13) demonstrating the spin squeezed state of the atomic ensemble. The quantum noise of I_- will contain also the shot noise of the probe, independent of the atoms, but this contribution can be suppressed by a suitable lock-in technique, e.g., by modulating the atomic medium. One can imagine many other geometries of the experiment for which different components of the (magnetic manifold) density matrix are squeezed.

We believe that the main assumptions in the paper are realistic: complete absorption is achievable, e.g., for cold atoms in a magneto-optical trap. Optical depth $\alpha = \lambda^2 nl/4\pi^2 \simeq 20$ for the density $n = 3 \times 10^{11}$ cm^{-3}, and the length $l = 0.5$ cm of the trap. The back action of the probe on the spin ensemble can be qualitatively addressed by assuming that every act of absorption of a probe photon destroys spin squeezing as an additional (to the spontaneous emission to level 0) channel of dissipation for SSS. This additional channel is irrelevant if $\gamma' s \ll \gamma$ where s is the saturation parameter of the probe. That is, if the probe transition and the nonclassical excitation transition have comparable rates $\gamma' \simeq \gamma$ we must fulfill the weak probe condition $s \ll 1$.

As mentioned at the beginning of this Letter the collective atomic correlations present in SSS can be important in diverse areas of atomic physics. The generation of quantum correlated atomic states via complete absorption may be of interest with other nonclassical states of light and with emphasis on other properties of the atomic sample, e.g., twin beam absorption may lead to reduced partition noise in a "quantum beam splitter" for atom interferometry. Besides the enhanced sensitivity of measurements the mapping of light statistics onto atomic states may be relevant for quantum computing and quantum communication, where our proposal is a "free-space" alternative to cavity QED methods [20].

Let us note that it is important for the type of spin squeezing of interest that the entire sample is considered. A single atom in a squeezed light field, or a thin layer of atoms in the sample, only absorbs a minute fraction of the light, and hence the squeezing properties of the light are not sufficiently transferred to the atoms.

As a closing remark we note that the limit of 50% efficiency of "quantum mapping" which is intrinsic for the direct absorption situation considered in the present paper may very well not be the general limit. For example, adiabatic population transfer or Raman-type transitions combined with our requirement of total absorption of the nonclassical input light could be promising candidates for further spin squeezing. Such a scheme involving two quantum correlated fields in two Raman-type transitions will be of direct relevance for spin squeezing of ground state substates in frequency standards and magnetometers.

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