Minimal SO(10) Unification

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It is shown that the doublet-triplet splitting problem can be solved in SO(10) with only a single adjoint Higgs field (as seems to be required by superstring theory), while at the same time avoiding the existence of light fields that would destroy the unification of gauge couplings. [S0031-9007(97)04770-4]

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The striking unification of gauge couplings [1] at about 10^{16} GeV in the minimal supersymmetric standard model (MSSM) points toward the possibility of a supersymmetric grand unified theory (SUSY GUT).

SO(10) is generally thought to be the most attractive grand unified group for a number of reasons. It achieves complete quark-lepton unification for each family, explains the existence of right-handed neutrinos and of "seesaw" neutrino masses, has certain advantages for baryogenesis, in particular since *B-L* is broken [2], and has the greatest promise for explaining the pattern of quark and lepton masses [3].

The greatest theoretical problem that any grand unified theory must face is the gauge hierarchy problem [4], and in particular that aspect of it called the "doublet-triplet splitting problem" [5], or $2/3$ splitting problem for short. The only way to achieve natural $2/3$ splitting in SO(10) is the Dimopoulos-Wilczek (DW) mechanism [6]. In Ref. [7] it was shown that realistic SO(10) models can be constructed using the DW mechanism.

One criticism that is sometimes made about SO(10) is that solving the $2/3$ splitting problem requires a somewhat complicated Higgs structure. The models constructed in Refs. [7] and [8] contained at least the following Higgs multiplets: three adjoints (**45**), two rank-two symmetric tensors (**54**), a pair of spinors ($\mathbf{16} + \overline{\mathbf{16}}$), two vectors (**10**), and several singlets. Aside from the issue of simplicity, there are some indications that it may be impossible to construct grand unified models with a multiplicity of adjoint fields from superstring theory [9]. What we show in this Letter is that a satisfactory $2/3$ splitting can be achieved with only a *single* adjoint.

The necessity of a complicated Higgs structure is largely traceable to one technical problem, namely the breaking of the rank of SO(10) from five to four without destabilizing the DW solution of the gauge hierarchy problem. The complete breaking of SO(10) to the standard model requires at least two sectors of Higgs: an adjoint sector and a spinor sector. The adjoint sector plays the central role in the DW mechanism for $2/3$ splitting, while the spinor sector both breaks the rank of SO(10) and gives right-handed neutrinos mass.

The dilemma is that if the spinor sector is coupled to the adjoint sector it tends to destabilize the DW form of the adjoint vacuum expectation value required for the $2/3$ splitting, while if the two sectors are *not* coupled (or coupled very weakly [10,11]) to each other in the superpotential there arise colored and charged (pseudo)goldstone fields which have a disastrous effect on the running of the gauge couplings [10].

In Refs. [7] and [8], an indirect way to couple the two sectors together without destabilizing the hierarchy was found. However, this solution to the problem involved a somewhat complicated Higgs structure including at least three adjoint fields.

In this Letter we show that there is a very simple way to couple the spinor and adjoint sectors together, with only a single adjoint, with a stable hierarchy, and with no pseudogoldstones. Before describing it, it will be helpful to review in more detail the problems that have been discussed above.

The problem of a stable hierarchy in SO(10).—The DW mechanism is based on the existence of an adjoint Higgs field, which we shall call A, getting a vacuum expectation value (VEV) in the *B-L* direction:

$$\langle A \rangle = \begin{pmatrix} 0 & & & & \\ & 0 & & & \\ & & a & & \\ & & & a & \\ & & & & a \end{pmatrix} \otimes i\tau_2 , \tag{1}$$

where $a \sim M_G$, the unification scale. The lower-right 3×3 block corresponds to SU(3) of color, and the upper-left 2×2 block to weak SU(2). When this adjoint is coupled to vector representations, which we will call T_1 and T_2, by terms such as $T_1 A T_2$ the color triplets in the vectors are given superlarge masses, while the weak doublets remain massless. (By having also a term of the form $M_T T_2^2$ it is ensured that only one pair of weak doublets remains light. The proton-decay amplitude

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from the exchange of colored Higgsinos is proportional to M_T/a^2, so that if $M_T/a \lesssim 10^{-1}$ it is sufficiently suppressed [7,8].) Such a "DW form" for the adjoint VEV is not possible in SU(5), where $\mathrm{tr}(A) = 0$.

An adjoint alone is not sufficient to break SO(10) to the standard model group, G_{SM}, and in particular cannot reduce the rank of the group. This requires either spinorial Higgs ($\mathbf{16} + \overline{\mathbf{16}}$) or rank-five antisymmetric tensor Higgs ($\mathbf{126} + \overline{\mathbf{126}}$). As the latter tend to destroy the perturbativity of the unified interactions below the Planck scale, we will assume that the rank-breaking sector has spinors, which we shall call C and $\overline{C}$. These spinors (also necessary to give mass to the right-handed neutrinos) have VEVs in the SU(5)-singlet direction.

The spinor VEVs break SO(10) down to SU(5), and thus the sector of the superpotential which depends on C and $\overline{C}$ but not on A, which we shall call W_C, leaves massless at least those components of the spinors in the coset SO(10)/SU(5). That is just a $\mathbf{10} + \overline{\mathbf{10}} + \mathbf{1}$ of SU(5), or a $[(3,2,\frac{1}{6}) + (\overline{3},1,-\frac{2}{3}) + (1,1,+1) + \mathrm{H.c.}] + (1,1,0)$ of G_{SM}.

The adjoint A, with the VEV shown in Eq. (1), breaks SO(10) down to $\mathrm{SU}(3)_c \times U(1)_{B-L} \times \mathrm{SO}(4)$. $[\mathrm{SO}(4) = \mathrm{SU}(2)_L \times \mathrm{SU}(2)_R.]$ The part of the superpotential that depends on A but not on the spinors, which we shall call W_A, thus leaves massless at least those components of the adjoint in the cosets $\mathrm{SO}(10)/[\mathrm{SO}(6) \times \mathrm{SO}(4)]$ and $\mathrm{SO}(6)/[\mathrm{SU}(3)_c \times U(1)_{B-L}]$. The first of these cosets consists of a $(\mathbf{6},\mathbf{4})$ of $\mathrm{SO}(6) \times \mathrm{SO}(4)$, which contains $[(3,2,\frac{1}{6}) + (3,2-\frac{5}{6}) + \mathrm{H.c.}]$ of G_{SM}. The second coset consists of $[(\overline{3},1,-\frac{2}{3}) + \mathrm{H.c.}]$ of G_{SM}.

Thus, with no coupling between the two sectors, there are extra, uneaten goldstone fields in $(3,2,\frac{1}{6}) + (\overline{3},1,-\frac{2}{3}) + \mathrm{H.c.}$ To avoid these, the adjoint must couple to the spinor. The obvious way to couple them together, by the term $g\overline{C}AC$, directly destabilizes the DW form assumed for $\langle A\rangle$. Let us assume that A has the form $\mathrm{diag}(b,b,a,a,a) \otimes i\tau_2$. $b = 0$ is the desired DW form. The VEVs of the spinors point in the SU(5)-singlet direction and have magnitude $c_0 \sim M_G$. Then $g\overline{C}AC = -\frac{g}{2}(2b + 3a)c_0^2$. The terms W_A must have a form that gives $\partial W_A/\partial b = O(M_G)b$, so that by themselves they would give $b = 0$. But taking into account also the coupling term $g\overline{C}AC$, one has $0 = -F_b^* = \partial W_{\mathrm{tot}}/\partial b = O(M_G)b - gc_0^2$, or $b \sim gM_G$. In the DW mechanism [7] the Higgs doublets get a seesaw mass of order b^2/M_{GUT}, so that g must be less than about 10^{-7}. This is the assumption made in Ref. [11]. This leads to the pseudogoldstone fields getting masses only of order $gM_{\mathrm{GUT}} < 10^9$ GeV, and thus to $\sin^2\theta_W = 0.2415$.

The spinor sector and adjoint sector must be coupled together in some more subtle way. In Ref. [7] such a way was proposed. There the spinor sector was assumed to contain a different adjoint Higgs, called A', whose VEV points in the SU(5) singlet direction. The two sectors (namely the A sector and the $(C,\overline{C},A')$ sector) were then coupled together by a term $\mathrm{tr}\, AA'A''$, where A'' was a third adjoint. Because the $AA'A''$ is totally antisymmetric under the interchange of any two adjoints (due to the fact that the adjoint is an antisymmetric tensor), there have to be three distinct adjoints in this term. This antisymmetry ensures, as it is easy to see, that this term does not contribute to any of the F terms as long as the VEVs of the three adjoints commute with each other. Therefore it does not destabilize the DW form of the VEV of A. And yet, it can also be shown that this trilinear term is sufficient to prevent the existence of any pseudogoldstone fields.

This has been the choice until now: to assume a complicated Higgs sector with at least three adjoint Higgs fields [7,12] or to assume that light pseudogoldstones exist which have a disastrous effect on the unification of couplings [10,11,13].

Solving the problem.—The solution to the above difficulty turns out to be remarkably simple. Let there be a single adjoint field A, and *two* pairs of spinors, $C + \overline{C}$ and $C' + \overline{C}'$. The complete Higgs superpotential is assumed to have the form

$$W = W_A + W_C + W_{ACC'} + (T_1AT_2 + ST_2^2)\,. \qquad (2)$$

The precise forms of W_A and W_C do not matter, as long as W_A gives $\langle A\rangle$ the DW form, and W_C makes the VEVs of C and $\overline{C}$ point in the SU(5)-singlet direction. For specificity we will take $W_C = X(\overline{C}C)^2/M_C^2 + f(X)$, where X is a singlet, and $f(X)$ is a polynomial in X containing at least a linear term. [Other forms for W_C are possible, such as $X(\overline{C}C - P_C^2)$, where $\langle P_C\rangle \sim M_G$.] We take $W_A = \frac{1}{4M}\,\mathrm{tr}\,A^4 + \frac{1}{2}M_A\,\mathrm{tr}\,A^2$. (There are arguments against explicit mass terms for adjoint fields in superstring theory [9], but one can simply replace M_A here by some singlet field.) The terms $(T_1AT_2 + ST_2^2)$ are a standard part of the 2/3 splitting mechanism in SO(10) models, as noted above.

The crucial terms that constitute the new mechanism we propose for coupling the spinor and adjoint sectors together have the form

$$W_{ACC'} = \overline{C}'[(P/M_1)A + Z_1]C + \overline{C}[(P/M_2)A + Z_2]C'\,, \qquad (3)$$

where Z_1, Z_2, and P are singlets, and $\langle P\rangle$ is assumed to be of order M_G. A critical point is that the VEVs of the primed spinor fields will vanish, and therefore the terms in Eq. (3) will not make a destabilizing contribution to $-F_A^* = \partial W/\partial A$.

The VEV of A is determined by the equation $0 = -F_A^* = \frac{1}{M}A^3 + M_AA$. If $\langle A\rangle = \mathrm{diag}(a_1,a_2,a_3,a_4,a_5) \otimes i\tau_2$, then for each i one has that $a_i^2 = 0$ or $MM_A (\equiv a^2)$. There is, therefore, a discrete vacuum degeneracy. The DW vacuum is obtained if two of the a_i's vanish and the other three have the same sign and magnitude a.

$F_X = 0$ implies that $\langle\overline{C}C\rangle^2 = -M_C^2f'$. The D terms and soft, SUSY-breaking terms will ensure that

$\langle \overline{C} \rangle \cong \langle C \rangle$. We take these VEVs to be of order the GUT scale. The F_C and $F_{\overline{C}}$ equations imply that $\langle X \rangle = 0$.

The most interesting equations are

$$0 = -F^*_{C'} = [(P/M_1)A + Z_1]C \,, \tag{4}$$

and

$$0 = -F^*_{\overline{C}'} = \overline{C}[(P/M_2)A + Z_2] \,. \tag{5}$$

It is necessary only to consider the first of these two equations, as they have the same structure. Let the VEV of C be decomposed as follows: $\langle C \rangle = \sum_K f_K C_K$, where the C_K are the irreducible multiplets of G_{SM} and the f_K are numerical coefficients. Since we have chosen the DW form for $\langle A \rangle$, Eq. (4) can be written

$$\left[\frac{3}{2}\, a(P/M_1)\,(B-L)_K + Z_1\right] f_K = 0 \,, \tag{6}$$

for all K. Since $\langle \overline{C} C \rangle \neq 0$, not all the f_K vanish. Suppose f_J does not vanish. Then Z_1 is fixed to be $Z_1 = -\frac{3}{2} a(\frac{P}{M_1})\,(B-L)_J$. This, in turn, implies that $f_K = 0$ for all K for which $(B-L)_K \neq (B-L)_J$. There are, therefore, a discrete number of solutions; in fact, four. One of them is $Z_1 = -\frac{3}{2} a\left(\frac{P}{M_1}\right)$, with $\langle C \rangle$ pointing in any direction which has $B - L = 1$. There is a two-complex-dimensional space of such directions. But actually these are all gauge equivalent. Thus, we can take the VEV of C to lie in the $SU(5)$-singlet direction without any loss of generality.

Now we will show that $\langle C' \rangle = \langle \overline{C}' \rangle = 0$. That C' and $\overline{C}'$ have no VEV in the SU(5)-singlet direction follows from $F_{Z_1} = F_{Z_2} = 0$. And from the SU(5)-nonsinglet components of the F_C and $F_{\overline{C}}$ equations it follows that C' and $\overline{C}'$ have no VEVs in the SU(5)-nonsinglet directions, either. All VEVs have now been fixed except for the one linear combination of P, Z_1, and Z_2 that is orthogonal to the linear combinations that are fixed by the $F_{C'}$ and $F_{\overline{C}'}$ equations. [See Eq. (6).] This VEV is not determined by the terms we have so far written down. One can add additional terms to W to fix this singlet VEV. It is also possible that it is determined by radiative effects when supersymmetry breaks. As noted above, we assume that it is of order M_G.

Knowing the VEVs, one can now read off the Higgsino mass matrices directly from W. For the representations $K = (3, 2, \frac{1}{6})$, $(\overline{3}, 1, -\frac{1}{3})$, and $(1, 1, +1)$, which are contained in the **10** of SU(5), one has 3×3 mass matrices, since such representations exist in the adjoint A and in the spinors C and C'. The masses come from the terms in Eq. (3), both through the VEVs of A, Z_1, and Z_2, and through the VEVs of the spinors C and $\overline{C}$.

$$W_{\mathrm{mass},10}(K) = (A_{\overline{K}}, \overline{C}_{\overline{K}}, \overline{C}'_{\overline{K}}) \begin{pmatrix} m_K & 0 & \langle \overline{C} \rangle \langle P \rangle / \sqrt{2}\, M_2 \\ 0 & 0 & \alpha_K a \langle P \rangle / M_2 \\ \langle C \rangle \langle P \rangle / \sqrt{2}\, M_1 & \alpha_K a \langle P \rangle / M_1 & 0 \end{pmatrix} \begin{pmatrix} A_K \\ C_K \\ C'_K \end{pmatrix} . \tag{7}$$

Here $\alpha_K \equiv \frac{3}{2}[(B-L)_K - 1]$, and takes the values -1, -2, and 0, respectively, for $K = (3, 2, \frac{1}{6})$, $(\overline{3}, 1, -\frac{1}{3})$, and $(1, 1, +1)$. The entry m_K vanishes for the color-triplet values of K, since W_A has goldstone modes in those directions, but is nonzero (and in fact equal to $a^2/2M$) in the $(1, 1, +1)$ direction. Thus for each K the 3×3 mass matrix has one vanishing eigenvalue, corresponding to a goldstone mode that gets eaten by the Higgs mechanism, and two nonvanishing GUT-scale eigenvalues. One sees also that the massless mode for $K = (1, 1, +1)$ is purely in the C direction, as it should be since only the spinor VEVs break that generator, while for the mass matrices of the color-triplet representations the massless mode is a linear combination of the adjoint and spinor as it should be.

As for the representations $(1, 2, -\frac{1}{2})$ and $(\overline{3}, 1, \frac{1}{3})$ that are contained in the $\overline{\mathbf{5}}$ of SU(5), and their conjugates, they are contained only in the spinors and obtain mass only from the VEVs of A, Z_1, and Z_2. It is easy to see that the weak doublets get mass of $3a\langle P \rangle / M_i$, while the color triplets get mass of $2a\langle P \rangle / M_i$.

In addition to these, the adjoint contains the $(8, 1, 0)$ and $(1, 3, 0)$, which get mass of $2a^2/M$ and a^2/M, respectively, and the $(3, 2, -\frac{5}{6})$ + H.c., which get eaten. There are also several singlets of G_{SM} which get superlarge mass. We have thus seen that no goldstone or pseudo-goldstones are left after symmetry breaking.

From the explicit spectrum given above one can compute the corrections to the low-energy gauge couplings due to the superheavy states. Since $\sin^2 \theta_W$ and α are better known, it is now usual to use them as inputs for a prediction of $\alpha_s(M_Z)$. The minimal SU(5) SUSY-GUT predicts [1] $\alpha_s(M_Z) = 0.127 \pm 0.005 \pm 0.002$, where the first error is the uncertainty in the low-energy sparticle spectrum, and the second is the uncertainty in the masses of the top quark and Higgs bosons. A global fit [14] to α_s from measurements at all energies gives $\alpha_s(M_Z) = 0.117 \pm 0.005$.

Using the notation of Ref. [8], we define $\epsilon_3 \equiv [\alpha_3(M_G) - \tilde{\alpha}_G]/\tilde{\alpha}_G$, where $\tilde{\alpha}_G \equiv \alpha_1(M_G) = \alpha_2(M_G)$. ($M_G$ is here defined to be the scale at which α_1 and α_2 are equal.) To obtain $\alpha_s(M_Z) \simeq 0.12$ requires, in general, that $\epsilon_3 \sim -0.02$ to -0.03. In the minimal SO(10) scheme presented here one finds that $\epsilon_3 \cong \frac{3}{5\pi} \tilde{\alpha}_G \ln\left[\frac{32}{9\sqrt{2}} \frac{\tilde{M}_T}{M_G}\right]$, where $\tilde{M}_T = a^2/2\langle S \rangle$ is the effective color-triplet Higgsino mass that comes into the Higgsino-mediated proton-decay amplitude. ϵ_3 thus comes out to be $+0.03$ if $\tilde{M}_T \simeq 10 M_G$, and $+0.06$ if $\tilde{M}_T \simeq 10^3 M_G$ (as is typically necessary if $\tan \beta$ is large).

It should be noted that this contribution to ϵ_3 is coming from the sector of the **10**'s of Higgs, and must be present in *any* SO(10) model. For comparison, in the absence of the mechanism proposed in this Letter, a model with only a single adjoint Higgs would have colored, pseudogoldstone fields that contribute an *additional* $+0.1$ to $+0.2$ to ϵ_3, so that the problem would be several times worse.

A complete discussion of GUT-scale threshold corrections to α_s would involve the sector of the theory responsible for fermion masses. This sector can in principle lower the value of α_s, but could also exacerbate the problem. There is one mechanism which is, however, guaranteed to lower the value of α_s. Roszkowski and Shifman [15] have emphasized that a gluino as the lightest SUSY particle provides weak-scale threshold corrections that lower the value of α_s. Moreover, in a recent paper [16] one of us (S. R.) has shown how to obtain a model with a gluino LSP in the context of gauge-mediated SUSY breaking and SO(10). This idea can easily be applied to the SO(10) model discussed here.

In any grand unified model the stability of the gauge hierarchy requires that certain operators allowed by the unified group—in particular HH'—be suppressed in the superpotential to sufficiently high order in M_P^{-1}. In SO(10) this means operators that contain T_1^2 (since $T_1 \supset H, H'$). Generally, these are assumed to be forbidden by a U(1) symmetry (or a Z_N subgroup) under which $T_1 \rightarrow e^{i\alpha}T_1$, $T_2 \rightarrow e^{-i\alpha}T_2$, $A \rightarrow A$, and $S \rightarrow e^{2i\alpha}S$. The term $T_1^2 S^\dagger$ is forbidden by holomorphy. (Of course, there must not be a field $\tilde{S}$ with opposite quantum numbers to S, as then $T_1^2\tilde{S}$ would be allowed.) Another term that must be forbidden in any SO(10) model is $T_1 T_2 A^2$. This can be ruled out (while allowing $T_1 A T_2$) by a Z_2 under which A, T_2, and P are odd.

The mechanism that we have proposed in this Letter involves introducing certain fields that could conceivably destabilize the hierarchy. In particular, if the field P did not appear in Eq. (3), Z_i would have the same quantum numbers as A, and $T_1 Z_i T_2$ would be allowed. The presence of the field P in Eq. (3) permits these dangerous terms to be forbidden by a symmetry under which P and Z_i have the same charge and A is neutral. In fact, this can be the same U(1) (or Z_N subgroup) that prevents T_1^2. For example, all dangerous terms can be forbidden by the symmetry $\mathrm{U}(1) \times Z_2 \times Z_2$, under which the fields have the following charges: $A(0^{+-})$, $T_1(1^{++})$, $T_2(-1^{+-})$, $C(0^{-+})$, $\overline{C}(0^{++})$, $C'(-1^{++})$, $\overline{C}'(-1^{-+})$, $P(1^{+-})$, $Z_i(1^{++})$, and $X(0^{++})$. The first Z_2 symmetry forbids the destabilizing terms like $\overline{C}CA^2$. However, $(\overline{C}C)^2 A^2/M_P^3$ is allowed, and can be shown to give a contribution of $O(M_G^5/M_P^4)$ to the μ parameter, thus solving the μ problem.

With this assignment of charges, the role of S in Eq. (2) can be played by P^2. If, instead, P and Z_i have charge 2, and C' and $\overline{C}'$ have charge -2, then the role of S can be played by Z_i. These are not the only possible symmetries or charge assignments that would stabilize the gauge hierarchy. There is a large range of possibilities, since the full global symmetry of the terms given in Eqs. (2) and (3) is $\mathrm{U}(1)^3 \times Z_2^2$.

In conclusion, we have demonstrated a simple supersymmetric SO(10) model which both breaks SO(10) to the standard model and solves the doublet-triplet splitting problem, with only one adjoint field, and without the appearance of extra light fields that disrupt the unification of couplings. In one version of the model a μ term is generated naturally.

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[1] S. Dimopoulos, S. Raby, and F. Wilczek, Phys. Rev. D **24**, 1681 (1981); C. Giunti, C. W. Kim, and U. W. Lee, Mod. Phys. Lett. A **6**, 1745 (1991); J. Ellis, S. Kelley, and D. V. Nanopoulos, Phys. Lett. B **260**,131 (1991); U. Amaldi, W. deBoer, and H. Furstenau, Phys. Lett. B **260**, 447 (1991); P. Langacker and M.-X. Luo, Phys. Rev. D **44**, 817 (1991); P. Langacker and N. Polonsky, Phys. Rev. D **47**, 4028 (1993).

[2] J. A. Harvey and M. S. Turner, Phys. Rev. D **42**, 3344 (1990); G. Lazarides and Q. Shafi, Phys. Lett. B **258**, 305 (1991); J. A. Harvey and E. W. Kolb, Phys. Rev. D **24**, 2090 (1981).

[3] S. M. Barr, Phys. Rev. D **24**, 1895 (1981); Phys. Rev. Lett. **64**, 353 (1990); K. S. Babu and S. M. Barr, Phys. Rev. Lett. **75**, 2088 (1995); G. Anderson, S. Raby, S. Dimopoulos, L. J. Hall, and G. D. Starkman, Phys. Rev. D **49**, 3660 (1994); L. J. Hall and S. Raby, Phys. Rev. D **51**, 6524 (1995); V. Lucas and S. Raby, Phys. Rev. D **54**, 2261 (1996); **55**, 6986 (1997).

[4] E. Gildener and S. Weinberg, Phys. Rev. D **13**, 3333 (1976); E. Gildener, Phys. Rev. D **14**, 1667 (1976).

[5] L. Maiani, in *Proceedings of Comptes Rendus de l'Ecole d'Etè de Physiques des Particules, Gif-sur-Yvette, 1979* (IN2P3, Paris, 1980), p. 3; S. Dimopoulos and H. Georgi, Nucl. Phys, **B150**, 193 (1981); M. Sakai, Z. Phys. C **11**, 153 (1981); E. Witten, Nucl. Phys. **B188**, 573 (1981).

[6] S. Dimopoulos and F. Wilczek, Report No. NSF-ITP-82-07.

[7] K. S. Babu and S. M. Barr, Phys. Rev. D **48**, 5354 (1993); **50**, 3529 (1994).

[8] Lucas and Raby (Ref. [3]).

[9] K. R. Dienes, Nucl. Phys. **B488**, 141 (1997).

[10] K. S. Babu and S. M. Barr, Phys. Rev. D **51**, 2463 (1995).

[11] J. Hisano, H. Murayama, and T. Yanagida, Phys. Rev. D **49**, 4966 (1994).

[12] Lucas and Raby, (Ref. [3]); Z. Berezhiani and Z. Tavartkiladze, hep-ph/9612232.

[13] S. Urano and R. Arnowitt, hep-ph/9611389.

[14] B. R. Webber, in *Proceedings of the 27th International Conference on High Energy Physics, Glasgow, Scotland, 1994*, edited by P. J. Bussey and I. G. Knowles (IOP, London, 1995).

[15] L. Roszkowski and M. Shifman, Phys. Rev. D **53**, 404 (1996).

[16] S. Raby, Phys. Rev. D **56**, 2852 (1997).