C, P, and Strong CP in Left-Right Supersymmetric Models

Rabindra N. Mohapatra,[1] Andrija Rašin,[2] and Goran Senjanović[2]
[1]*Department of Physics, University of Maryland, College Park, Maryland 21218*
[2]*International Center for Theoretical Physics, 34100 Trieste, Italy*
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We systematically study the connection between P, C, and strong CP in the context of both nonsupersymmetric and supersymmetric left-right theories. We find that the solution to the strong CP problem requires both supersymmetry and parity breaking scales to be around the weak scale. [S0031-9007(97)04761-3]

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There are two possible ways of solving the strong CP problem. The first is the dynamical relaxation mechanism, such as the celebrated Peccei-Quinn symmetry which promotes the strong CP phase into a dynamical variable [1].

The second idea is to utilize some discrete symmetry [2–4] to make the strong CP phase vanish at the tree level. It then becomes calculable in perturbation theory, and a viable solution to the problem requires that these perturbative corrections be below the experimental upper limit. The most appealing candidates for this job are the fundamental space-time symmetries: parity (P) and time reversal (CP).

Rather natural candidates are left-right (LR) symmetric theories [5] which provide a framework for the spontaneous breakdown of parity. Furthermore, CP can be spontaneously broken even in the minimal version of these theories [6]. In addition they can be embedded in SO(10) grand unified theories (GUTs), which are the minimal truly unified models of quarks and leptons. In this letter, we focus our attention on these natural candidates for the solution of the strong CP problem, both in ordinary and supersymmetric versions [7].

It is well known that the strong CP problem contains two aspects, that is the smallness of θ_{QCD}, the coefficient of the $F\tilde{F}$ term, and the smallness of $\theta_{\mathrm{QFD}} = \mathrm{ArgDet}\mathbf{M}$, where $\mathbf{M}$ is the mass matrix of colored fermions. It is highly suggestive to use parity since it implies both $\theta_{\mathrm{QCD}} = 0$ and $\mathbf{M} = \mathbf{M}^\dagger$ which in turn gives $\theta_{\mathrm{QFD}} = 0$.

This would be sufficient if parity were an exact symmetry of nature. However, parity must be broken and the real challenge in these theories is to keep $\theta \equiv \theta_{\mathrm{QCD}} + \theta_{\mathrm{QFD}}$ small to all orders in perturbation theory. Without supersymmetry this is an impossible task. Essentially, the problem is that the requirement of weak CP violation destroys the hermiticity of the quark mass matrices already at the tree level which induces, in general, large θ_{QFD}. Another way to see it is to note that the constraint of parity invariance alone allows for complex couplings in the Higgs potential which lead to complex vacuum expectation values (VEVs) for the Higgs fields and thereby destroy the hermiticity of the quark mass matrices even at the tree level. Recently, it has been argued [7] that making the left-right symmetric model supersymmetric leads to a Higgs potential, where all coupling parameters are real thus giving us a CP-conserving vacuum. Furthermore, the perturbative one-loop contributions to $\bar{\theta}$ can be shown to be small under certain circumstances [8,9].

These observations have inspired us to revisit the supersymmetric left-right (SUSYLR) model and carefully discuss under what circumstances $\bar{\theta}$ in this model is guaranteed to be acceptably small. We find that supersymmetry and parity symmetry by themselves are not sufficient to control the one loop contributions. One needs charge conjugation invariance (C) for the purpose. It then turns out that in general the hermiticity of the quark mass matrices can only be preserved at the expense of weak CP violation, thus making the theory unrealistic. We find one exception: *low scale of parity breaking M_R and parity breaking achieved only through nonrenormalizable operators*. In this case the smallness of $\bar{\theta}$ is achieved by a soft violation of CP and is controlled by the small ratio of M_R/M_{Planck}. We find it rather interesting that the requirement of smallness of the strong CP phase requires an experimentally accessible scale of parity restoration. This is the major new result of our paper.

In order to set the framework for our discussion we first analyze the essential features of parity and charge conjugation and their role in the strong CP problem.

No supersymmetry.—We start with the minimal left-right symmetric theory based on the gauge group $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R \times \mathrm{U}(1)_{B-L} \times P$ and the following fermionic content:

$$Q_L = \begin{pmatrix} u \\ d \end{pmatrix}_L (2,1,1/3), \qquad Q_R = \begin{pmatrix} u \\ d \end{pmatrix}_R (1,2,1/3),$$
$$L_L = \begin{pmatrix} \nu \\ e \end{pmatrix}_L (2,1,-1), \qquad L_R = \begin{pmatrix} \nu \\ e \end{pmatrix}_R (1,2,-1), \quad (1)$$

with gauge quantum numbers spelled out in brackets.

Under parity these fields transform as usual

$$Q_L \leftrightarrow Q_R; \qquad L_L \leftrightarrow L_R, \quad (2)$$

and similarly under charge conjugation

$$Q_L \leftrightarrow (Q^c)_L \equiv C\bar{Q}_R^T; \qquad L_L \leftrightarrow (L^c)_L \equiv C\bar{L}_R^T. \quad (3)$$

We will stick to the somewhat conservative assumption that there are no new quarks and leptons. It should be

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mentioned that if this assumption is relaxed it is possible to construct viable models based on parity only which predict calculably small θ [10–12]. Similarly, with additional fermions, one can use both P and C symmetries and CP can be used to exchange the two SU(2) groups [13].

With the above fermion content the field that provides quark and lepton masses is the Higgs bidoublet, $\phi\,(2,2,0)$, which under parity transforms as $\phi \leftrightarrow \phi^{\dagger}$. Keeping in mind eventual SO(10) embedding, we allow a sign ambiguity in the charge conjugation transformation of ϕ,

$$\phi \leftrightarrow \phi^{T}. \tag{4}$$

The imposition of parity and the gauge symmetry determine the Yukawa couplings,

$$L_y = \mathbf{h}_q \bar{Q}_L \phi Q_R + \mathbf{h}_l \bar{L}_L \phi L_R , \tag{5}$$

to be Hermitian i.e.,

$$\mathbf{h}_q = \mathbf{h}_q^{\dagger}; \qquad \mathbf{h}_l = \mathbf{h}_l^{\dagger}. \tag{6}$$

Clearly, since the quark mass matrices are given by

$$\mathbf{M}_q = \mathbf{h}_q \langle\phi\rangle , \tag{7}$$

they will be Hermitian if and only if $\langle\phi\rangle$ is real ($\langle\phi\rangle$ real obviously preserves parity). But $\langle\phi\rangle$ can be real only if the Higgs potential is CP conserving. Here lies the crux of the problem. Now, $\mathbf{h}$ is either real or complex. If it is real then $\langle\phi\rangle$ must be complex in order for CP to be broken, in which case $\mathbf{M}_q$ cannot be Hermitian. If, on the other hand, $\mathbf{h}$ is complex then unfortunately there are complex couplings in the Higgs potential and $\langle\phi\rangle$ itself becomes complex, destroying again the Hermiticity of $\mathbf{M}_q$.

Let us demonstrate this in some detail. Consider first the minimal case with a single bidoublet ϕ. It is a simple exercise to show that the potential which depends on ϕ only has all the couplings real due to parity symmetry. However, in order to break $\mathrm{SU}(2)_R$ symmetry at the scale $M_R \gg M_W$ we need other Higgs fields, χ_L and χ_R, which are nontrivial representations under $\mathrm{SU}(2)_L$ and $\mathrm{SU}(2)_R$, respectively. The troublesome couplings in the schematic representation are

$$(\alpha \chi_L^{\dagger} \chi_L + \beta \chi_R^{\dagger} \chi_R) \phi^{\dagger} \phi + \text{H.c.}. \tag{8}$$

Parity imposes only $\alpha = \beta^*$ so that α is, in general, complex. Only if one imposes *charge conjugation on top of parity* are these couplings made real. As we will see, this additional requirement of C invariance in addition to parity happens also in the supersymmetric version. Now, however, $\langle\phi\rangle$ must be complex in order to have nonvanishing weak CP violation since C invariance also makes the Yukawa couplings real. The hermiticity of $\mathbf{M}_q$ required for $\bar{\theta}$ to vanish is then lost.

One could imagine a possible way out along the lines of Ref. [8]. Suppose that there are two bidoublets with opposite transformation properties under C

$$\phi_1 \rightarrow \phi_1^{T}, \qquad \phi_2 \rightarrow -\phi_2^{T}. \tag{9}$$

This implies that $h_1 = h_1^T$ and real, and $h_2 = -h_2^T$ and purely imaginary. In the context of SO(10), ϕ_1 would belong to 10-dimensional representation and ϕ_2 to 120-dimensional representation. It is noteworthy that in SUSY one *must* have at least two bidoublets in order to have nonzero quark mixing angles.

Now the quark mass matrices become

$$M_q = h_1 \langle\phi_1\rangle + h_2 \langle\phi_2\rangle . \tag{10}$$

Notice that $\langle\phi_2\rangle \rightarrow -\langle\phi_2\rangle^*$ under CP, so that real $\langle\phi_2\rangle$ breaks CP invariance. Obviously if both $\langle\phi\rangle_i$ are real, M is Hermitian and complex. This would guarantee weak CP violation without the strong one. At this point, all seems well since, as before, the interaction terms between the ϕs in the potential are real. However, again there are complex couplings, with χ_L and χ_R fields of the type,

$$i(\alpha \chi_L^{\dagger} \chi_L + \beta \chi_R^{\dagger} \chi_R) \phi_1 \phi_2 + \text{H.c.} \tag{11}$$

Parity imposes $\beta = -\alpha^*$, and charge conjugation makes α real. Obviously, the presence of both real and imaginary couplings in the potential will render the VEVs of the bidoublets complex. This in turn kills the hermiticity of the mass matrices and implies a strong CP phase already at the tree level. We should stress that this problem is generic and does not depend on the choice of χ fields, i.e., whether they are doublets, triplets or higher representations.

In supersymmetry it is the superpotential that defines the theory, and one might hope that at least at the renormalizable level such dangerous terms may be absent [7]. However, the issue is more subtle and now we discuss it in detail.

Supersymmetry.—It is well known that in supersymmetry one needs at least two bidoublets to get realistic fermion mass matrices so that the above scenario finds here its natural place. There are, however, new CP problems in supersymmetry: The relevant one for us is that the masses of gauginos are complex in general. Here P and C again play a fundamental role: P makes gluino mass real but not the masses of the left and right winos. At the one-loop level, these complex masses lead to a finite but unacceptable contribution to $\bar{\theta}$ of order $\alpha/4\pi$. In Ref. [8], one appeals to SO(10) grand unified extension in order to make these masses real. The point is simply that parity and charge conjugation suffice: P makes gluino mass real, and P and C ensure the same for weak gaugino masses. Thus we impose both of them and study the consequences.

Interestingly enough, we find that complete consistency of the theory requires that the W_R mass must be in the TeV range.

Let us go back to Eq. (9). In the minimal left-right model with the seesaw [14] mechanism, the χ fields are taken to be triplets Δ and Δ^c under $\mathrm{SU}(2)_L$ and $\mathrm{SU}(2)_R$ groups, respectively [15]. Of course, anomaly cancellation in supersymmetry requires the doubling of such fields ($\overline{\Delta}$ and $\overline{\Delta}^c$). It is easy to see that at the renormalizable level there are no such dangerous complex couplings in the superpotential. The problem is that in this model there can be no spontaneous breakdown of left-right symmetry and if the theory is augmented by the parity-odd gauge singlet field, this gets cured at the expense of the breakdown of electromagnetic charge invariance [16]. The way out of this impasse is either to include more Higgs fields [16,17] or to resort to nonrenormalizable dimension four terms in the superpotential [7]. In either case, one necessarily generates the imaginary couplings described above.

Now we discuss these cases step by step starting with the renormalizable superpotential.

(a) If one adds B-L neutral triplets Ω and Ω_c, one finds a consistent and realistic theory with a possibility of phenomenologically interesting low B-L scale without the need for the parity-odd singlet [17]. However, in this theory the new terms in the superpotential (we are only schematic here in notation; for exact expressions see [17]),

$$i\alpha\Omega\phi_1\phi_2 + i\beta\Omega_c\phi_1\phi_2\,, \tag{12}$$

are of the type discussed above and thus $\alpha = -\beta$ real. Next, it can be shown that Ω_c VEV is real. The terms in the superpotential which are relevant are the couplings of the right-handed triplet fields,

$$W = m_\Delta[\mathrm{Tr}(\Delta\overline{\Delta}) + \mathrm{Tr}(\Delta_c\overline{\Delta}_c)] + a[\mathrm{Tr}(\Delta\Omega\overline{\Delta}) + \mathrm{Tr}(\Delta_c\Omega\overline{\Delta}_c)]\,. \tag{13}$$

It can be easily seen that C and P render the above couplings real. In the P breaking and electromagnetic charge preserving vacuum $\langle\Delta\rangle = \langle\overline{\Delta}\rangle = \langle\Omega\rangle = 0$ and $\langle\Omega_c\rangle = M_R\mathrm{diag}(1,-1)$ with M_R being a real number

$$M_R = \frac{m_\Delta}{a}\,. \tag{14}$$

This induces the imaginary μ-type effective mixing term between ϕ_1 and ϕ_2, thus making it impossible to keep both bidoublet VEVs real. This just as in the nonsupersymmetric case, destroys the hermiticity of the quark mass matrices.

(b) Alternatively, one can work without Ω fields, assuming that there are nonrenormalizable terms in the superpotential to achieve the spontaneous breakdown of parity. Again, one can do without the parity-odd singlet [18]. In this case, the analog of the mixing of Ω and ϕ fields (14) is achieved through following the $d = 4$ terms in the superpotential,

$$i\,\frac{\alpha}{M_{\mathrm{Pl}}}\,\Delta\overline{\Delta}\phi_1\phi_2 + i\,\frac{\beta}{M_{\mathrm{Pl}}}\,\Delta^c\overline{\Delta}^c\phi_1\phi_2\,. \tag{15}$$

Again $\alpha = -\beta$ is real. Now clearly the complex mixing term between ϕ_1 and ϕ_2 is suppressed by $\frac{M_R}{M_{\mathrm{Pl}}}$. It is easy to see that the relative phase between the ϕ_1 and ϕ_2 VEVs can be controlled by the same suppression factor. It is a simple exercise to show that the strong CP phase is of order

$$\theta = \frac{M_R^2}{m_S M_{\mathrm{Pl}}}\,, \tag{16}$$

where m_S is the scale of SUSY breaking in the light particle sector of the theory. At this level, clearly this parameter is completely undetermined.

On the other hand, in this version of the theory, the splitting of the bidoublets is achieved through the above $d = 4$ terms and thus, besides the usual two light doublets of the minimal supersymmetric standard model (MSSM) above the scale M_R^2/M_{Pl}, there will appear the other two doublets. It has been shown in Ref. [8] that the running of the Yukawa couplings below M_R quickly generates sizeable θ when four doublets are present.

Thus one is forced to the low parity breaking scale scenario. One way to get this is to introduce a parity-odd singlet superfield σ [16]. In this case, in order not to introduce complex couplings in the superpotential, an additional parity-even singlet X is needed. Namely, a parity-odd singlet has imaginary couplings with the bidoublets, and thus one must ensure that its VEV be imaginary also. This can be achieved if one chooses a superpotential for the singlets of the form,

$$W_s = X(\alpha\sigma^2 + M^2)\,, \tag{17}$$

where α and M^2 are real by parity.

One can also obtain a desired pattern of symmetry breaking using only nonrenormalizable operators, as long as the neutrino Yukawa couplings satisfy $f \leq 10^{-2} - 10^{-3}$ and $m_S \approx M_R \approx 1$ TeV. Namely, in this case the nonrenormalizable terms should lower the energy, at the parity broken extremum, to be the minimum of the potential. The couplings f, due to running between the scale M_U of assumed universality of soft terms and the scale of right-handed neutrinos M_{ν_R}, cause the difference between the VEVs $v = \langle\Delta_c\rangle$ and $\overline{v} = \langle\overline{\Delta}_c\rangle$,

$$v^2 - \overline{v}^2 \approx \frac{f^2}{16\pi^2}\ln\left(\frac{M_U}{M_{\nu_R}}\right)m_S^2\,. \tag{18}$$

We find that the condition for the parity breaking and electric charge conserving minimum is

$$\frac{g^2}{2}\,(v^2 - \overline{v}^2)^2 < \frac{m_\Delta}{M_{\mathrm{Pl}}}\,v^2\overline{v}^2 \tag{19}$$

for a range of values of parameters that characterize the nonrenormalizable terms in the superpotential. The condition on f noted above follows from this inequality.

Next, in order to break CP, we need nonzero (and real) VEVs of both bidoublets. This can happen only if there is a mixing term between ϕ_1 and ϕ_2. This term (real due to parity) breaks C softly. This result is a reflection of a general theorem regarding the impossibility of spontaneous CP violation in the supersymmetric model with four Higgs doublets [19]. If one does want to stick to spontaneous violation, this is easily achieved with two singlets, as in the above example. In the presence of this soft C-breaking term, one expects finite contributions to the phase of the left and right gaugino masses. There are no one-loop contributions to such phases. If they arise at the two- or higher-loop level, their contribution to the

Θ is $\leq \alpha^3/(4\pi)^3$ which is of order 10^{-9} and is therefore small. Also we repeat that there is a one-loop contribution to quark masses due to the soft SUSY breaking terms that has already been evaluated in Ref. [9] and is shown that it can be at about the 10^{-9} to 10^{-10} level.

The main implication of our work is the low scale of parity breaking, necessary for the solution to the strong *CP* problem. Let us briefly comment on the implications of this model for neutrino masses. The smallness of the neutrino masses in our model is of course guaranteed by the seesaw mechanism. As far as the values of the neutrino masses are concerned, it depends on the precise model for the Dirac neutrino masses in the theory. In order for the neutrino masses to be below the upper experimental bounds, one must assume the neutrino Dirac mass terms as an order of magnitude or so smaller than the charged lepton masses. This, in turn, implies that ν_μ and ν_τ have to decay, and both the ν_τ and ν_μ can decay only through the exchange of the neutral component of the left-handed triplet Δ [20] rapidly enough to satisfy necessary cosmological constraints. This scenario is phenomenologically completely consistent and has interesting predictions of rare μ decays and M-$\overline{M}$ conversions. Another possibility for being in accord with cosmological limits on neutrino masses is to suppress the neutrino Dirac mass terms as a higher order loop effect [21]. The model in this case has to be supplemented by the addition of extra color triplet fields coupling to quark fields, which does not affect the discussion of the strong *CP* problem given above.

It is well known that in left-right models with low M_R, there are tree level neutral Higgs contributions to the flavor changing neutral current effects. Present observations require that the mass of these neutral Higgs bosons must be more than 5 TeV or so. Since these masses are proportional to M_R, this is consistent with our result that puts M_R also in the same TeV range.

Since our results heavily depend on the imposition of charge conjugation on top of parity it is natural to consider the SO(10) GUT extension of *LR* models. Namely, in SO(10), charge conjugation is an automatic gauge symmetry; and, furthermore, as we remarked before, our choice of the C-transformation properties of bidoublets would simply imply that ϕ_1 lies in the 10-dimensional representation, and ϕ_2 lies in the 120-dimensional representation. On the other hand, it is hard, if not impossible, to achieve low M_R in the supersymmetric SO(10), at least in the minimal version of the theory.

In conclusion, we stress that this is a natural solution to the strong *CP* problem since low M_R scale (order m_S) can be achieved naturally in the process of minimization of the potential. Consistency with the hierarchy problem suggests then that M_R is of a few orders of TeVs. We find it rather appealing that the smallness of θ in left-right symmetric theories is linked to both supersymmetry and M_R being at the low scale. This provides to date the strongest theoretical motivation for a low mass W_R, which has long been of great phenomenological and experimental interest.

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