The Case against Scaling Defect Models of Cosmic Structure Formation

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We calculate predictions from defect models of structure formation for both the matter and cosmic microwave background over all observable scales. Our results point to a serious problem reconciling the observed large-scale galaxy distribution with the Cosmic Background Explorer normalization, a result which is robust for a wide range of defect parameters. We conclude that standard scaling defect models are in conflict with the data, and show how attempts to resolve the problem by considering nonscaling defects would require radical departures from the standard scaling picture. [S0031-9007(97)04752-2]

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Defect models offer an elegant explanation of the origin of cosmic structure. The idea is that some distribution of defects—or more generally, field disorder—is produced during a cosmic phase transition [1]. The defects then start a process of "coarsening" which continues through to the present day, contributing a component to the matter in the Universe which evolves in a highly nonlinear way. Cosmic strings, for example, move at relativistic velocities, periodically self-intersect, and break off loops, which themselves eventually decay into gravity waves. Such processes can seed the onset of gravitational collapse in a universe which is initially perfectly homogeneous.

In contrast with other models of cosmic structure formation, calculations for defects require the modeling of highly nonlinear processes from very early times (e.g., the time of grand unification), right up to the present day. During this period the Universe increases by around 25 orders of magnitude in size. Only recently has it become practical to solve the full Boltzmann equations for the matter and radiation perturbations in the presence of defect sources consistently modeled over such a length of time [2]. Accurate large-scale numerical simulations are currently the best source of information on the details of the defect evolution, but even using state of the art technology it is still necessary to extrapolate with scaling arguments [1] to achieve anything like the required dynamic range. Here we describe work that is not as closely linked to specific defect simulations and is therefore able to explore a wider range of possible defect models. Thus we can systematically investigate the robustness of the clash between defect models and observations.

Our calculations use the fact that if only the power spectra are to be calculated, then one only needs two-point functions of the defect stress energy [3]. Scaling arguments can then be used to increase the dynamic range. This scaling behavior has been observed to some degree in numerical simulations and has become part of the standard lore of defect evolution, although the extent to which it is valid over a factor of 10^{25} in cosmic expansion is not yet clear.

For the current calculations this approach was incorporated into a version of CMBFAST [4] modified to include source stress energy for the scalar, tensor, and vector contributions, which are generic in defect based models [2,5,6]. In order to do this, we model the components of the defect stress energy under a number of simple assumptions which maintain causality and conserve stress energy. The source is approximated by a network of linelike segments with correlation length $\xi\eta$ at conformal time η and velocity taken from a Gaussian distribution with rms v, truncated to prevent $v > c$. The number of lines is reduced causally, so as to maintain a constant density with respect to the horizon. This approach is similar to the model used in Ref. [7], which was shown to give two-point functions in good agreement with string simulations for certain stress energy components, but we have updated it to include all the components required and an improved decay mechanism [8].

Any active source which creates perturbations inside the horizon is likely to be incoherent, leading to the absence or suppression of secondary Doppler peaks [9]. Hence, the form of unequal-time correlators (UETC) is also important. Our approach, which contrasts with that used in Ref. [2], is not to calculate the UETC directly, but to create an ensemble of source histories with the correct statistics. Then in order to calculate the ensemble average of the matter power spectrum and the cosmic microwave background (CMB), one must use the Boltzmann code for each source history and average the resulting spectra. The results shown here used 100–400 realizations which give very small statistical errors, but runs with just 40 realizations sufficed to establish the basic picture.

In Fig. 1 we plot the angular power spectrum of the CMB for what we shall call the standard string model (solid line). This uses string model parameters $\xi = 0.3$ and $v = 0.65$ as suggested by simulations, an assumption of perfect scaling from defect formation to the present day, and a flat background cosmology with $\Omega_c = 0.95$, $\Omega_b = 0.05$, and $h = 0.5$ where $H_0 = 100h$ km sec^{-1} Mpc^{-1}. Included also are the standard cold dark matter (CDM) model based on inflation (dot-dashed line) and all the current published data points with error bars based on the assumption of Gaussianity [10]. The main features to note are the absence of any discernible Doppler peak in the defect spectrum and the apparent conflict with the data

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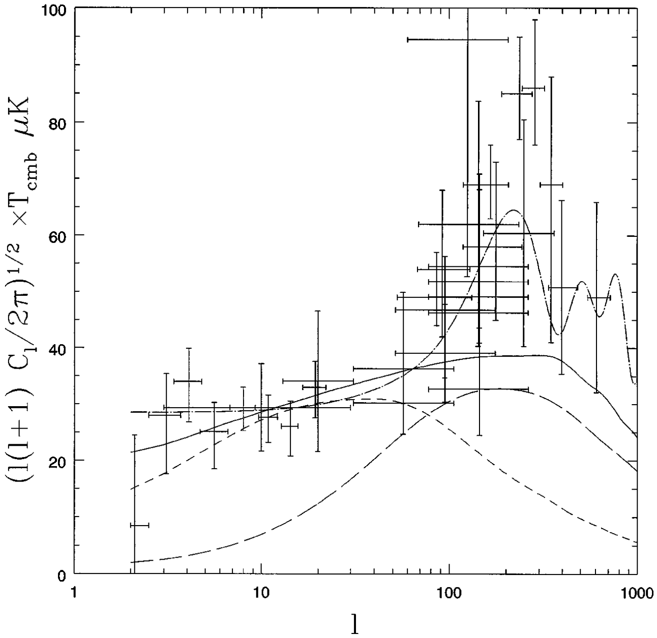

FIG. 1. The (COBE normalized) angular power spectrum of CMB anisotropies for the standard cosmic string model (solid) plotted with the current observational data, the standard CDM curve (dotted). The two dashed curves give the partial contributions from two time windows to either side of $z = 100$.

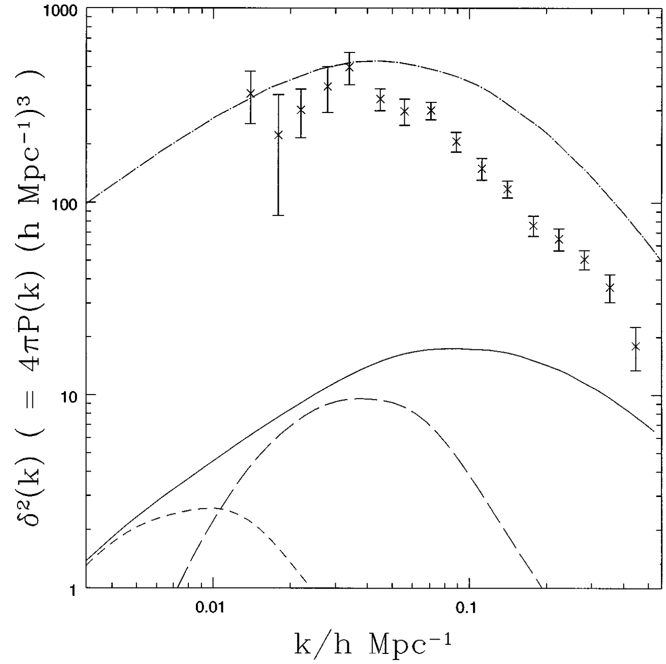

FIG. 2. The power spectrum of the dark matter perturbations for the same models and windows shown in Fig. 1, plotted with the data.

points. We have repeated these calculations for various different values of ξ and v and also for sensible variations of the cosmological parameters h and Ω_b. The spectrum is modified by these variations, but none manage to increase the amount of power at angular scales with $l = 200-400$ by very much. Clearly the situation looks bad for defect models, although it is worth noting that the plotted error bars are 1σ, and deviations from the assumed Gaussianity may require even larger error bars due to the small sky coverage. We expect the situation to be much clearer when the new CMB data arrive in the near future.

Figure 1 also shows the partial results which come from integrating the defect contributions over two time windows: Window 1 ($z = 1300$ to $z = 100$, during which η increases by a factor of 5) gives the long-dashed curve, and window 2 ($z = 100$ to $z = 1.6$, during which η increases by a factor of 7) gives the short-dashed curve. This information will be helpful in the subsequent discussion of nonscaling defect models.

Using the same annotation as Fig. 1, Fig. 2 shows the Cosmic Background Explorer (COBE) normalized cold dark matter density perturbation power spectrum predicted for the standard string model and that for standard CDM, compared with the data [11]. The contributions from the same two time windows are also included, as in Fig. 1. Theory and data are often compared using σ_8, the variance of the fractional matter overdensity in a ball of radius $8h^{-1}$ Mpc. For standard CDM $\sigma_8 = 1.2$ for $h = 0.5$, while the value favored by observations is $\sigma_8 = 0.5$. If one were to compare with the string model at these scales, one calculates $\sigma_8 = 0.31$ and hence the bias on these scales between the galaxy distribution, which is largely baryonic matter, and the cold dark matter is $b_8 = \sigma_8/\sigma_8^{\mathrm{DM}} \approx 1.5$. This not unreasonable value can even be slightly reduced by changing parameters.

However, these comparisons ignore the fact that there is a woeful absence of power on larger scales. We quantify the conflict for low k by calculating the hypothetical bias $b_{100} \equiv \sigma_{100}/\sigma_{100}^{\mathrm{DM}}$ where σ_{100} is defined in analogy to σ_8, but for spheres of $100h^{-1}$ Mpc and the favored value ($\sigma_{100} = 3.7 \times 10^{-2}$) is calculated for a smooth curve which gives a good fit to the data points. The standard string model has $b_{100} = 5.4$ which cannot be improved substantially by any of the variations already discussed. The chances of a real physical model having such a large value of b_{100} are remote [12], and there is no observational evidence for a large b_{100} [13,14]. We conclude that the standard string model is in conflict with the observations at an unacceptable level on scales around $100h^{-1}$ Mpc. Variations relating to possible systematic uncertainties in simulations and our knowledge of the baryon density and Hubble constant cannot alleviate the discrepancy. We also note that these conclusions do not depend on the stringy nature of the model that we have used. For instance, the results are very similar if we impose a sharp subhorizon cutoff on the source stress energy, mimicking behavior closer to that of cosmic textures.

By far the most effective way of addressing the large b_{100} problem is to exploit the uncertainty in the overall scaling behavior of the string network. It is this behavior, after all, that relates the contributions from defects on different scales, and has simply been put into our calculation by hand. Although scaling has been observed to some degree in simulations, it is not completely clear that simple laws are valid over a wide dynamic range and events during the history of the Universe may lead to deviations. For example, the radiation-matter transition is known at the very least to cause a shift; one could speculate that this is not yet well understood.

We have extensively probed the possible deviations from the standard picture, and found it very difficult to get around the large b_{100} problem. This can be understood by looking at the contributions from the two time windows illustrated in Figs. 1 and 2. The first window provides essentially all the contributions to the COBE normalization, while the second window provides the dominant contribution to σ_{100}. The problem is that these two windows span a sufficiently narrow period in the defect history that something dramatic must happen to the scaling behavior to shift their relative contributions sufficiently. An extreme (and unmotivated) "solution" which suggests itself is to simply turn off the string network at $z = 100$, hence preserving perturbations which contribute to σ_{100}, but removing the highest possible fraction of the contributions to COBE scales. The result of doing this is illustrated in Figs. 3 and 4 (solid line), and manages to give $b_{100} = 1.2$.

There are two simple types of deviations from scaling which may be more acceptable. The first, which has been observed to occur to some degree at the radiation-matter transition [15], is just a step in the string density from one value to another, occurring in a smooth way over some period of time. Such a deviation can be quantified by the ratio $\chi = (\eta^{1/2}\theta_{00})^{\rm rad}/(\eta^{1/2}\theta_{00})^{\rm mat}$ (where $\chi = 1$ gives "standard scaling"). For instance, if the radiation-matter transition gives rise to a difference in the amount of small-scale structure on the strings in the two eras, then χ could be interpreted as the ratio of the renormalized string tensions. The second type of deviation we consider is a power law deviation from scaling quantified by a parameter α via $\theta_{00} \sim \eta^{-(1/2+\alpha)}$, for which the density in strings $\rho \sim 1/\eta^{2+2\alpha}$, with the choice $\alpha = 0$ corresponding to a standard scaling law. This may model the behavior, for example, in an open universe or in one dominated by a cosmological constant [16,17].

In order to illustrate the problem, we show the results from four further models for the CMB in Fig. 3 and for the matter power spectrum in Fig. 4. The first two are mild deviations from scaling which one might imagine are plausible: Model *A* (dotted curve, $b_{100} = 3.4$) is a transition of $\chi = 2$, with the transition beginning at $8\eta_{\rm eq}$ and ending at $10\eta_{\rm eq}$ where $\eta = \eta_{\rm eq}$ is the time of equal matter and radiation. Model *B* (long-dashed curve, $b_{100} = 2.9$) is a power law deviation from scaling with $\alpha = 0.25$.

The other two examples are much more extreme; their virtue being that they can fit the data points in the matter power spectrum at around $100h^{-1}$ Mpc: Model *C* (long-short dashed curve, $b_{100} = 1.0$) is a transition between the same times as for model *A* but with $\chi = 10$ and model *D* (short-dashed curve, $b_{100} = 0.7$) is a power law deviation from scaling with $\alpha = 0.75$. While models *C* and *D* fit the matter spectrum at scales of around $100h^{-1}$ Mpc, they completely fail to fit smaller scale galaxy data, and give a large excess of small-scale power in the CMB spectrum. One might hope that changes to the ionization history and matter content of the Universe could solve some of these problems. However, the fact remains that there is

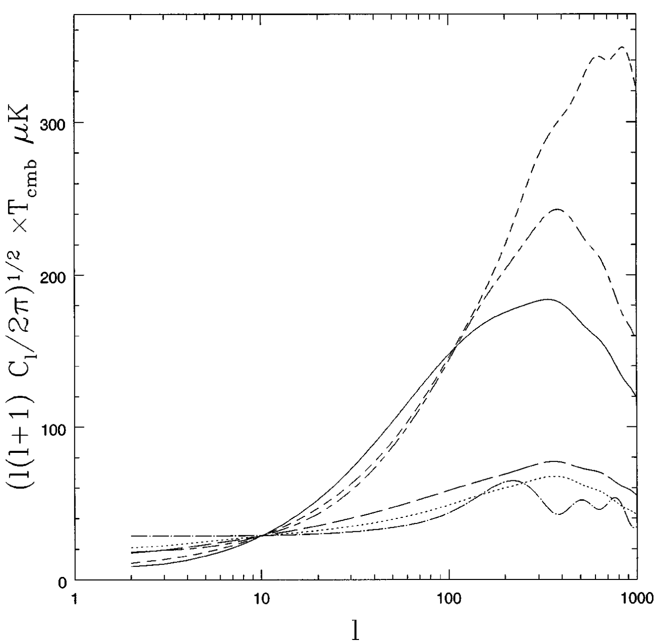

FIG. 3. The angular power spectrum of CMB anisotropies for the various nonscaling models discussed in the text. The three most extreme models (which have reasonable values of b_{100}) have the highest peaks. Standard CDM is included for reference (dash-dotted curve).

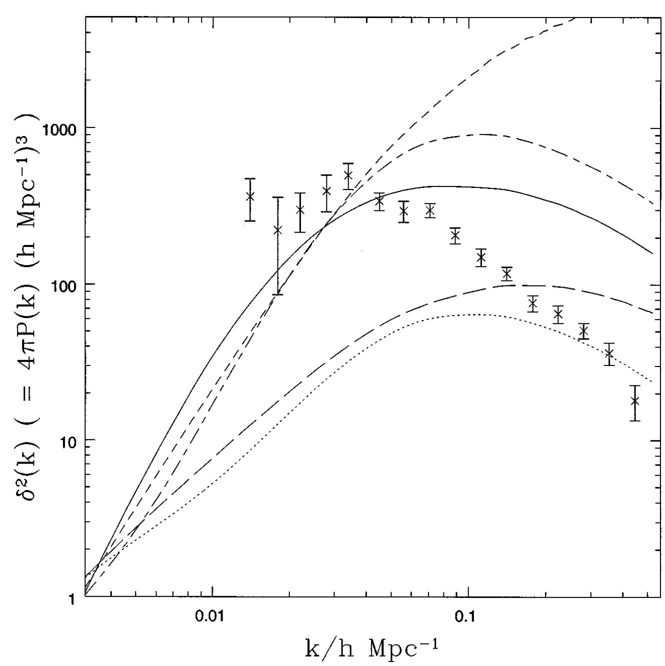

FIG. 4. The power spectrum of the dark matter perturbations plotted with the data for the same models as Fig. 3.

no evidence to suggest that such extreme deviations from scaling could occur in the standard defect scenario.

We now relate our results to previous work on the subject. Several papers have discussed the bias in COBE normalized defect models. In Ref. [18] a serious bias problem was noted on scales up to $20h^{-1}$ Mpc, but concerns remained that the simulations were not including all the relevant contributions (particularly to the density fluctuations) because of their limited dynamic range. The compilation of Refs. [3] and [19] in Ref. [20], although looking very much like our Fig. 2, involved very different treatments of the defects at scales relevant to COBE vs b_{100}, and it is not clear that a straightforward compilation is valid. Our work avoids these uncertainties by solving the full Boltzmann equations with a single source model to compute perturbations consistently on all scales. The scaling assumption translates into essentially "infinite" dynamic range.

There has also been work more recently which is on equal footing in this respect [2]. Conceptually, the main difference between this work and ours is that they extract the UETC's directly from numerical defect simulations. Both groups scale the correlation functions to gain dynamic range. In addition, Ref. [2] uses some sophisticated methods to work efficiently with the UETCs. Bearing in mind these differences, it should be noted that the two results look very similar. A strength of the simulation based approach is that the UETC's are associated with well defined defect scenarios. On the other hand, our approach is more flexible, allowing us to explore a wide range of UETC's in order to test the robustness of the results against variation among defect models as well as possible systematic uncertainties in the simulations.

Even at a more technical level there is a large degree of similarity between our results and those in Ref. [2]. We find that on superhorizon scales the scalar, vector, and tensor anisotropic stresses are in the simple ratio $\langle|\Theta^S|^2\rangle:\langle|\Theta_i^V|^2\rangle:\langle|\Theta_{ij}^T|^2\rangle = 3:2:4$ as imposed by causality and isotropy. Around $l = 10$, we find that $C_l^S:C_l^V:C_l^T$ is approximately $3:1:0.4$. Our value of $C_l^V:C_l^T$ is very close to that in Ref. [2], while our $C_l^S:C_l^V$ is somewhat larger. The degree of similarity is striking given that we use a simple model while large simulations were used in Ref. [2]. We believe our larger $C_l^S:C_l^V$ is due to the relatively large value of Θ_{00} compared to Θ^S in our model. We have not directly compared our sources with those in [6] (describing local strings), but our scalar component seems to be larger. These differences (and those between [6] and [2]) suggest that the relative strength of the scalar component may vary noticeably from one type of defect to another. We should note that even if there were only a scalar component the comparison with the current data would be very bleak; the vector and tensor components only make things worse. While it has been suggested that models with highly suppressed anisotropic stresses might achieve improved values for the COBE normalized bias [21], no concrete defect model has been proposed which has this feature.

Therefore, we conclude that the predictions of standard scaling defect scenarios are in serious conflict with the current data and that this situation can only be remedied by extreme modifications to the scaling law.

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