Possible Understanding of Hyperdeformed $^{144-146}$Ba Nuclei Appearing in the Spontaneous Fission of ^{252}Cf

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The dependence of the stability of prolate deformed nuclei on the neck parameter is studied from the point of view of the interplay between order and chaos. The statistical properties of level spectra for the deformed space explored by the fission process are presented. It is found that on saddles the single particle motion is unstable against chaos, whereas at minimums it is stable. Based on this investigation a possible understanding of hyperdeformed $^{144-146}$Ba appearing in the spontaneous fission of ^{252}Cf is provided. [S0031-9007(97)04689-9]

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Searching for hyperdeformed (HD) nuclei, i.e., nuclei with axis ratios around 3:1, nowadays has become one of the most challenging subjects. Hyperdeformation of ^{144,145,146}Ba was observed by Ter-Akopian *et al.* [1]. It was shown that the HD ^{144,145,146}Ba nuclei were produced at the scission point of the spontaneous fission of ^{252}Cf. In fact, the HD states in actinide nuclei were already studied theoretically [2]. It has been found that in certain heavy nuclei the second saddle point is split, leading to an excited reflection-asymmetric configuration with extremely large quadrupole and octupole deformations, which may be responsible for the HD states. Therefore, the search for hyperdeformation in heavy nuclei is closely related to the study of the fission process. A popular theoretical approach is to examine the potential energy surface in order to study the underlying mechanism for the formation of HD states [2,3]. But it is even more interesting and inspiring to explore the association of the stability of a large deformed configuration of nuclei with the statistical properties of the single particle spectra by way of which a possible understanding of hyperdeformation can be achieved. For the spherical nuclei the statistical features of spectra are better known [4,5]. Recently, it has been reported, based on a simplified model [6], that for a prolate deformation virtually no chaos is discernible while for the oblate case the motion shows strong chaos when the octupole term is turned on. The extension of this investigation to a very large deformed space explored by the fission process and the hyperdeformation is a new direction for nuclear structure study. The aim of this Letter is to develop an approach to understand the fission and the hyperdeformation from the point of view of the interplay between order and chaos. We first illustrate the dependence of statistical features of level spectra on the neck degree of freedom, which plays an important role in the fission process. Then we explore how the stability of nuclear configurations along the fission path from the ground state to scission is governed by the nearest neighbor distribution of level spectra. Finally, we provide a possible understanding of the appearance of hyperdeformed $^{144-146}$Ba nuclei in the spontaneous fission of ^{252}Cf.

The two center shell model (TCSM) [7], in which the neck degree of freedom is included as a shape parameter, is adoped in this study. This model is very favorable for the description of extremely large deformed nuclei, especially for the description of the scission configuration. The potential of the TCSM is axially symmetric with respect to the z axis and is taken to be

$$V(\rho,z)=\begin{cases}\frac{1}{2}m_0\omega_{z1}^2z'^2+\frac{1}{2}m_0\omega_{\rho1}^2\rho^2, & z<z_1\,,\\ \frac{1}{2}f_0m_0\omega_{z1}^2z'^2(1+c_1z'+d_1z'^2)+\frac{1}{2}m_0\omega_{\rho1}^2(1+g_1z'^2)\rho^2, & z_1<z<0\,,\\ \frac{1}{2}f_0m_0\omega_{z2}^2z'^2(1+c_2z'+d_2z'^2)+\frac{1}{2}m_0\omega_{\rho2}^2(1+g_2z'^2)\rho^2, & 0<z<z_2\,,\\ \frac{1}{2}m_0\omega_{z2}^2z'^2+\frac{1}{2}m_0\omega_{\rho2}^2\rho^2, & z>z_2\end{cases}$$

with the abbreviation of $z'=z-z_1$ for $z<0$ and $z'=z-z_2$ for $z>0$. We denote the positions of the centers of the two fragments by z_1 and z_2, with $z_1\leq0\leq z_2$. All parameters in the above formula are related to five shape parameters, by which the nuclear shape can be described very well. They are the separation of the two centers Δz, the neck parameter ϵ ($\epsilon=0$ corresponds to ovaloids, $\epsilon=1$ to well necked-in shapes), the mass asymmetry $X_i=(A_1-A_2)/(A_1+A_2)$ with A_1 and A_2 the mass numbers of the fragments (X_i ranges from 0 to 1), and finally the ellipsoidal deformations of the fragments, β_1 and β_2. Based on this realistic model, we calculate the nearest neighbor level spacing distribution of single particle spectra. Since the level density of proton (or neutron) greatly changes with the level energy, the unfolding procedure, which can be found in Ref. [8], is performed in order to study local statistical properties of levels, such as spacing distribution.

In Fig. 1 we show the nearest neighbor level spacing distribution of single particle spectra (for neutron) for the prolate configuration (with considerable mass asymmetry) as a function of the neck parameter. From this figure it can be seen that when the neck parameter is very small (no necked-in shape) the level spacing distribution approaches the typical Poisson distribution and stable prolate nuclei are expected. This conclusion is just the same as in Ref. [6]. However, as soon as the neck parameter increases, the level spacing distribution starts to deviate from the Poisson distribution and approaches a Wigner type. When the neck parameter is taken to be 0.6, i.e., with a considerable necked-in shape, the quantum statistical results of spectra display a typical Wigner distribution. It means that the stability of prolate nuclear systems against chaos is destroyed by neck formation. If the neck parameter further increases the level spacing distribution gradually comes back to the Poisson distribution. For example, when the neck parameter is taken to be 0.95, i.e., the system approaches to the scission configuration, the statistical behavior of the spectra demonstrates a Poisson-like distribution and the stability against chaos will be restored. Thus we can conclude that the stability of prolate nuclear systems against chaos also depends strongly on the neck parameter in addition to the quadrupole and octupole deformations. Consequently, when we study the fission process and the hyperdeformation in which the neck formation plays a crucial role, it is necessary to consider the influence of the neck parameter explicitly.

In order to establish the general connection between the stability of prolate deformed configuration and the statistical property of level spectra, we illustrate the nearest neighbor level spacing distribution of spectra along a fission path of ^{252}Cf in Fig. 2. This fission path is obtained by the potential energy surface calculation in the TCSM. It starts from the ground state, goes through the first saddle point, the second minimum and the second saddle point, then passes through a well developed mass asymmetric and necked-in third minimum and third saddle point, and finally reaches the scission point. At the ground state a Poisson-like distribution of spectra has been obtained, which is shown in Fig. 2(a). As the system approaches the first saddle point from the ground state, the statistical property of spectra starts to deviate from the Poisson-like distribution as shown in Fig. 2(b) (which corresponds to the configuration in between the ground state and first saddle point). At the first saddle point a Wigner-like distribution appears [Fig. 2(c)]. On the second and third saddle points the statistical property of level spectra is more likely a Wigner-like distribution, as shown in Figs. 2(e) and 2(g), respectively. The level spacing distributions associated with the second and third minimums as well as the scission point are found to be a Poisson-like distribution as shown in Figs. 2(d), 2(f), and 2(h). Therefore, we can summarize as follows: on saddles a Wigner-like distribution of spectra has been observed, which means that the single particle motion at

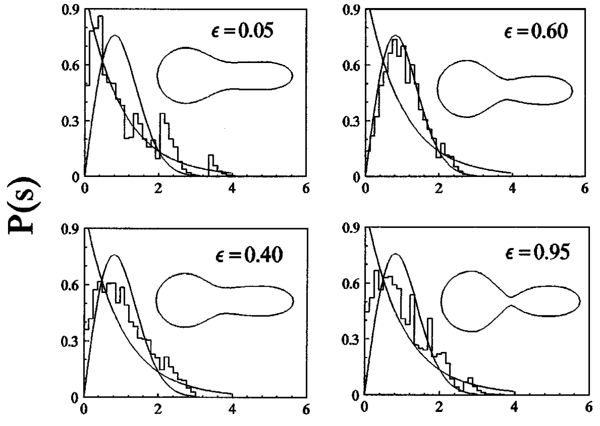

FIG. 1. The nearest neighbor level spacing distributions of spectra as a function of unfolded energies at different neck parameters (the other parameters are fixed to be $\Delta z = 8.0$ fm, $\beta_1 = 0.3$, $\beta_2 = 3.0$, $X_i = 0.3$). The solid lines denote Wigner distribution and Poisson distribution; the histograms are our numerical results. The corresponding nuclear shapes are denoted.

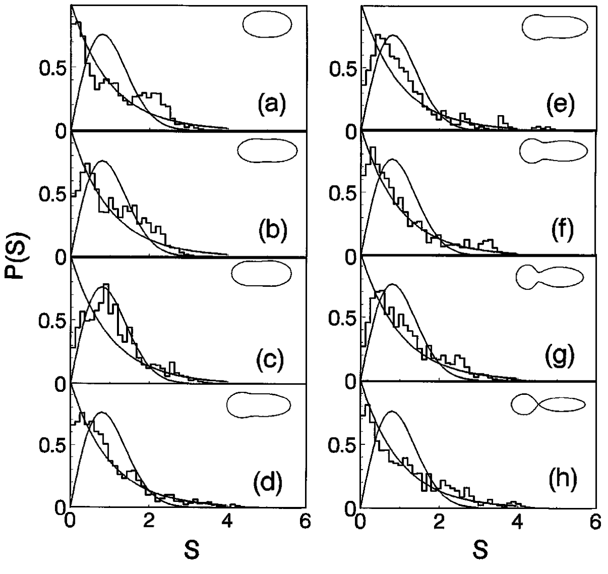

FIG. 2. The level spacing distributions of spectra for the deformed space explored by the fission process of ^{252}Cf: (a) the ground state, (b) the configuration between the ground state and the first saddle point, (c) the first saddle point, (d) the second minimum, (e) the second saddle point, (f) the third minimum, (g) the third saddle point, and (h) the scission point. The corresponding nuclear configurations are denoted.

those configurations is unstable against chaos; however, at the minimums of fission potential a Poisson-like distribution has been demonstrated and the motion of single particles is quite regular. From this study we have tried to establish a general relation between the stability of nuclear deformations and the statistical property of level spectra. Based on this, we have deep insight into properties of fission statics and we further understand why the second minimum is responsible for the superdeformation and the third minimum or scission configuration is responsible for the hyperdeformation in heavy nuclei. In addition, our study can also provide important information about the dependence of nuclear dissipation on the deformed space in heavy nuclei. According to Ref. [9], the concept of nuclear friction can be considered from the point of view of regular versus chaotic motion and the rate of dissipation is drastically reduced if the single particle motion is regular. In this Letter we have explored the dependence of statistical properties of level spectra on nuclear deformations. Therefore, we can learn that the nuclear dissipation depends strongly on nuclear configurations along the fission path. For example, the dissipation on saddles seems to be much larger than that in minimums. This conclusion, in principle, supports the statement that the dissipation at configurations from the saddle to scission point is larger than that in the ground state well [10].

Based on the above general investigation on statistical properties of spectra for the deformed space explored by the fission process, we try to understand the hyperdeformation appearing at the scission of ^{252}Cf. We know that during the fission process the systems of ^{252}Cf evolve from the ground state and go through saddles, then reach the scission. During this process, the energy levels of the system will be split into two sets, one for the normal deformed Mo and the other for the hyperdeformed Ba. This behavior is in fact quantitively reflected in Fig. 3(a) where the relevant statistical analyses are shown. The nearest neighbor level spacing distribution appears closer to an integrable case than to the typical Wigner distribution. Of physical interest are the pronounced new shell structures that emerge for the configuration of the normal deformed Mo and hyperdeformed Ba pair at scission. This pattern is also directly related to the periodic orbits on the Poincaré section map indicated in Fig. 3(b), in which the surface of section is chosen to be on the plane z-P_z at $\rho = 0$ and $P_\rho > 0$ (z is an axially symmetric coordinate and ρ is perpendicular to z in the cylinder coordinate system). Here, the z component of angular momentum is fixed to be zero, which corresponds to the case of $L_z = 0$ in quantum mechanical treatment. Since we have left out terms like l^2 and spin-orbit coupling in the classical treatment, we cannot claim that such structures arise exactly where we have found them. However, the essential point is the fact that such structures will always emerge. From above classical and quantum mechanical treatments, it has been implied that there is a locally stable structure against chaos for the configuration of a normal deformed Mo and hyperdeformed Ba pair at scission of ^{252}Cf. To prove this statement we have also calculated the potential energy surface (PES) of ^{252}Cf by using the shell correction approach with TCSM and liquid-drop model [11]. In Fig. 4 we plot the PES with respect to the shape parameters of distance between two fragments and deformation of the barium nucleus. A local minimum on the PES has been found at the distance between two fragments $\Delta Z \approx 18.2$ fm and the deformation of Ba $\beta_2 \approx 3.1$. The corresponding configuration of this minimum is close to the scission point of ^{252}Cf. This behavior is qualitatively consistent with the statistical behavior of single particle spectra.

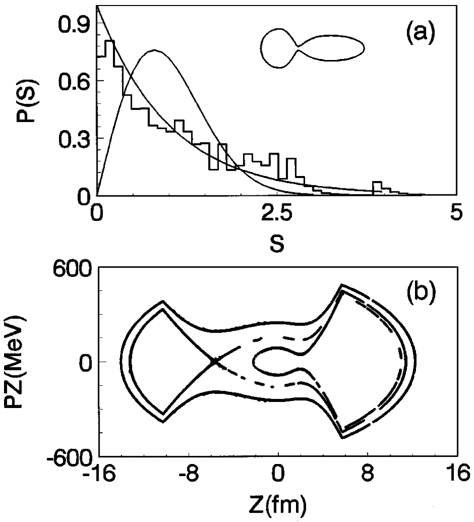

FIG. 3. (a) The level spacing distribution of spectra as a function of unfolded energies for the system consisting of a normal deformed Mo and hyperdeformed Ba at scission. The solid lines denote Wigner distribution and Poisson distribution; the histogram is our numerical results. The corresponding nuclear configuration is denoted. (b) The Poincaré surface of section z-P_z for the classical analog of the system indicated in (a).

In conclusion, we have found that the stability of prolate deformed nuclei strongly depends on the neck parameter in addition to the quadrupole and octupole deformations. It has been pointed out that when the neck parameter increases to a certain value the stability of prolate nuclei against chaos is destroyed and if it reaches a quite large necked-in case the stability of these systems against chaos will be restored again. This feature greatly influences the fission process and therefore it is necessary to take the neck parameter into account in the investigation of the fission and nuclear structure with a large deformation. The statistical properties of spectra for the deformed space explored by the fission process have been further presented, for example, the chaotic motion has been observed on saddles while the ordered motion has been found in minimums. This provides us with important information not only for fission statics,

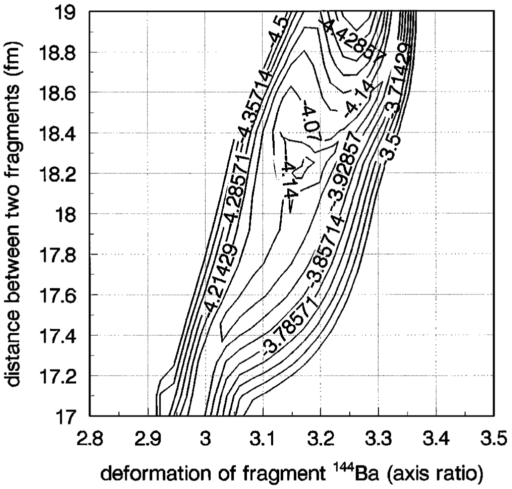

FIG. 4. Potential energy surface near the scission configuration of the system of ^{252}Cf as a function of distance between two fragments and deformation of fragment Ba.

but also for fission dynamics. Based on this investigation the classical and quantum mechanical results are reported for the single particle motion in a complex system which consists of a normal deformed nucleus Mo and hyperdeformed nucleus Ba at the scission configuration during the spontaneous fission of ^{252}Cf. The regular character of the single particle motion has been observed, which means that a kind of stable structure exists in the system. The calculation of the potential energy surface lends further support to the statistical explanation on the HD shape of Ba.

From above investigations we may start a quite general problem about a relation between the stability of nuclear configuration and the statistical property of level spectra, which is associated with the investigation on nuclear dynamics and structure under extremely large deformations with a strong necked-in shape and mass asymmetry. The significance of this kind of study can go far beyond nuclear physics, as was pointed out by Mottelson in Ref. [5].

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